boundary displacement by integrating the line load solution, which requires analysis similar to that of Equations (36)-(38). To obtain quantitative values, a realistic viscoelastic model should be used throughout the solution—in contrast to the paper where a combination of the "standard linear solid" and the "constant loss tangent model" were used to illustrate the method. It is easily seen that the boundary tangential displacement distribution is not symmetric about the rolling cylinder, and, in fact, the point under the cylinder axis is displaced in the opposite direction to the motion, but this is chiefly due to the inelastic properties of the material, here a "memory," and not to the lack of symmetry in the pressure distribution which the paper shows is very small. In contrast to plastic or viscoplastic materials, the displacements in the viscoelastic material recover completely after the cylinder has rolled on sufficiently far.

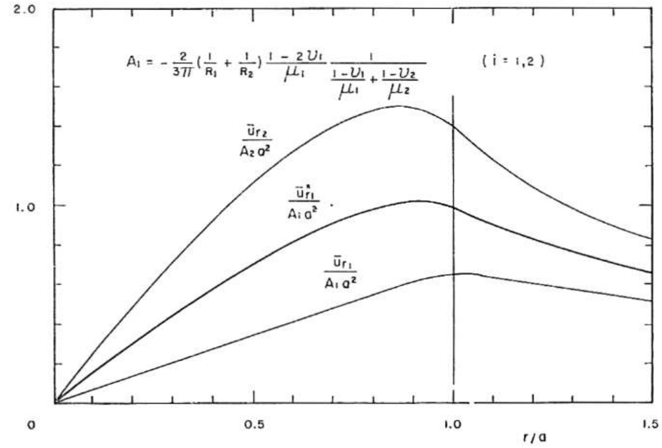

Fig. 1

Contact Stress Analysis of Normally Loaded Rough Spheres[1]

G. C. FENG.[2] In his paper on the contact stress analysis of normally loaded rough spheres, Goodman employs the assumption that the normal displacements due to the surface shearing stresses in the contact region may be neglected in comparison with the normal displacements due to the normal stresses. This assumption is justified in the paper by reference to certain theoretical considerations and to experimental results. Once a solution has been achieved on this basis, it is possible to check the initial assumption. If we substitute the expressions for $h(\xi)$ and $h^*(\xi)$ given by the author's equations (53) and (60) into the formulas (25) and (40) and integrate, we find for the ratio of surface normal displacement due to shear, $\bar{w}_{si}$, to surface normal displacement due to normal stress, $\bar{w}_{ni}$, the expression

$$\frac{\bar{w}_{si}}{\bar{w}_{ni}} = (-1)^i \frac{1}{\pi K} \frac{1-2\nu_i}{1-\nu_i}\left(a^2 - \frac{1}{2}r^2\right)^{-1}\left\{\frac{1}{2}a\,(a^2 - r^2)^{1/2} - \frac{1}{2}r^2 \ln\frac{a + (a^2 - r^2)^{1/2}}{r} + a^2\int_{r/a}^{1}(1-x^2)\frac{\tanh^{-1}x}{(x^2 - r^2/a^2)^{1/2}}\,dx\right\}$$

$$(i = 1, 2;\ r < a)$$

Here

$$K = \pi\left[\frac{1-\nu_1}{\mu_1} + \frac{1-\nu_2}{\mu_2}\right] \div \left[\frac{1-2\nu_1}{\mu_1} - \frac{1-2\nu_2}{\mu_2}\right]$$

as in the paper. The superscript i takes on the numerical value 1 or 2 depending on which of the two bodies in contact is under discussion. We note that, even in the worst possible case, $\nu_1 = 0$, $\nu_2 = 1/2$, $\mu_1/\mu_2 \ll 1$, the ratio $\bar{w}_s/\bar{w}_n$ nowhere exceeds 0.125 in magnitude. In most practical cases it is considerably smaller. The assumption that the normal displacement due to shear is negligible compared with the normal displacement due to normal pressure is therefore consistent with the results of the analysis.

On the other hand, the *lateral* displacements due to the shear loading on the interface, correctly accounted for in the paper, are of the same order of magnitude as the lateral displacements due to the Hertzian normal load. We find, for example, with the aid of the author's expressions (24), that in the contact region the surface lateral displacement in body 1 is given by the expression

$$\bar{u}_{ri} = -\frac{2}{3K}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{(1-2\nu_1)/\mu_1}{\dfrac{1-\nu_1}{\mu_1} + \dfrac{1-\nu_2}{\mu_2}}\left\{\frac{4}{3\pi}\frac{1-\nu_1}{1-2\nu_2}r^2 + \left(\frac{K}{\pi} - \frac{1-\nu_1}{1-2\nu_1}\right)\frac{a^3 - (a^2 - r^2)^{3/2}}{r}\right\}$$

This expression (and its counterpart for $r > a$) is pictured in Fig. 1. The heavy central curve gives the value which the lateral displacement would have were there no friction or no difference in elastic constants; i.e., it gives the Hertzian value of $\bar{u}_r$. The upper and lower curves give the values of $\bar{u}_{r1}$ and $\bar{u}_{r2}$ which occur when friction completely prevents slip. It may be seen that the difference in ordinates from the central to the outer curves is of the same order of magnitude as the ordinates of the central curve. This difference is due to the presence of shear on the interface.

Author's Closure

THE computation of normal and radial displacements on the surface of the spheres in contact, within the loaded region, is a valuable addition to the paper. The author is indebted to Mr. Feng for accomplishing this. It appears from his analysis that the solution given in the paper is self-consistent. The displacement along the z-axis due to surface shears in the x-y plane is negligible in comparison with the z-displacement due to surface pressures which are in the z-direction. This assumption, in one form or another, is common in three-dimensional contact stress analysis, though not all writers say explicitly what it is that they are neglecting. As Mr. Feng's equations show, the lateral (i.e., radial) displacement due to surface shear stresses is of the same order of magnitude as the lateral displacement due to normal pressure on the surface. This effect of the surface shear stresses cannot be neglected.

[1] By L. E. Goodman, published in the September, 1962, issue of the JOURNAL OF APPLIED MECHANICS, vol. 29, TRANS. ASME, vol. 84, Series E, pp. 515–522.

[2] Research Assistant, Department of Aeronautics and Engineering Mechanics, University of Minnesota, Minneapolis, Minn.

Random Vibration of Beams[1]

J. L. BOGDANOFF[2] and J. E. GOLDBERG[3]

In an early paper (see reference [1] of authors' paper), van Lear and Uhlenbeck studied the response of Bernoulli-Euler

[1] By S. H. Crandall and Asim Yildiz, published in the June, 1962, issue of the JOURNAL OF APPLIED MECHANICS, vol. 29, TRANS. ASME, vol. 84, Series E, pp. 267–275.

[2] Professor of Aero and Engineering Sciences, Purdue University, Lafayette, Ind. Mem. ASME.

[3] Professor of Civil Engineering, Purdue University.