**β-model and cooling flows in X-ray clusters of galaxies**

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A B S T R A C T

The spatial emission from the core of cooling-flow clusters of galaxies is inadequately described by a β-model. Spectrally, the central region of these clusters is well approximated with a two-temperature model, where the inner temperature represents the multiphase status of the core and the outer temperature is a measure of the ambient gas temperature. Following this observational evidence, I extend the use of the β-model to a two-phase gas emission, where the two components coexist within a boundary radius \( r_{\text{cool}} \) and the ambient gas alone fills the volume shell at a radius above \( r_{\text{cool}} \). This simple model still provides an analytic expression for the total surface brightness profile

\[
S(b) = S_{\text{cool}}(0) \left[ 1 - \left( \frac{b}{r_{\text{cool}}} \right)^{2(0.5+3\beta_{\text{cool}})} \right] + S_{\text{amb}}(0) \left[ 1 + \left( \frac{b}{r_{c}} \right)^{2} \right]^{0.5-3\beta_{\text{amb}}}.
\]

(Note in the first term the different sign with respect to the standard β-model.) Based upon a physically meaningful model for the X-ray emission, this formula can be used (i) to improve significantly the modelling of the surface brightness profile of cooling flow clusters of galaxies when compared to the standard β-model results, (ii) to constrain properly the physical characteristics of the intracluster plasma in the outskirts, like, e.g., the ambient gas temperature.

Key words: galaxies: clusters: general – X-rays: galaxies.

1 INTRODUCTION

To constrain the physical parameters of extended X-ray sources (e.g. groups and clusters of galaxies), the observed surface brightness can be either geometrically de-projected or, more simply, fitted with a model obtained from an assumed distribution of the gas density.

Given the hydrostatic equilibrium within the cluster, the gravitational potential supports both the gas and the galaxies distribution. If the latter is approximated via the King approximation (1962) to the inner portions of an isothermal sphere (Lane-Emden equation in Binney & Tremaine 1987), the gas density is then written as

\[
\rho_{\text{gas}} = \rho_{0}(1 + x^{2})^{-3\beta/2},
\]

where \( x = r/r_{c} \) and \( r_{c} \) is the core radius of the distribution.

The surface brightness profile observed at the projected radius \( b, S(b) \), is the projection on the sky of the plasma emissivity, \( \epsilon(r) \)

\[
S(b) = \int_{b}^{\infty} \frac{\epsilon(r) r^{2} dr}{\sqrt{r^{2} - b^{2}}}.
\]

The emissivity is equal to

\[
\epsilon(r) = \Lambda(T_{\text{gas}}) n_{p}^{2} \text{ergs}^{-1} \text{cm}^{-3},
\]

where \( n_{p} = \rho_{\text{gas}}/(2.21 \mu m_{p}) \) is the proton density and the cooling function, \( \Lambda(T_{\text{gas}}) \), depends upon the mechanism of the emission (mainly owing to bremsstrahlung at \( T_{\text{gas}} > 2.5 \text{ keV} \)).

Assuming isothermality and a β-model for the gas density (equation 1), the surface brightness profile has an analytic solution (equation 3.196.2 in Gradshteyn & Ryzhik 1965)

\[
S(b) = n_{0}^{2} r_{c} \Lambda(T_{\text{gas}}) B(3\beta - 0.5, 0.5) \left[ 1 + \left( \frac{b}{r_{c}} \right)^{2} \right]^{0.5-3\beta}
\]

\[
= S_{0}(1 + x^{2})^{0.5-3\beta},
\]

where the validity of the beta function \( B(a,b) \) puts the strict constraint \( 3\beta > 0.5 \) and the cooling function \( \Lambda(T_{\text{gas}}) \) does not change radially.

The β-model (Cavaliere & Fusco-Femiano 1976, 1978) provides a good representation of the observed surface brightness and has the advantage to easily constrain the gas density distribution.

Elsewhere (Ettori 2000), I consider the effect of the presence of a temperature gradient in the estimate of the β-model parameters. In this paper, I will focus on the deficiency of the β-model in...
modelling in a satisfactory way the central emission from cooling flow clusters of galaxies.

The cooling flows (e.g. Sarazin 1988, Fabian 1994) result in an enhancement of the gas density in the central region owing to the high cooling efficiency in the cluster core. Recently, there have been attempts to model this excess in emission with a generic double β-model, i.e. the sum of two components of surface brightness (Ikebe et al. 1996; Xu et al. 1998; Mohr, Mathiesen & Evrard 1999; Reiprich & Böhinger 1999). The correlation between the presence of a cooling flow and the necessity for a second β-model is well indicated from this figure: Peres et al. (1998) find that 40 per cent of a flux limited sample of 55 clusters of galaxies has a deposition rate of more than 100 M⊙ yr⁻¹; this is the same percentage of the clusters in the Mohr et al. sample that are better modelled with a double β-model instead of a single one (18 out of 45 clusters).

The double β-model as sum of two \( S(b) \) in equation (4), however, is not physically meaningful. In fact, a single isothermal temperature is usually assumed for both different density components that, therefore, are not in equilibrium. Moreover, data with high spatial resolution do not show evidence of a second inner core radius.

I present in this work a simple geometrical and physical model of the emission from cooling flow clusters of galaxies. This model relies on recent spectral evidence that the cluster plasma can be described as a gas with two phases, one related to the cooling gas and the other to the ambient medium. Assuming that the extended intracluster gas density, \( n_{\text{gas}} \), is well described by a β-model, I show in the following section that an analytic expression for \( S(b) \) can be obtained to describe the surface brightness from cooling flow clusters of galaxies. In Section 3, I apply this model to real data of clusters with or without cooling flows. This model allows to handle the emissivity due to each component. I discuss the physical implications of this in Section 4. In Section 5, I present some concluding remarks.

2 THE TWO-PHASE EMISSION MODEL

Recent spectral analyses of cooling-flow clusters of galaxies (Allen et al. 2000; White 2000) have shown how the spectral capabilities of the present instruments are unable to resolve all the fine structures of a multiphase gas, allowing just a modelling with a two-phase component, one that describes the emission from the central cooling gas and the other that takes into account the extended emission from the ambient medium.

These observational results provide us with a simple and natural model for the total cluster emission: an inner cold phase confined within \( r = r_{\text{cool}} \) and overlapping the diffuse, ambient gas (see Fig. 1).

We assume that the two components coexist within \( r_{\text{cool}} \), whereas only the ambient plasma fills the cluster volume shell at radius above \( r_{\text{cool}} \). The total cluster emissivity is then \( \epsilon(r, T) = \epsilon_{\text{cool}} + \epsilon_{\text{amb}} \), where

\[
\epsilon = \begin{cases} 
\lambda(T_{\text{cool}}) n_{\text{p,cool}}(r^2) + \lambda(T_{\text{amb}}) n_{\text{p,amb}}(r^2), & r < r_{\text{cool}}, \\
\lambda(T_{\text{amb}}) n_{\text{p,amb}}(r^2), & r > r_{\text{cool}},
\end{cases}
\]

from the definition in equation (3).

This simple model provides an analytic expression for the surface brightness profile defined in equation (2)

\[
S(b) = \int_{r_{\text{cool}}}^{r_{\text{amb}}} \epsilon(r, T) \frac{\epsilon}{\sqrt{r^2 - b^2}} \, dr, 
\]

where the integration limits in \( S_{\text{cool}}(b) \) contains the boundary of the inner region at \( r = r_{\text{cool}} \).

Now, I integrate the emissivity along the line of sight. \( S_{\text{amb}}(b) \) is still equation (4). To integrate \( S_{\text{cool}}(b) \), one needs the assumption that the only scale parameter of the gas density is the dimension of the cooling region, \( r_{\text{cool}} \). Considering that we are in the regime \( r/\lambda < 1 \), I can move the ‘±’ sign from the exponent to the radic and derive a β-model in the form of

\[
n_{\text{p,cool}} = n_{\text{p,cool}} \left[ 1 - \left( \frac{r}{r_{\text{cool}}} \right)^{2(2\beta_{\text{cool}})} \right].
\]

The behaviour of this profile ensures that the gas density within the cooling region has no other parameter scale than the dimension of the region itself and \( \epsilon_{\text{cool}} \) goes to zero when \( r \to r_{\text{cool}} \).

Then, I can integrate analytically \( S_{\text{cool}}(b) \) (equation 3.196.3 in Gradsteyn & Ryzhik 1965)

\[
S_{\text{cool}}(b) = S_{\text{cool}}(b) \left[ 1 - \left( \frac{b}{r_{\text{cool}}} \right)^{2(4.5)} \right],
\]

where

\[
S_{\text{cool}}(b) = n_{\text{p,cool}}^{2} \lambda_{\text{cool}} \sqrt{2} \left( T_{\text{cool}} - T_{\text{amb}} \right) \Delta T_{\text{cool}} (0.5, 3\beta_{\text{cool}} + 1),
\]

where \( \Delta T_{\text{cool}} \) and \( \Delta T_{\text{amb}} \) represent the two gas temperatures corresponding to the cooling region and to the ambient of the cluster, respectively.

3 COMPARISON WITH THE DATA

I have applied this model to observations of clusters of galaxies
that can map the emission in regions well beyond the cluster core to disentangle the effect of the two components. Moreover, I have considered clusters with evidence of a large cooling flow and the Coma cluster (ROR: rp80005n00, exposure time: 20.0 ks, considered clusters with evidence of a large cooling flow and the disentangle the effect of the two components. Moreover, I have

Figure 2. The cluster surface brightness profile is fitted here with a single $\beta$-model (dashed line) and a double $\beta$-model (solid line; the thickest indicates the model with five parameters). The dotted lines show the two components of the double $\beta$-model with five parameters. The panel below shows the residuals in unit of $\sigma$.

Table 1. Best-fitting results using the models discussed in the text. The core (cooling) radii are in $h_0^{-1}$ Mpc. The values for the F-test represent the level of significance. The symbols $2\beta_2$ and $2\beta_6$ indicate the double $\beta$-model with five and six parameters, respectively.

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$R_{\text{core}}$</th>
<th>$\beta$</th>
<th>$2\beta - 5$ parameters</th>
<th>$2\beta - 6$ parameters</th>
<th>F-test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mpc/arcmin</td>
<td>$\beta$</td>
<td>$\beta_{\text{cool}}^2$</td>
<td>$\beta_{\text{cool}}^2$</td>
<td>$1\beta \rightarrow 2\beta_5$</td>
</tr>
<tr>
<td>A1795</td>
<td>1.49/15.2</td>
<td>0.10 ± 0.01, 40.9(28)</td>
<td>0.26 ± 0.01, 8.6(26)</td>
<td>0.64 ± 0.15, 0.28 ± 0.01, 8(25)</td>
<td>&gt;0.99</td>
</tr>
<tr>
<td>A2199</td>
<td>1.50/30.2</td>
<td>0.08 ± 0.01, 23.1(35)</td>
<td>0.13 ± 0.01, 47.5(56)</td>
<td>0.74 ± 0.06, 0.13 ± 0.01, 3.6(55)</td>
<td>&gt;0.99</td>
</tr>
<tr>
<td>Coma</td>
<td>1.49/38.2</td>
<td>0.57 ± 0.01, 8.2(74)</td>
<td>0.65 ± 0.01, 6.5(72)</td>
<td>3.37 ± 0.05, 0.60 ± 0.03, 8.6(71)</td>
<td>0.83</td>
</tr>
</tbody>
</table>

fit, i.e. $r_c = r_{\text{cool}}$. This is not in contradiction with the present observational results. Allen (2000) quotes the cooling radii obtained from deprojection analysis of 30 cooling flow clusters images. Ettori & Fabian (1999) estimate the core radii for 23 of these clusters using a single $\beta$-model over the radial range [0.1, 1] $R_{500}$, the radius at which the mean cluster density is $500$ the background value. The distribution of the ratio, $r_c/r_{\text{cool}}$, has a median value of 1.33, an average of 1.61 and a standard deviation of 1.19, and can be considered consistent with $\sim 1$. For the clusters in this study, the distribution of $r_c/r_{\text{cool}}$ has a median value of 1.32 ($\pm 0.24$, 90 per cent confidence level) and 0.84 ($\pm 0.15$) for A1795 and A2199, respectively. I remind, however, that I am using a different definition of $r_{\text{cool}}$ than the one adopted in the standard spatial analysis: in the latter, $r_{\text{cool}}$ is the radius where the cluster cooling time first exceeds the Hubble time, whereas in this work $r_{\text{cool}}$ defines the boundary of the central cool phase of the gas.

It is worth noting that another version of a ‘five-parameter’ fit, in which $\beta_{\text{cool}}$ is fixed equal to $\beta_{\text{amb}}$ and $r_c/r_{\text{cool}}$ are left free to vary, provides a significantly worse $\chi^2$ than the one obtained by using the ‘five-parameter’ fit adopted here.

Finally, the $\chi^2$ obtained with the models above does not vary significantly from the $\chi^2$ measured after the fit with other models which are not strictly based upon a physical framework, like the sum of two standard $\beta$-models with six free parameters (e.g. Reiprich & Böhringer 1999) or, with the slope fixed to a common value, with five free parameters (Mohr et al. 1999).
4 DISCUSSION

The use of this physically meaningful model allows us to directly handle each of the two gas distributions, one that describes the profile of the gas related to the cooling flow and the other that model the ambient gas. As shown above, there is no statistical justification for using the six-parameter fit. Therefore, I consider hereafter the case $r_c = r_{cool}$.

We use in the following discussion the definition of the central gas density for a cluster at redshift $z$, that is

$$n_p(0) = \left[ \frac{4\pi(1+z)^2S(0)}{1.21r_c\Lambda(T)b} \right]^{0.5}$$

(9)

where $n_p(0)$ is in $\text{cm}^{-3}$, $r_c$ is the core or cooling radius in cm, $S(0)$ is the central surface brightness in count $s^{-1} \text{sr}^{-1}$, $b$ is the proper beta function and the cooling function $\Lambda(T)$ is in units of count $s^{-1} \text{cm}^2$.

Using the condition that the two phases have to be in pressure equilibrium, I can now put constraints on their temperatures. To do this, I consider the mean properties of each phase to handle integrated values instead of differential ones, because of the simple assumption that each phase is represented by a single temperature that does not depend on the radius. Therefore, each phase density, $n(<r)$, is the integral of the radial density, $n(r)$, over the volume occupied from that phase (between 0 and $r_{cool}$ for the inner phase; between 0 and $r = X r_{cool}$ for the outer component) divided by the integrated volume. Then,

$$T_{cool} = T_{amb} \left( \frac{n_{amb}}{n_{cool}(r)} \right) = T_{amb}g^{1/(2-a)}f^{2/(2-a)} = T_{amb}f$$

(10)

where I have made use of the relation in equation (9) $\Lambda(T)n_0^2 \sim S(0)/B(a,b)$. I have assumed $\Lambda(T) \sim T^\alpha$ ($\alpha \approx 0.5$ for only bremsstrahlung emission observed by broad-band instruments), and I have defined

$$g = \frac{\Lambda(T) n_{amb}(0)^2}{\Lambda(T) n_{cool}(0)} = \frac{S_{amb}(0) B_{cool}}{S_{cool}(0) B_{amb}}$$

$$I = \frac{n_{amb}(<r)}{n_{cool}(0)} \frac{n_{cool}(0)}{n_{cool}(<r)} = \frac{\int_0^X (1 + x^2)^{-1.5} \beta_{cool} x^2 \, dx}{\frac{1}{3}(1 + X)}$$

$$f = g^{1/(2-a)}f^{2/(2-a)}$$

with $\int_0^X (1 + x^2)^{-1.5} \beta_{cool} x^2 \, dx = B(1.5,1.5\beta_{cool} + 1)/2$ and $I_X[a] = \int_0^X (1 + x^2)^{-a} \beta_{cool} x^2 \, dx$.

However, one generally measures a single emission-weighted temperature, $\bar{T}$. Given the considerations above, I can now disentangle the two components (if any) using equation (10) in the following relation

$$T = \int T \, dV = \sum_{i=1}^N \int T_i \, dV_i$$

$$= T_{amb} \left[ 1 + f B(1.5,3\beta_{cool} + 1)/(2g I_X[3])^{-1} \right]$$

$$= T_{amb} F$$

(12)

where I still use the relation $\Lambda(T)n_0^2 \sim S(0)/B(a,b)$, calculate $\int_0^X (1 + x^2)^{-1.5} \beta_{cool} x^2 \, dx = B(1.5,1.5\beta_{cool} + 1)/2$ and adopt the symbols $f; g; I_X$ defined in equation (11).

In the equations above, both the function $f$ and $F$ have to be smaller than 1 by definition. Their behaviour, however, depends strongly upon $X$, the radius in unit of $r_{cool}$ up to where the outer phase extends and can be represented with a single temperature. Fig. 3 shows how the function $f$ and $F$ depend upon $X$: $f$ diminishes significantly owing to the presence of $X^3$ in $I$ (equation 11), whereas $F$ converges quite rapidly (at $X \geq 4$), providing a robust estimate on $T_{amb}/T_{cool}$ ratio. Therefore, even if we are not able to constrain the ratio between the temperature of the two phases owing to the uncertainty of the extension of the outer component, we can assess the ambient temperature, $T_{amb}$, in a cooling flow cluster with an emission-weighted temperature, $\bar{T}$, just using the azimuthally averaged surface brightness profile.

I show in Table 2 the constraints on $F = T/T_{amb}$ obtained from the spatial fit of the surface brightness profiles of the clusters in exam and compare these values to the two-temperature spectral results in Allen et al. (2000), Markevitch et al. (1998, 1999) and White (2000). The agreement is remarkably good with the results of Markevitch and collaborators and White, which assume a two-phase gas for their spectral model in a way similar to the one I.
have adopted for the physical framework described above. (Note that Markevitch et al. measure the isothermal ambient temperature excising the cooling region, whereas White adds a cooling flow component in the spectral fit). On the other hand, the disagreement with the results of Allen and collaborators can be explained with the more complex model that they adopt, where an absorption intrinsic to the cluster is combined with the cooling flow only.

Several aspects of the cluster physical characteristics are affected from the inclusion of a cooling flow in the modelling of the surface brightness with a $\beta$-model. With respect to the single $\beta$-model, one expects (i) excess in the gas density in the cooling region, (ii) change in the $\beta_{amb}$ value, (iii) variations in the gas ambient temperature.

I present in Table 3 some of the more interesting physical quantities that can be evaluated with the equations above and given an emission-weighted temperature, $\bar{T}$ (from Allen et al. 2000; $k\bar{T} = 5.40 \pm 0.05$ and $4.16 \pm 0.03$ keV for A1795 and A2199, respectively).

For example, if one identifies the inner component with the cooling flow, a proper description of its gas distribution is now available through equation (7). The luminosity of the intracluster gas can then be estimated without the contribution of the emission from the cool phase

$$L_{amb} = \int 1.21 n_p(r) \Lambda(T) 4\pi r^2 dr,$$

where the integral is computed upon the cluster volume and $\Lambda(T)$ is here in erg s$^{-1}$ cm$^3$. Using only the cluster surface brightness profile and a broad-band emission-weighted gas temperature, and applying equations (12) and (13), I will investigate in a forthcoming paper the effects on the clusters luminosity–temperature relation of the presence of significant cooling flows (see, e.g., the results from spectral analyses in Allen & Fabian 1998, Markevitch 1998).

Appreciable corrections can also affect $M_{gas}$, $M_{cool}$ and the terms of the so-called $\beta$-problem (Mushotzky 1984; Edge & Stewart 1991) owing to the variation of the $\beta$ value (for A1795 and A2199, $\beta_{amb}$ increases by 20 and 10 per cent when compared with the $1 - \beta$ model fit results, respectively). In the two-phase model described here, it is simple to calculate the gas and the total mass:

$$M_{gas}(r > r_{cool}) = M_{gas,cool} + \int_0^{r_{cool}} \rho_{amb} 4\pi r^2 dr,$$

where $M_{gas,cool} = 2\pi \rho_{cool} x_{cool}^3 B(1.5, 1.5) \beta_{cool} + 1 \approx 27(3) \times 10^{11} M_\odot$ for A1795 (A2199) for an assumed $T_{cool} = (f/\bar{T}) \bar{T} = 0.27$; the total gravitating mass is given by the application of the hydrostatic equilibrium,

$$M_{tot} = \frac{\int r^2 d(T_{cool}\rho_{cool} + T_{amb}\rho_{amb})}{\mu m_p G \rho_{gas}},$$

and is proportional to $r_{cool} \beta_{amb} T_{amb} x(1 + x^2)^{-1}$ at $r > r_{cool}$.

5 CONCLUSIONS

I have presented a new analytic formula to model the total surface brightness profile of clusters of galaxies where a two-phases intracluster gas can be assumed. This scenario is consistent with the present results of spectral analyses of the central regions of clusters that harbour a cooling flow.

The use of this formula allows to properly disentangle the contribution of the cooling flow to the cluster emissivity using only the spatial distribution of the X-ray photons. After removing the contamination from the cooling flow, I show how some relevant physical parameters are affected, like, for example, the ambient gas temperature (see Table 2). In a forthcoming paper, I will investigate the systematic changes in the temperature, luminosity and mass (cf. Table 3) of a sample of clusters of galaxies and how these variations affect the relations among these quantities.

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