Are 17 deg and 8 deg deviations from orthogonality bad or moderate? We believe these deviations to be moderate. In Fig. 1*a* we plot the second derivatives Nf_2'', Nf_4'', where N are normalization factors. We also plot the rigorously orthogonal Legendre polynomials

$$S_2 = N(-1 + 3y^2), \qquad S_4 = N\left(1 - 10y^2 + \frac{35}{3}y^4\right)$$

($S_2 \equiv Nf_2''$.) The distinction between S_4 and Nf_4'' is almost imperceptible. In Fig. 1*b* we plot the first derivatives Nf_2', Nf_4', and the polynomials

$$T_2 = N(y - y^3), \qquad T_4 = N(y - 4y^3 + 3y^5)$$

obtained by constructing the orthogonal family T_k of Nf_2'. The disagreement between T_4 and Nf_4' is pronounced, but still not alarming. In the two figures we show, for comparison, also the $x = 0$ boundary values

$$t_2(y) = b_{22}K^\circ_{2,xy}, \qquad t_4(y) = b_{44}K^\circ_{4,xy} + b_{42}K^\circ_{2,xy}$$

$$s_2(y) = a_{22}J^\circ_{2,yy}, \qquad s_4(y) = a_{44}J^\circ_{4,yy} + a_{42}J^\circ_{2,yy}$$

of the orthonormalized derivatives of the rigorous eigenfunctions ($K^\circ \equiv K(0, y), \Re \equiv$ real part)

$$K_2(x, y) = 4\Re \sum z_k^{-2}[2 + \cos^2 z_k]\Phi_k$$

$$K_4(x, y) = 8\Re \sum z_k^{-4}[42(3 + 2\cos^2 z_k) - z_k^2(34 + 3\cos^2 z_k)]\Phi_k$$

$$J_2(x, y) = 6\Re \sum z_k^{-1}(\sin^2 z_k/\cos^3 z_k)\,\Phi_k$$

$$J_4(x, y) = -30\Re \sum z_k^{-3}(\sin^2 z_k/\cos^3 z_k)[14 + 7\cos^2 z_k - 3z_k^2]\Phi$$

$$\Phi_k(x, y) \equiv e^{-z_k y}(\cos z_k y - y \cot z_k \sin z_k y)$$

which were established in an earlier paper.[9] The summation is carried out over the first quadrant roots z_k of $\sin 2z_k + 2z_k = 0$.

Professor Williams' comment about the nonuniqueness of the weight factor of the radial polynomials is very timely. In the reference[7] mentioned earlier the authors did encounter a difficulty with the choice of the r exponent, $p = 2$. They found that $p = 2 + \epsilon$ is a preferable choice, where $\epsilon > 0$ may be arbitrarily small. But in the lack of a specific rule for determining ϵ they retained their original choice, $\epsilon = 0$.

The authors are also in agreement with the comments of Dr. Silverman. Clearly, for problems involving displacement boundary conditions the results will depend on Poisson's ratio. The authors have not attempted to extend their method to mixed boundary-value problems, but such an extension may well require a switch from stress functions to displacement functions. (See, e.g., Equation [11] of reference in footnote 8.)

Cylindrical Shells Under Line Load[1]

Yi-Yuan Yu.[2] A re-examination of the problem of a cylindrical shell under line load on the basis of equations including the effect of transverse shear deformation, as is given in the present paper, is important as well as interesting. However, it should be pointed out that, from the standpoint of the theory of the cylindrical shell (not necessarily shallow), an operator of the form

$$\left(1 - \frac{1-\nu}{2}\,\frac{h^2}{12\kappa_1}\,\nabla^2\right)$$

which should be operating on the left-hand member of Equation [3*a*] appears to be missing. A similar operator is missing from the corresponding dynamic equation given as Equation [5*a*] in reference (6) of the paper, by P. M. Naghdi and the author. As a consequence, Equation [3*a*] and its dynamic counterpart are of the eighth instead of tenth order, and they are thus not quite equivalent to the originally coupled equations from which the latter was and the former also could have been deduced. That these equations are somewhat incomplete may be seen from the fact that only four branches of the dispersion curve for the propagation of waves in cylindrical shells are obtainable from the dynamic counterpart of Equation [3*a*], although five are predicted by the theory which includes the effects of transverse-shear deformation and rotational inertia. A more appropriate form of the Donnell dynamic equations as well as another still more complete set of Donnell-type dynamic equations, both of which include the shear and rotation effects, have been derived by the writer in a forthcoming paper,[3] and the latter equations have been applied to the vibration problems of cylindrical shells.[4]

Author's Closure

The author wishes to thank Professor Yu for his interest in the paper. The following remarks are in order:

First, the system of Equations [3] of this paper are uncoupled from the five original displacement equations of equilibrium to the extent that homogeneous solutions are required for only two equations ([3*a*] and the second-order partial differential equation to which [3*d*] and [3*e*] can be reduced). Thus the complete solution depends on the solution of the *system* of Equations [3] and, as Professor Yu has observed, Equation [3*a*] is not the single (tenth-order) partial differential equation to which the original displacement equations can be reduced (even though w is the only dependent variable in [3*a*]). That is, the system of Equations [3] is not completely uncoupled in the sense to which Professor Yu is alluding when describing the equations which he has derived using the Donnell type approximations. However, it should be noted that such a single tenth-order equation can be obtained from the product of the factors operating on the two homogeneous equations which must be solved in obtaining the complete solution of Equations [3]. The dynamical counterpart of such an equation has been given in a recent paper[5] (the equation preceding [IIE] where a similar situation is evident.

[9] G. Horvay, "Biharmonic Eigenvalue Problem of the Semiinfinite Strip," *Quarterly of Applied Mathematics*, vol. 15, 1957, p. 65.

[1] By R. M. Cooper, published in the December, 1957, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 79, pp. 553–558.

[2] Professor, Department of Mechanical Engineering, Polytechnic Institute of Brooklyn, Brooklyn, N. Y. Mem. ASME.

[3] "On the Donnell Equations and Donnell-Type Equations of Thin Cylindrical Shells," by Yi-Yuan Yu, to be presented at the Third U. S. National Congress of Applied Mechanics.

[4] "Vibrations of Thin Cylindrical Shells Analyzed by Means of Donnell-Type Equations," by Yi-Yuan Yu, presented at the Twenty-Sixth Annual Meeting of the Institute of the Aeronautical Sciences, New York, N. Y., January, 1958.

[5] "Propagation of Nonaxially Symmetric Waves in Elastic Cylindrical Shells," by R. M. Cooper and P. M. Naghdi, *Journal of the Acoustical Society of America*, vol. 29, 1957, pp. 1365–1373.