Case 2: Axisymmetric Steady Rotation

$$\epsilon^x = 0 \quad \dot{\epsilon}^x = 0 \quad \omega^x = 0$$
$$\epsilon^y = 0 \quad \dot{\epsilon}^y = 0 \quad \omega^y = 0$$
$$\epsilon^z = \epsilon \quad \dot{\epsilon}^z = 0 \quad \omega^z = \omega$$

so the right-hand side of the Reynolds equation (B2) vanishes identically. Boundary conditions for a hemispherical cup centered on the z-axis are

$$p(\pi/2, \phi) = 0$$

so the pressure and also the resultant film force vanish identically.

Case 3: Transverse Steady Rotation

$$\epsilon^x = 0 \quad \dot{\epsilon}^x = 0 \quad \omega^x = \omega$$
$$\epsilon^y = 0 \quad \dot{\epsilon}^y = 0 \quad \omega^y = 0$$
$$\epsilon^z = \epsilon \quad \dot{\epsilon}^z = 0 \quad \omega^z = 0$$

Boundary conditions for a hemispherical cup centered on the y-axis are

$$p(\theta, 0) = 0$$

giving for an isoviscious lubricant a pressure distribution

$$p = \frac{6\mu R^2 \omega \epsilon (2 - \epsilon \cos\theta) \sin\theta \sin\phi}{C^2(4 + \epsilon^2)(1 - \epsilon\cos\theta)^2}$$

which is everywhere positive (or negative).
Appropriate limits of integration for such a cup are

$$\theta_1 = 0 \quad \phi_1 = 0$$
$$\theta_2 = \pi \quad \phi_2 = \pi$$

giving resultant film forces

$$F^x = 0$$

$$F^y = \frac{3\pi\omega\mu R^2}{(C/R)^2\epsilon^2(4 + \epsilon^2)}\{(1 + \epsilon^2)\ \ln\ [(1 + \epsilon)/(1 - \epsilon)] - 2\epsilon\}$$

$$F^z = -\frac{12\pi\omega\mu R^2}{(C/R)^2\epsilon^2(4 + \epsilon^2)}\{1 + \epsilon^2/2 - 1/(1 - \epsilon^2)^{1/2}\}$$

These results have been previously reported in part by Wannier [3] and more extensively by Tipei [6].

DISCUSSION

F. A. Martin[9]

The authors have made a commendable effort in showing how the finite element approach can be suitably applied to the complicated analysis of spherical bearings and also in introducing new performance data.

The general presentation of results in Fig. 11 is a useful format for designers, giving a relationship between the nondimensional resultant force, the span angle (related to L/D) and the eccentricity ratio.

Have the authors any background experience indicating where spherical hydrodynamic bearings may be usefully applied? Linkages in mechanical handling and earth moving equipment comes to mind, where shock loads and squeeze action would be relevant (although there will also be some oscillatory motion). Other applications (with pure rotation) may possibly be found in the textile industry.

The general scope of analysis using the finite element method is considerable. In this one paper we see solutions for squeeze films, pure rotation and transient behavior. Suggested future work on spherical bearings could be to predict load capacity with relative oscillatory motion of the bearing-sphere combination and also to study their capability of supporting thrust loads. It is appreciated, however, that this is a very specialized field and one would have to be very selective in studying the many options available.

NOTE: Further discussion by F. A. Martin was directed toward Table 3 and Fig. 11 of the paper in preprint form, but is no longer applicable to the modified version of the paper presented here.

P. K. Das[10]

The paper is a significant contribution to both the marginally explored area of spherical bearings and further use of the finite element method in lubrication. The finite element method allows studying the effect of geometry changes to the ideal sphere. Such studies can show whether geometric shapes different from the ideal have superior load carrying capability in certain circumstances. Indeed, the human joints are not perfectly spherical and perhaps for good reasons. Have the authors conducted any study to determine the desirable geometric features for an application?

The authors have hinted at the spherical bearings operating in a mode somewhat similar to a elastohydrodynamic point-contact. Under this condition, a principal mode of failure is likely to be scuffing due to high friction and associated temperature rise. It appears desirable to extend the calculations to include both the steady state temperature distribution and the instantaneous local temperature at the point of contact. What are the authors' views on the need for heat transfer calculations?

The paper has shown the importance of mesh size and distribution in lubrication analysis. Is there any empirical guide in choosing the mesh size and distribution for a desired accuracy? Furthermore, the authors have chosen to use linear elements. Considering the highly nonlinear nature of the pressure field, would a second order shape function be more appropriate for lubrication analysis? Of course, this will increase computation cost, particularly under dynamic condition which is already highly expensive to analyze. Can the authors recommend ways to maintain the cost of dynamic analysis to a reasonable level?

Authors' Closure

The authors appreciate the thoughtful written discussion published here as well as the oral discussion offered at the Conference.

We are particularly grateful to Mr. Martin for calling our attention to an inadvertent inconsistency in the preprinted version of this paper. As he suggests, a number of present and potential applications for spherical bearings involve oscillatory motion under nonreversing loads. Examples range from industrial rocker arms to artificial human hips.

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As suggested by Dr. Das, departures from ideal surface regularity can apparently have strong (30:1) beneficial effects on film thickness in such bearings, which are notorious for their conventionally low load capacities. Our recent work [12, 13], based largely on the finite element analysis procedures of the present paper, then addresses the search for *optimum* film geometries. The basic objective of this work is substantially enhancing lubricant film thickness, thus avoiding *both* the scuffing mode of failure *and* the local temperature calculation suggested by Dr. Das for near-point-contact conditions.

In other applications we have used second-order shape functions as suggested by Dr. Das. Particularly attractive is the 6-node triangular element with mid-side nodes. As expected, the quadratic element offers a significant increase in accuracy over the same number of linear elements. (The accuracy advantage is less marked when the number of *nodes* is kept fixed.) However, increased computational complexities, with or without numerical quadrature, negate much of the apparent advantage of the second-order element. Furthermore, with the quadratic element the non-negativity constraints (2) and (6) are *not* formally equivalent, and the practical numerical consequences of this have not been explored for ruptured lubricant films in which the constraints are active. In this regard, we might cite an additional general reference [14] for the quadratic programming problem of minimizing functional (4) subject to constraint (6).

Dr. Das has raised again the matter of mesh selection. It would appear that optimal mesh density distribution dpends on distributions of both film thickness and film pressure. Since the latter is initially unknown, some sort of iterative trial-and-error approach is apparently required for steady-state problems. (We have ourselves done this quite informally in developing meshes such as Fig. 5). For dynamic problems the previous solution is always at hand; however mesh recasting at every time step can be quite inefficient.

Oral discussion by Professor Allaire and Dr. Rohde addressed the possibility of automated adaptive mesh selection, based on smoothing a composite variable ph^n (where 1 and 3/2 were suggested as possible exponents).

Whether such refinements as optimized meshes and higher-order elements will be cost-effective remains to be seen. We do believe, however, that computational efficiency can be improved by very careful attention to program organization and choice of numerical methods. No *single* device (such as the mobility method) seems to promise a cost breakthrough for finite element analysis of dynamically loaded bearings, either in spherical or cylindrical geometries; even so, prospects for gradual improvement seem excellent.

Additional References

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14 Cryer, C. W., "The Solution of a Quadratic Programming Problem Using Systematic Overrelaxation," *J. SIAM Control*, Vol. 9, 1971, pp. 385–392.