

Fig. 10 Tori $S = 0.036$

Acknowledgments

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DISCUSSION

V. O'Brien²

It is always interesting to see a careful experimental series which can be compared to theoretical expressions for some range, and also extends insight into ranges where theory is still lacking. A connection between linear objects of length L and tori with mean circumference L is not obvious.

I. Viscous Regime

For low Reynolds numbers, $Re_l = (2lU/\nu) < 1$, where $2l$ is the longest particle dimension, Fig. 11, there has to be a distinction between steady Stokes drag ($Re_l \rightarrow 0$, unbounded domain) and the measured drag. Stokes drag for a variety of bodies is known analytically or can be estimated by inscribing/circumscribing bodies after Hill and Power [30]. Long cylinders fall stably broadside, as do straight strings of spherical particles [31], prolate ellipsoids and a rectangular parallelepiped, Fig. 1(a). The frontal area, F , is important. The drag of the prolate spheroid is less than that of the cylinder or prism. The open tori, solidity $S < 1$, have $F = S \cdot \pi l^2$, less than a solid closed particle ($S=1$) with the same side projection, and less Stokes drag.

The "Oseen" effect, the inertial increase of drag is a simple linear function of Re_l dependent on the Stokes drag for arbitrary solid particles³ or circulating fluid spheroids [33]. The approximation is only valid to $O(Re_l)$, i.e., $Re_l < 1$.

The boundary effect for particles traversing the centerline of a fluid container is a function of container shape [34, 35]. For an outer cylinder of radius R_0 and axisymmetric particles [36], general solid particles⁵ or circulating fluid spheroids [33], it is a linear function of $\lambda = l/R_0$, to $O(\lambda^2)$, where the coefficient multiplying λ is also a function of Stokes drag.

In the linear perturbation range then for all test particles

$$\text{Drag} = \text{Stokes drag} [1 + k_1 Re_l + k_2 \lambda] \quad [1]$$

where the constants k_1 and k_2 are functions of particle shape

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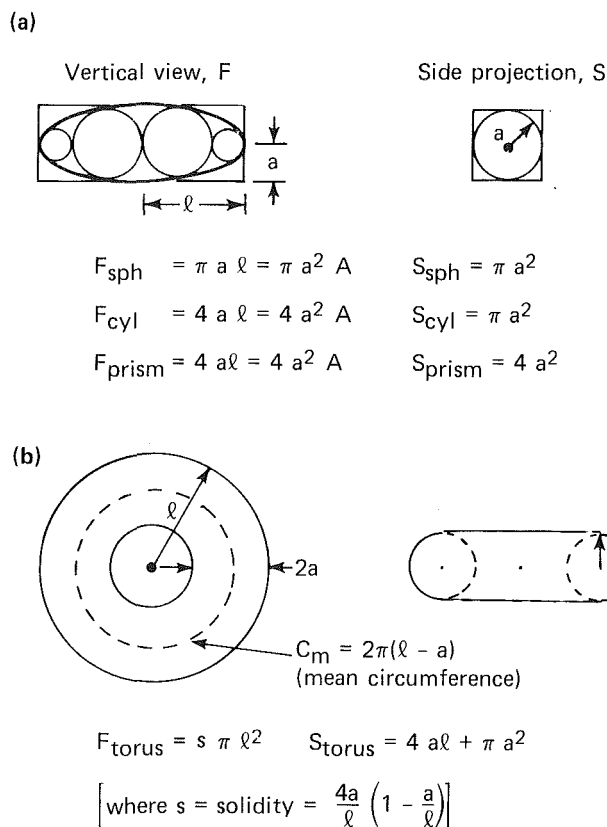


Fig. 11 Projected area of particles.

and internal circulation for fluid spheroids. The settling factor $K_\infty = U_\infty/U_s$, where U_s pertains to the Stokes drag on an "equivalent" solid sphere (same volume and density), involves other shape functions but allows the use of the solid spherical coefficient 2.1044 for extrapolating $\lambda \rightarrow 0$ for very low Re_l results at small λ (author's Fig. 1). The bracket in equation (1) can be inverted with k_i change of sign to the order of estimation. (The second method can be ignored.) The results are consistent with the exact torus theory, Fig. 2.

Extrapolations are sensitive to data accuracy and the percentages quoted seem to be in terms of $K_b = 1$ not K_∞ . (The data point for $S = 0.036$, $Re_d = 0.803$ should not have been included. The value is large because of Re_d .) An average error of 0.998 must be a mistake.

Since all shape factors can be related, one could use A_e , an "equivalent prolate spheroid" instead of a sphere. However, I doubt the statement, about two "exact" theories showing a prolate spheroid has a higher drag than the a cylinder of the same (large) aspect ratio. The frontal area of the spheroid is always less by a factor $\pi/4$ for all values of A . Slender-body theory, all singularities on the axis, does not take into account the flat ends of a cylinder. See references [37, 38].

II. Inertial Regime

Here all the curves are empirical. Figure 4 shows smooth curves for the cylinder data, each A value represented by a linear C_D versus $\log Re_c$ relationship at small Re_c (less C_D for greater A), curving up as $Re_c \rightarrow 1000$ (inverse order for A). The equation (29), for the deviation of measured C_D from the linear (Stokes) C_D , obviously cannot be used for $1 > Re_c \rightarrow 0$. The "transition region" where standing vortices develop, become unstable and detach themselves as $Re_c \rightarrow \infty$ is dependent on boundary proximity even if the measured drag is not. The attached vortex behind a finite cylinder moving broadside must be three-dimensional from its initial ap-

pearance. The vortex arrangement behind an open torus will be quite different, but initially axisymmetric. As the Reynolds number increases, the vortex shedding pattern depends on solidity S . The critical Reynolds number for shift of pattern from the solid spheroidal loops to ring-pairs is a phenomenon worth pursuing in more depth experimentally and theoretically.

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R. E. Johnson³

Although the title of this paper emphasizes that it will study the effects of curvature on the drag and vortex shedding of elongated bodies, this paper seems to concentrate its effort on determining which theoretical or empirical equation best describes the drag on cylinders and tori. Consequently, the paper gives the reader a fairly good overview of our ability to predict the drag on cylinders and tori for Reynolds numbers below 1000.

I found the observations and photographs of vortex shedding from tori to be the most interesting part of the paper. Some of the interesting features observed, however, may be due to the unique body symmetry and absence of body ends. One must question whether a cylinder bent into a segment of a torus would produce any of the same vortex shedding patterns. The existence of two possible vortex patterns, a counterrotating ring vortex pair and a helical counterrotating vortex pair, in the medium solidity range is an intriguing observation. Unfortunately the present study gave little indication of why these two patterns exist.

R. G. Hussey⁴

The paper by D. R. Monson is interesting and well done. His drag results and his striking photographs of vortex shedding from tori are valuable contributions. Some of his results for cylinders are different from those found by me and my co-workers, so I welcome the opportunity to comment on these differences. The author points out that his boundary effect observations for cylinders in the inertial regime are different from those of Huner, and for $Re_L = 2.9$, his coefficient of 1.37 is closer to de Mestre's 1.339 than to Stalnaker's 0.886. Huner, de Mestre, and Stalnaker treated the case of a cylinder moving midway between parallel walls, whereas a circular outer boundary is used in Monson's experiment. The author obtains a relation between the two

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geometries by using the concept of hydraulic diameter. He correctly points out that the solution of Happel and Bart for a square outer boundary indicates that the use of the hydraulic diameter concept in that instance gives a boundary coefficient that is too large by 10 percent. Another example is the sphere moving midway between infinite parallel plane walls, where use of the hydraulic diameter gives a coefficient of 1.052, which is larger than the correct value of 1.004. It is possible that when the object is a slender cylinder moving broadside rather than a sphere, the difference in the coefficient may be larger. Therefore, in the absence of a more complete justification for the use of the hydraulic diameter concept in the Stokes flow region and the inertial region, I am inclined to attribute our different experimental results to differences in the geometry of the outer boundary. The author may wish to compare his results to those of C. M. White [39], who also used a circular outer boundary.

The author is correct in stating that Stalnakar's empirical inertial correction is based on a curve fit to experimental points that have considerable scatter. In a subsequent paper, Yang-Jen Chen [40] obtained a better correlation that extends to higher Reynolds number, but is consistent with Stalnakar's results. However, I think that the difference between these empirical correlations and Brenner's first order theory is not as large as the author indicates. In order to use Brenner's theory, it is necessary to know the Stokes drag. For long cylinders ($A \geq 20$), the slender body calculations of Russel (or Batchelor) can be used for the Stokes drag. Then at $Re_L = 1$, Brenner's theory gives a correction factor of 1.046 for $A = 100$ and 1.067 for $A = 20$, while Stalnakar's expression gives 1.062 (independent of A) and Chen's expression gives 1.051 for $A = 100$ and 1.068 for $A = 20$. It is true for $Re_L < 1$, both Chen's and Stalnakar's empirical functions give larger values for the correction factor than Brenner's formula, but the correction is small in this region (less than 0.7 percent at $Re_L = 0.1$). At values of Re_L larger than 1, Brenner's theory is not valid. Finally, since Stalnakar's and Chen's correlations were obtained with cylinders of large aspect ratio, they should not be expected to apply to short cylinders ($A = 1$ or $A = 4$).

Huner found that over the range of Reynolds number of his experiment ($0.22 < Re_c < 2.6$), the dimensionless drag ($\text{drag}/4\pi\mu UL$) consistently decreased with increasing cylinder length, and he used this observation to devise a means of extrapolating to infinite cylinder length. The author's results at $Re_c = 1$ imply that the dimensionless drag increases with increasing cylinder length for $A \geq 100$. I am unable to explain the difference between our results. I think it is unlikely that the difference is due to the larger aspect ratio used by the author, because (at fixed Re_c) larger aspect ratio implies larger Re_L and the larger Re_L the more valid is the physical model (small end effects) proposed by Huner to explain his observations.

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Author's Closure

I would like to thank Professors Hussey and Johnson and Dr. O'Brien for their interest in and comments on this paper. Their discussions and additional references add further insight to the subject of this paper. I will comment on a few of their points.

In order to detect the effect of transverse curvature on the drag and vortex shedding of elongated bodies, one must have a reliable noncurved body data base with which to compare. Even for the simple straight cylinder family, such needed information was lacking, and thus considerable effort focused on establishing this reference baseline. The study of transverse curvature effects in planes other than normal to the flow direction was beyond the stated scope of the paper. There are no doubt many further discoveries to be made in this area of study.

Viscous Regime

Dr. O'Brien seems to be implying that in the viscous regime study, comparisons were made between measured drag and various theoretical Stokes drag solutions which apply only for $Re_L \rightarrow 0$ and an unbounded fluid. This is not the case. Every effort was made to eliminate boundary and inertial terms from the measured drag values. With the present definition of shape factor, the Stokes drag on a sphere of the same volume as any arbitrary elongated body is used as a reference geometry. Then e.g., for a cylinder falling broadside within a container, equation (2) will give the measured drag as

$$\text{Meas. Drag} = \left[\frac{\text{Stokes drag on sphere of equal vol. at same velocity}}{\left(\frac{1}{K_\infty}\right)\left(\frac{1}{K_b}\right)\left(\frac{1}{K_{be}}\right)} \right] \quad (33)$$

The shape factor K_∞ may contain some inertial terms which may be estimated by Brenner's relation, equation (11). For the linear perturbation range, we solve equation (11) for $K_{\infty v}$, then use series expansions of the solution and retain only the first term. The result can be expressed in the form $K_\infty = K_{\infty v}/(1 + k_1 Re_L)$. Note that if Re_L (or Re_d for the case of the torus) is significant, K_∞ will be smaller than $K_{\infty v}$, not larger as supposed by Dr. O'Brien. K_b is approximated by equation (3a) which can be expressed as $K_b = (1 + k_2 \lambda)^{-1}$. Equation (3) can be converted to the same form as discussed by Dr. O'Brien, however, I find equation (3) to be accurate over a broader range of λ than is equation (3a). Similarly, the approximate "endwall" correction factor K_{be} , equation (6), can be expressed as $K_{be} = 1.0$.

I agree with Prof. Hussey that the constant in equation (6) may be a little low for the cylindrical boundary application, however, even in the $A = 100$ case, which had the largest endwall correction, if we increase the constant 0.011, say as much as 20 percent, the endwall correction $1 - K_{be}$ would only change from 0.6 to 0.7 percent. The question remains whether Brenner's equation (3) remains accurate for greatly elongated particles. I believe this requires further verification.

When these linearized expressions are substituted into equation (33), we obtain

$$\text{Meas. Drag} = \left[\frac{\text{Stokes drag} \dots}{K_{\infty v}} \right] (1 + k_1 Re_L + k_2 \lambda), \quad (34)$$

which is the same as Dr. O'Brien's linearized equation. For solid particles, k_1 is a function of the particle shape and k_2 and is a function of the particle shape and the boundary shape. The original relations from which equation (34) was derived are accurate over a somewhat wider range of Re_L , λ , and D_h/L than would be this linearized version.

The value $K_{\infty v}$, the Stokes regime shape factor for an unbounded fluid, is then extracted from equation (33), hopefully with a minimum of error if we have met all the conditions on the various relations. In the alternate graphical extrapolation method, to eliminate the boundary effect K_b ,

the experimental measured quantity ($K_\infty K_b K_{be}$) obtained from equation (33) is plotted for various values of λ and extrapolated back to zero λ using the slope predicted by Brenner's theory as a guide at the intercept if the curve is not linear. The intercept value yields the quantity ($K_\infty K_{be}$) since when $\lambda = 0$, $K_b = 1$. then ($K_\infty K_{be}$) was divided by K_{be} as estimated by equation (6) and corrected to the Stokes regime utilizing Brenner's equation (11) evaluated at the Reynolds number of the lowest value of λ tested. Be advised that in 4, the test container diameter was fixed and the model size (and hence Re) was varied. This yields an estimate for $K_{\infty v}$ which is compared to theoretical values. It is the error $1 - (K_{\infty v} / K_{\infty v \text{ theory}})$ in percent that is being quoted.

C. M. White measured the drag on cylinders having aspect ratios from 4.9 to 158. He used a combined empirical extrapolation relation to simultaneously correct to infinite aspect ratio and an unbounded fluid. Only in his Table 2 tests did he give uncorrected drag coefficients which could be corrected to finite aspect ratio. Choosing two values close to the present tests, $A = 4.9$ and 11, I reduced the data to $K_{\infty v}$ as was done for the cylinders of $A < 10$ in the present paper. The results are $K_{\infty v} = 0.697$ and 0.559, respectively, compared to 0.727 and 0.570, respectively, for the semiempirical cylinder relation, equations (22), (27), and (28).

Dr. O'Brien's discussion of the "equivalent prolate spheroid" has pointed out a misleading statement in the paper. As long as we are comparing bodies of equal volume but different shapes at a given velocity, equal shape factors will ensure equal drag. The ad hoc modification of prolate spheroid theory adjusts its shape factor to be equal to that of a cylinder or torus of a given aspect ratio. The drag on the two bodies will be different at a given velocity since the volumes of the two bodies are not the same. The ratio of the volume of the prolate spheroid of equal shape factor to that of the geometry of interest is given by $V_e/V_i = 2/(3\sigma_i^2)$. The ratio of equivalent spherical diameters would be the cube root of this ratio. Thus, the subscript e and the definition of σ in the

become increasingly inaccurate with decreasing aspect ratio since they neglect the drag of the ends. Oddly, inclusion of this term would increase the differences between experimental data and, e.g., the Batchelor theory at the low aspect ratio end of its range of validity.

Dr. O'Brien has cited references which lead to a multitude of alternate a priori analyses which attempt to predict the Stokes drag of arbitrary bodies, especially elongated bodies. Except for the Hill and Power extremum principles method, all the other analysis are oriented toward estimating drag for the case of fall along the major axis rather than broadside. Although some of the methods have been suggested to be applicable for the broadside fall case as well, their accuracy and range of validity has not been demonstrated. They all appear to lose accuracy and/or become unwieldy for large aspect ratios and thus would not be useful over the full range of aspect ratio. I find the ad hoc modification of the prolate spheroid equation to be a relatively easy procedure which allows approximations to data or other theories over the full range of aspect ratio since it has the correct asymptotic form for both large and small aspect ratios.

For the torus, $K_{be} = 1$ in equation (33) since it has no ends. The $S = 0.036$ data point had little influence on how the curve was extrapolated back to zero λ , Fig. 11. As can be seen in the paper, when this data point was not included in the second method, results were comparable to the first method. The "error" of 0.998 should be corrected to read "correlation."

Inertial Regime

From a theoretical standpoint, Dr. O'Brien is correct in that transitional expressions developed for $Re_c > 1$ which fit the form of equation (29) are not necessarily correct for $1 > Re_c \rightarrow 0$. This is especially true for large A since we know β is a function of Reynolds number even though it may be constant over a limited range of Reynolds number. The status of transitional relations for the cylinder family is as follows:

| $[C_D/C_{Dv} - 1]_\infty$ | Range of Study | Author |
|---|--|---|
| $0.062 Re_L^{1/2}$ | $20 < A < 380$ $0.02 < Re_L < 10$ | Stalnaker and Hussey [1] |
| $0.12 A^{-1/4} Re_L^{0.75}$ | $11 < A < 49$ $0.01 A^{1/4} < Re_L < 30 A^{1/4}$ | Chen and Hussey (Hussey's references [40]) |
| $0.114 A^{-0.279} Re_L^{0.779}$ | $A < 13$ } $A = 100$ } $\frac{3}{1 + \log A} < Re_L < \frac{1200}{1 + \log A}$ | Present |
| $0.825 A^{-0.779} Re_L^{0.779}$ | | |
| $\frac{0.215}{K_{\infty v c}} A^{-0.67} Re_L$ | Theory $Re_L < 0.05$ | Brenner and Cox [10] |

paper should be taken to represent an ellipsoid of equal shape factor rather than drag. d_s for the geometry of interest is to be used in equations (2) and (10) to estimate the drag. Comparisons on the basis of equal frontal areas are equally valid but would require a redefinition of the drag equation to reflect this change of reference.

Dr. O'Brien has reminded us that the cylinder drag theories

For the purpose of making engineering estimates, the overlaps in aspect ratio and Reynolds number of these studies and the similarities in form and values of the constants in these relations suggest that Chen's relation may be acceptable for $A < 11$ and the present relation for $A < 13$ may be usable for $Re_L < 3/(1 + \log A)$. Stalnaker's relation appears to be most accurate in the $A = 10-20$ range and the present relation

for $A = 100$ may be acceptable for $Re_L < 1$. In the range $0.05 < Re_L < 1$ most of these relations agree within a percent or two. For $Re_L < 0.1$, Prof. Hussey has shown that the difference between these relations are small and Brenner's theory suggests that the drag will be less than one percent greater than the Stokes drag for $Re_L < 0.05$ for all A .

At $Re_c = 1$, the $A = 100$ drag coefficient is 17 percent less than the drag projected by Huner [22] for $A = \infty$. There would have to be an error in this data point several times the stated accuracy for this point to fall above the curve for infinite aspect ratio. Presently, I have no reason to suspect such an error in the data. This result points out the need for more

data at large aspect ratios over a range of Reynolds number spanning this point.

Dr. O'Brien suggests, without documentation, that the vortex shedding patterns may be sensitive to boundary proximity. If this is correct, it points out the need for test containers which are large compared to the largest dimension for greatly elongated bluff bodies. This need was also made clear for obtaining accurate viscous drag data as well. The reasons for the dual vortex patterns in the medium solidity range will no doubt be revealed in a stability analysis of coannular shear layers of opposite vorticity. To my knowledge, such a study has never been done.