Check for updates

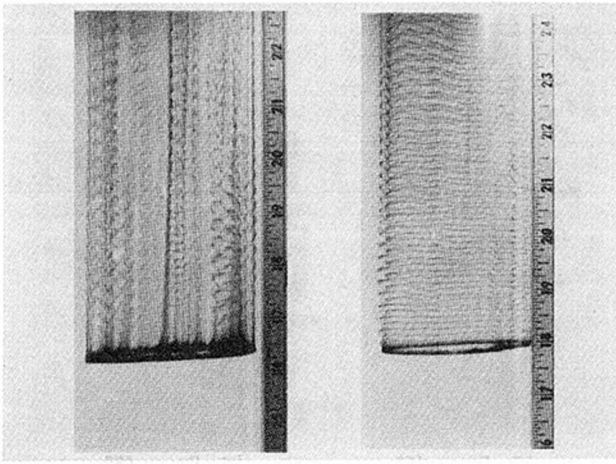

Fig. 10 Tori $S = 0.036$

Acknowledgments

The author wishes to acknowledge the consultation and assistance of B. N. McDonald of the Donaldson Co. in the design and setting up of the experimental timing system. The assistance of Jayne Enthoven and Judy Neufeld, work-study students, during testing and data reduction is gratefully acknowledged.

References

1 Stalnaker, J. F., and Hussey, R. G., "Wall Effects on Cylinder Drag at Low Reynolds Number," *Physics of Fluids*, Vol. 22, No. 4, Apr. 1979, pp. 603–613.

2 Amarakoon, A. M. D., Hussey, R. G., Good, B. J., and Grimsal, E. G., "Drag Measurements for Axisymmetric Motion of a Torus at Low Reynolds Number," *Physics of Fluids*, Vol. 25, No. 9, Sept. 1982, pp. 1495–1501.

3 Monson, D. R., "The Effect of Transverse Curvature on the Drag and Vortex Shedding of Elongated Bluff Bodies at Low Reynolds Number," ASME Paper 81/WA/FE-4, 1981.

4 Monson, D. R., "Experimental Drag Characteristics of Tori, Ducted-Spheres, and Other Shapes at Low Reynolds Numbers," Masters thesis, Department of Aerospace and Engineering Mechanics, University of Minnesota, 1965.

5 Tchen, C. M., "Motion of Small Particles in Skew Shape Suspended in a Viscous Liquid," *Journal of Applied Physics*, Vol. 25, No. 4, Apr. 1954, pp. 463–473.

6 Johnson, R. E., "An Improved Slender-Body Theory for Stokes Flow," *Journal of Fluid Mechanics*, Vol. 99, Part 2, 1980, pp. 411–431.

7 Brenner, H., "Effect of Finite Boundaries on the Stokes Resistance of an Arbitrary Particle, Part 1," *Journal of Fluid Mechanics*, Vol. 12, Jan. 1962, pp. 35–48.

8 Chwang, A. T., and Wu, T. Y., "Hydromechanics of Low-Reynolds-Number Flow. Part 4. Translation of Spheroids," *Journal of Fluid Mechanics*, Vol. 75, Part 4, 1976, pp. 677–689.

9 Brenner, H., "The Oseen Resistance of a Particle of Arbitrary Shape," *Journal of Fluid Mechanics*, Vol. 11, 1961, pp. 604–610.

10 Brenner, H., and Cox, R. G., "The Resistance to a Particle of Arbitrary Shape in Translational Motion at Small Reynolds Numbers," *Journal of Fluid Mechanics*, Vol. 17, 1963, pp. 561–595.

11 Happle, J., and Bart, E., "The Settling of A Sphere Along the Axis of a Long Square Duct at Low Reynolds Number," *Applied Scientific Research*, Vol. 29, 1974, pp. 241–258.

12 Roos, F. W., and Willmarth, W. W., "Some Experimental Results on Sphere and Disk Drag," *AIAA Journal*, Vol. 9, No. 2, Feb. 1971, pp. 285–291.

13 McNown, J. S., Lee, H. M., McPherson, M. B., and Engez, S. M., "Influence of Boundary Proximity on the Drag of Spheres," *Proc. VII Intern. Cong. of Appl. Mech.*, London, 1948, pp. 17–27.

14 McNown, J. S., and Malaika, J., "Effects of Particle Shape on Setting Velocity at Low Reynolds Numbers," *Trans. of Am. Geophys. Union*, Vol. 31, 1950, pp. 74–82.

15 Majumdar, S. R., and O'Neill, M. E., "On Axisymmetric Stokes Flow Past a Torus," *Journal of Applied Mathematics and Physics (ZAMP)*, Vol. 28, 1977, pp. 541–550.

16 Dorrepaal, J. M., Majumdar, S. R., O'Neill, M. E., and Ranger, K. B., "A Closed Torus in Stokes Flow," *Q. J. Mech. Appl. Math.*, Vol. 29, 1976, pp. 381–397.

17 Goren, S. L., and O'Neill, M. E., "Asymmetric Creeping Motion of an Open Torus," *Journal of Fluid Mechanics*, Vol. 101, Part 1, 1980, pp. 97–110.

18 Johnson, R. E., and Wu, T. Y., "Hydromechanics of Low-Reynolds Number Flow. Part 5. Motion of a Slender Torus," *Journal of Fluid Mechanics*, Vol. 95, Part 2, 1979, pp. 263–277.

19 Heiss, J. F., and Coull, J., "The Effect of Orientation and Shape on the Settling Velocity of Non-Isometric Particles in a Viscous Medium," *Chem. Eng. Progr.*, Vol. 48, 1952, p. 133.

20 Batchelor, G. K., "Slender-Body Theory for Particles of Arbitrary Cross-Section in Stokes Flow," *Journal of Fluid Mechanics*, Vol. 44, Part 3, 1970, pp. 419–440.

21 Russel, W. B., Hinch, E. J., Leal, L. G., and Tieffenbruck, G., "Rods Falling Near a Vertical Wall," *Journal of Fluid Mechanics*, Vol. 83, Part 2, 1977, pp. 273–287.

22 Huner, B., an Hussey, R. G., "Cylinder Drag at Low Reynolds Number," *Physics of Fluids*, Vol. 20, No. 8, 1977, pp. 1211–

23 Pruppacher, H. R., LeClair, B. P., and Hamielec, A. E., "Some Relations Between Drag and Flow Pattern of Viscous Flow Past a Sphere and a Cylinder at Low and Intermediate Reynolds Numbers," *Journal of Fluid Mechanics*, Vol. 44, Part 4, 1970, pp. 781–790.

24 Tritton, D. J., "The Flow Past a Circular Cylinder at Low Reynolds Numbers," *Journal of Fluid Mechanics*, Vol. 6, 1959, pp. 547–567.

25 Shi, Y., "Low Reynolds Numbers Flow Past An Ellipsoid of Revolution of Large Aspect Ratio," *Journal of Fluid Mechanics*, Vol. 23, Part 4, 1965, pp. 657–671.

26 Schiller, and Nauman, *Ver. Deut. Ing.*, Vol. 77, 1933, pp. 318–320.

27 Roshko, A., "On the Development of Turbulent Wakes from Vortex Streets," NACA Report 1191, 1954.

28 Takamoto, M., and Izumi, K., "Experimental Observation of Stable Arrangement of Vortex Rings," *Physics of Fluids*, Vol. 24, No. 8, Aug. 1981, pp. 1582–1583.

20 Hardin, J. C., "The Role of the Helical Jet Mode in Aerodynamic Noise Generation," AIAA Paper No. AIAA-82-1963, 1981.

DISCUSSION

V. O'Brien[2]

It is always interesting to see a careful experimental series which can be compared to theoretical expressions for some range, and also extends insight into ranges where theory is still lacking. A connection between linear objects of length L and tori with mean circumference L is not obvious.

I. Viscous Regime

For low Reynolds numbers, $Re_l = (2lU/\nu) < 1$, where $2l$ is the longest particle dimension, Fig. 11, there has to be a distinction between steady Stokes drag ($Re_l \to 0$, unbounded domain) and the measured drag. Stokes drag for a variety of bodies is known analytically or can be estimated by inscribing/circumscribing bodies after Hill and Power [30]. Long cylinders fall stably broadside, as do straight strings of spherical particles [31], prolate ellipsoids and a rectangular parallelpiped, Fig. 1(a). The frontal area, F, is important. The drag of the prolate spheriod is less than that of the cylinder or prism. The open tori, solidity $S < 1$, have $F = S \cdot \pi l^2$, less than a solid closed particle ($S = 1$) with the same side projection, and less Stokes drag.

The "Oseen" effect, the inertial increase of drag is a simple linear function of Re_l dependent on the Stokes drag for arbitrary solid particles[3] or circulating fluid spheroids [33]. The approximation is only valid to $0(Re_l)$, i.e., $Re_l << 1$.

The boundary effect for particles traversing the centerline of a fluid container is a function of container shape [34, 35]. For an outer cylinder of radius R_0 and axisymmetric particles [36], general solid particles[5] or circulating fluid spheroids [33], it is a linear function of $\lambda = l/R_0$, to $0(\lambda^2)$, where the coefficient multiplying λ is also a function of Stokes drag.

In the *linear* perturbation range then for all test particles

$$\text{Drag} = \text{Stokes drag}\,[1 + k_1\, Re_l + k_2 \lambda] \qquad [1]$$

where the constants k_1 and k_2 are functions of particle shape

[2] M. S. Eisenhower Research Center, Applied Physics Laboratory, The Johns Hopkins University, Laurel, Md. 20707.

Copyright © 1983 by ASME

Downloaded from http://solarenergyengineering.asmedigitalcollection.asme.org/fluidsengineering/article-pdf/105/3/320/5520360/318_1.pdf by guest on 18 July 2024

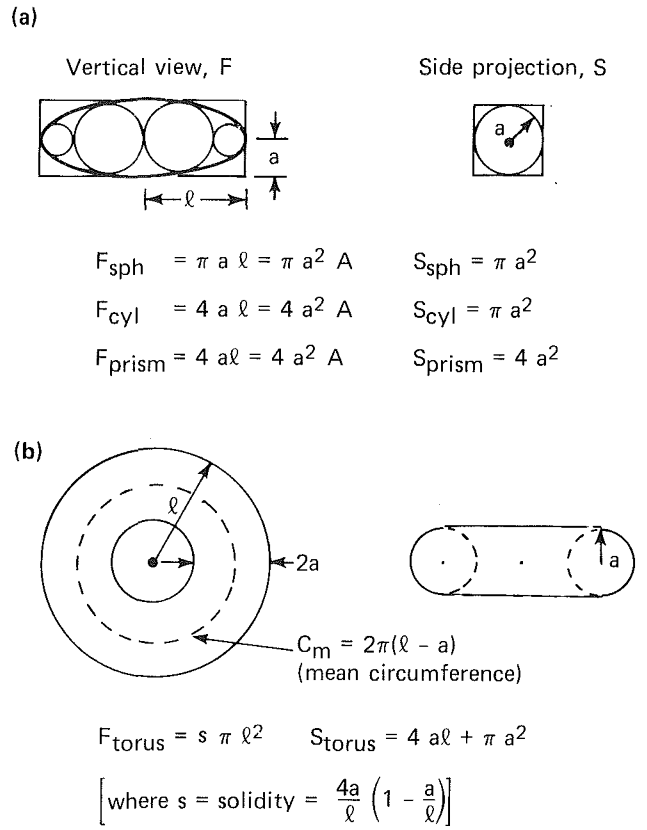

Fig. 11 Projected area of particles.

and internal circulation for fluid spheroids. The settling factor $K_\infty = U_\infty / U_s$, where U_s pertains to the Stokes drag on an "equivalent" solid sphere (same volume and density), involves other shape functions but allows the use of the solid spherical coefficient 2.1044 for extrapolating $\lambda \to 0$ for very low Re_l results at small λ (author's Fig. 1). The bracket in equation (1) can be inverted with k_i change of sign to the order of estimation. (The second method can be ignored.) The results are consistent with the exact torus theory, Fig. 2.

Extrapolations are sensitive to data accuracy and the percentages quoted seem to be in terms of $K_b = 1$ not K_∞. (The data point for $S = 0.036$, $\mathrm{Re}_d = 0.803$ should not have been included. The *value* is large because of Re_d.) An average *error* of 0.998 must be a mistake.

Since all shape factors can be related, one could use A_e, an "equivalent prolate spheroid" instead of a sphere. However, I doubt the statement, about two "exact" theories showing a prolate spheroid has a higher drag than the a cylinder of the same (large) aspect ratio. The frontal area of the spheroid is always less by a factor $\pi/4$ for all values of A. Slender-body theory, all singularities on the axis, does not take into account the flat ends of a cylinder. See references [37, 38].

II. Inertial Regime

Here all the curves are empirical. Figure 4 shows smooth curves for the cylinder data, each A value represented by a linear C_D versus log Re_c relationship at small Re_c (less C_D for greater A), curving up as $\mathrm{Re}_c \to 1000$ (inverse order for A). The equation (29), for the deviation of measured C_D from the linear (Stokes) C_D, obviously cannot be used for $1 > \mathrm{Re}_c \to 0$. The "transition region" where standing vortices develop, become unstable and detach themselves as $\mathrm{Re}_c \to \infty$ is dependent on boundary proximity even if the measured drag is not. The attached vortex behind a finite cylinder moving broadside must be three-dimensional from its initial appearance. The vortex arrangement behind an open torus will be quite different, but initially axisymmetric. As the Reynolds number increases, the vortex shedding pattern depends on solidity S. The critical Reynolds number for shift of pattern from the solid spheroidal loops to ring-pairs is a phenomenon worth pursuing in more depth experimentally and theoretically.

Acknowledgment

This work was supported by the U. S. Navy (Naval Sea Systems Command) under Contract N00024-83-C-5301.

Additional References

30 Hill, R., and Power, G., *Quart. J. Mech. Appl. Math.*. Vol. 9, 1956, p. 313.

31 Gluckman, M. J., Weinbaum, S., Pfeffer, R., *J. Fl. Mech.*, Vol. 55, 1972, p. 677.

32 Brenner, H., *J. Fl. Mech.*, Vol. 11, 1961, p. 604.

33 O'Brien, V., APL/JHU Rpt. CM-1003, "'Axisymmetric Viscous Flows Correct to First-Order in Reynolds Number," Oct. 1961.

34 Brenner, H., *J. Fl. Mech.*, Vol. 12, 1962, p. 35.

35 Happel, J., and Bart, E., *Appl. Sci. Res.*, Vol. 29, 1974, p. 241.

36 I-D. Chang, *ZAMP*, Vol. 12, 1961, p. 6.

37 Amarakoon, A. M. D., Hussey, R. G., Good, B. J., and Gremsal, E. G., *Phys. Fl.*. Vol. 25, 1982, p. 1495.

38 O'Brien, V., *Can. J. Chem. Engr'g.*, Vol. 51, 1973, p. 793.

R. E. Johnson[3]

Although the title of this paper emphasizes that it will study the effects of curvature on the drag and vortex shedding of elongated bodies, this paper seems to concentrate its effort on determining which theoretical or empirical equation best describes the drag on cylinders and tori. Consequently, the paper gives the reader a fairly good overview of our ability to predict the drag on cylinders and tori for Reynolds numbers below 1000.

I found the observations and photographs of vortex shedding from tori to be the most interesting part of the paper. Some of the interesting features observed, however, may be due to the unique body symmetry and absence of body ends. One must question whether a cylinder bent into a segment of a torus would produce any of the same vortex shedding patterns. The existence of two possible vortex patterns, a counterrotating ring vortex pair and a helical counterrotating vortex pair, in the medium solidity range is an intriguing observation. Unfortunately the present study gave little indication of why these two patterns exist.

R. G. Hussey[4]

The paper by D. R. Monson is interesting and well done. His drag results and his striking photographs of vortex shedding from tori are valuable contributions. Some of his results for cylinders are different from those found by me and my co-workers, so I welcome the opportunity to comment on these differences. The author points out that his boundary effect observations for cylinders in the inertial regime are different from those of Huner, and for $\mathrm{Re}_L = 2.9$, his coefficient of 1.37 is closer to de Mestre's 1.339 than to Stalnaker's 0.886. Huner, de Mestre, and Stalnaker treated the case of a cylinder moving midway between parallel walls, whereas a circular outer boundary is used in Monson's experiment. The author obtains a relation between the two

[3]Assistant Professor, Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana-Champaign, Ill.

Professor of Physics, Dept. of Physics and Astronomy, Louisiana State Univ., Baton Rouge, La. 70803.

Downloaded from http://solarenergyengineering.asmedigitalcollection.asme.org/fluidsengineering/article-pdf/105/3/320/5520360/318_1.pdf by guest on 18 July 2024

geometries by using the concept of hydraulic diameter. He correctly points out that the solution of Happel and Bart for a square outer boundary indicates that the use of the hydraulic diameter concept in that instance gives a boundary coefficent that is too large by 10 percent. Another example is the sphere moving midway between infinite parallel plane walls, where use of the hydraulic diameter gives a coefficient of 1.052, which is larger than the correct value of 1.004. It is possible that when the object is a slender cylinder moving broadside rather than a sphere, the difference in the coefficient may be larger. Therefore, in the absence of a more complete justification for the use of the hydraulic diameter concept in the Stokes flow region and the interial region, I am inclined to attribute our different experimental results to differences in the geometry of the outer boundary. The author may wish to compare his results to those of C. M. White [39], who also used a circular outer boundary.

The author is correct in stating that Stalnaker's empirical inertial correction is based on a curve fit to experimental points that have considerable scatter. In a subsequent paper, Yang-Jen Chen [40] obtained a better correlation that extends to higher Reynolds number, but is consistent with Stalnaker's results. However, I think that the difference between these empirical correlations and Brenner's first order theory is not as large as the author indicates. In order to use Brenner's theory, it is necessary to know the Stokes drag. For long cylinders ($A \geq 20$), the slender body calculations of Russel (or Batchelor) can be used for the Stokes drag. Then at $\mathrm{Re}_L = 1$, Brenner's theory gives a correction factor of 1.046 for $A = 100$ and 1.067 for $A = 20$, while Stalnaker's expression gives 1.062 (independent of A) and Chen's expression gives 1.051 for $A = 100$ and 1.068 for $A = 20$. It is true for $\mathrm{Re}_L < 1$, both Chen's and Stalnaker's empirical functions give larger values for the correction factor than Brenner's formula, but the correction is small in this region (less than 0.7 percent at $\mathrm{Re}_L = 0.1$). At values of Re_L larger than 1, Brenner's theory is not valid. Finally, since Stalnaker's and Chen's correlations were obtained with cylinders of large aspect ratio, they should not be expected to apply to short cylinders ($A = 1$ or $A = 4$).

Huner found that over the range of Reynolds number of his experiment ($0.22 < Re_c < 2.6$), the dimensionless drag ($\mathrm{drag}/4\pi\mu UL$) consistently decreased with increasing cylinder length, and he used this observation to devise a means of extrapolating to infinite cylinder length. The author's results at $\mathrm{Re}_c = 1$ imply that the dimensionless drag increases with increasing cylinder length for $A \geq 100$. I am unable to explain the difference between our results. I think it is unlikely that the difference is due to the larger aspect ratio used by the author, because (at fixed Re_c) larger aspect ratio implies larger Re_L and the larger Re_L the more valid is the physical model (small end effects) proposed by Huner to explain his observations.

Additional References

39 White, C. M., "The Drag of Cylinders in Fluids at Slow Speeds," *Proc. Royal Soc. London*, Series A, Vol. 186, 1946, pp. 472-479

40 Chen, Yang-Jen, and Hussey, R. G., "Effect of a Horizontal Plane Boundary on a Falling Horizontal Cylinder at Low Reynolds Number," *Phys. Fluids*, Vol. 23, No. 5, 1980, pp. 853-857.

Author's Closure

I would like to thank Professors Hussey and Johnson and Dr. O'Brien for their interest in and comments on this paper. Their discussions and additional references add further insight to the subject of this paper. I will comment on a few of their points.

In order to detect the effect of transverse curvature on the drag and vortex shedding of elongated bodies, one must have a reliable noncurved body data base with which to compare. Even for the simple straight cylinder family, such needed information was lacking, and thus considerable effort focused on establishing this reference baseline. The study of transverse curvature effects in planes other than normal to the flow direction was beyond the stated scope of the paper. There are no doubt many further discoveries to be made in this area of study.

Viscous Regime

Dr. O'Brien seems to be implying that in the viscous regime study, comparisons were made between measured drag and various theoretical Stokes drag solutions which apply only for $\mathrm{Re}_L \to 0$ and an unbounded fluid. This is not the case. Every effort was made to eliminate boundary and inertial terms from the measured drag values. With the present definition of shape factor, the Stokes drag on a sphere of the same volume as any arbitrary elongated body is used as a reference geometry. Then e.g., for a cylinder falling broadside within a container, equation (2) will give the measured drag as

$$\text{Meas. Drag} = \begin{bmatrix}\text{Stokes drag on}\\ \text{sphere of equal}\\ \text{vol. at same velocity}\end{bmatrix}\left(\frac{1}{K_\infty}\right)\left(\frac{1}{K_b}\right)\left(\frac{1}{K_{be}}\right). \tag{33}$$

The shape factor K_∞ may contain some inertial terms which may be estimated by Brenner's relation, equation (11). For the linear perturbation range, we solve equation (11) for $K_{\infty v}$, then use series expansions of the solution and retain only the first term. The result can be expressed in the form $K_\infty = K_{\infty v}/(1 + k_1\,\mathrm{Re}_L)$. Note that if Re_L (or Re_d for the case of the torus) is significant, K_∞ will be *smaller* than $K_{\infty v}$ not larger as supposed by Dr. O'Brien. K_b is approximated by equation (3*a*) which can be expressed as $K_b = (1 + k_2\,\lambda)^{-1}$. Equation (3) can be converted to the same form as discussed by Dr. O'Brien, however, I find equation (3) to be accurate over a broader range of λ than is equation (3*a*). Similarly, the approximate "endwall" correction factor K_{be}, equation (6), can be expressed as $K_{be} = 1.0$.

I agree with Prof. Hussey that the constant in equation (6) may be a little low for the cylindrical boundary application, however, even in the $A = 100$ case, which had the largest endwall correction, if we increase the constant 0.011, say as much as 20 percent, the endwall correction $1 - K_{be}$ would only change from 0.6 to 0.7 percent. The question remains whether Brenner's equation (3) remains accurate for greatly elognated particles. I believe this requires further verification.

When these linearized expressions are substituted into equation (33), we obtain

$$\text{Meas. Drag} = \left[\frac{\text{Stokes drag}\ldots}{K_{\infty v}}\right](1 + k_1\,\mathrm{Re}_L + k_2\,\lambda), \tag{34}$$

which is the same as Dr. O'Brien's linearized equation. For solid particles, k_1 is a function of the particle shape and k_2 and is a function of the particle shape and the boundry shape. The original relations from which equation (34) was derived are accurate over a somewhat wider range of Re_L, λ, and D_h/L than would be this linearized version.

The value $K_{\infty v}$, the Stokes regime shape factor for an unbounded fluid, is then extracted from equation (33), hopefully with a minimum of error if we have met all the conditions on the various relations. In the alternate graphical extrapolation method, to eliminate the boundary effect K_b,

Downloaded from http://solarenergyengineering.asmedigitalcollection.asme.org/fluidsengineering/article-pdf/105/3/320/5520360/318_1.pdf by guest on 18 July 2024

the experimental measured quantity ($K_\infty\ K_b\ K_{be}$) obtained from equation (33) is plotted for various values of λ and extrapolated back to zero λ using the slope predicted by Brenner's theory as a guide at the intercept if the curve is not linear. The intercept value yields the quantity ($K_\infty\ K_{be}$) since when $\lambda = 0$, $K_b = 1$. then ($K_\infty\ K_{be}$) was divided by K_{be} as estimated by equation (6) and corrected to the Stokes regime utilizing Brenner's equation (11) evaluated at the Reynolds number of the lowest value of λ tested. Be advised that in 4, the test container diameter was fixed and the model size (and hence Re) was varied. This yields an estimate for $K_{\infty v}$ which is compared to theoretical values. It is the error $1 - (K_{\infty v}/K_{\infty v\,\text{theory}})$ in percent that is being quoted.

C. M. White measured the drag on cylinders having aspect ratios from 4.9 to 158. He used a combined empirical extrapolation relation to simultaneously correct to infinite aspect ratio and an unbounded fluid. Only in his Table 2 tests did he give uncorrected drag coefficients which could be corrected to finite aspect ratio. Choosing two values close to the present tests, $A = 4.9$ and 11, I reduced the data to $K_{\infty v}$ as was done for the cylinders of $A < 10$ in the present paper. The results are $K_{\infty v} = 0.697$ and 0.559, respectively, compared to 0.727 and 0.570, respectively, for the semiempirical cylinder relation, equations (22), (27), and (28).

Dr. O'Brien's discussion of the "equivalent prolate spheriod" has pointed out a misleading statement in the paper. As long as we are comparing bodies of equal volume but different shapes at a given velocity, equal shape factors will ensure equal drag. The ad hoc modification of prolate spheriod theory adjusts its shape factor to be equal to that of a cylinder or torus of a given aspect ratio. The drag on the two bodies will be different at a given velocity since the volumes of the two bodies are not the same. The ratio of the volume of the prolate spheriod of equal shape factor to that of the geometry of interest is given by $V_e/V_i = 2/(3\ \sigma_i^2)$. The ratio of equivalent spherical diameters would be the cube root of this ratio. Thus, the subscript e and the definition of σ in the paper should be taken to represent an ellipsoid of equal shape factor rather than drag. d_s for the geometry of interest is to be used in equations (2) and (10) to estimate the drag. Comparisons on the basis of equal frontal areas are equally valid but would require a redefinition of the drag equation to reflect this change of reference.

Dr. O'Brien has reminded us that the cylinder drag theories become increasingly inaccurate with decreasing aspect ratio since they neglect the drag of the ends. Oddly, inclusion of this term would increase the differences between experimental data and, e.g., the Batchelor theory at the low aspect ratio end of its range of validity.

Dr. O'Brien has cited references which lead to a multitude of alternate a priori analyses which attempt to predict the Stokes drag of arbitrary bodies, especially elongated bodies. Except for the Hill and Power extremum principles method, all the other analysis are oriented toward estimating drag for the case of fall along the major axis rather than broadside. Although some of the methods have been suggested to be applicable for the broadside fall case as well, their accuracy and range of validity has not been demonstrated. They all appear to lose accuracy and/or become unwieldy for large aspect ratios and thus would not be useful over the full range of aspect ratio. I find the ad hoc modification of the prolate spheriod equation to be a relatively easy procedure which allows approximations to data or other theories over the full range of aspect ratio since it has the correct asymptotic form for both large and small aspect ratios.

For the torus, $K_{be} = 1$ in equation (33) since it has no ends. The $S = 0.036$ data point had little influence on how the curve was extrapolated back to zero λ, Fig. 11. As can be seen in the paper, when this data point was not included in the second method, results were comparable to the first method. The "error" of 0.998 should be corrected to read "correlation."

Inertial Regime

From a theoretical standpoint, Dr. O'Brien is correct in that transitional expressions developed for $\text{Re}_c > 1$ which fit the form of equation (29) are not necessarily correct for $1 > \text{Re}_c \rightarrow 0$. This is especially true for large A since we know β is a function of Reynolds number even though it may be constant over a limited range of Reynolds number. The status of transitional relations for the cylinder family is as follows:

$[C_D/C_{Dv} - 1]_\infty$	Range of Study	Author
$0.062\ \text{Re}_L^{1/2}$	$20 < A < 380$ $0.02 < \text{Re}_L < 10$	Stalnaker and Hussey [1]
$0.12\, A^{-1/4}\text{Re}_L^{0.75}$	$11 < A < 49$ $0.01\, A^{1/4} < \text{Re}_L < 30\, A^{1/4}$	Chen and Hussey (Hussey's references [40])
$0.114\, A^{-0.279}\text{Re}_L^{0.779}$ $0.825\, A^{-0.779}\text{Re}_L^{0.779}$	$A < 13$, $A = 100$ $\frac{3}{1+\log A} < \text{Re}_L < \frac{1200}{1+\log A}$	Present
$\frac{0.215}{K_{\infty vc}} A^{-0.67}\ \text{Re}_L$	Theory $\text{Re}_L < 0.05$	Brenner and Cox [10]

For the purpose of making engineering estimates, the overlaps in aspect ratio and Reynolds number of these studies and the similarities in form and values of the constants in these relations suggest that Chen's relation may be acceptable for $A < 11$ and the present relation for $A < 13$ may be usable for $\text{Re}_L < 3/(1 + \log A)$. Stalnaker's relation appears to be most accurate in the $A = 10\text{-}20$ range and the present relation

Downloaded from http://solarenergyengineering.asmedigitalcollection.asme.org/fluidsengineering/article-pdf/105/3/320/5520360/318_1.pdf by guest on 18 July 2024

for $A = 100$ may be acceptable for $\mathrm{Re}_L < 1$. In the range $0.05 < \mathrm{Re}_L < 1$ most of these relations agree within a percent or two. For $\mathrm{Re}_L < 0.1$, Prof. Hussey has shown that the difference between these relations are small and Brenner's theory suggests that the drag will be less than one percent greater than the Stokes drag for $\mathrm{Re}_L < 0.05$ for all A.

At $\mathrm{Re}_c = 1$, the $A = 100$ drag coefficient is 17 percent less than the drag projected by Huner [22] for $A = \infty$. There would have to be an error in this data point several times the stated accuracy for this point to fall above the curve for infinite aspect ratio. Presently, I have no reason to suspect such an error in the data. This result points out the need for more data at large aspect ratios over a range of Reynolds number spanning this point.

Dr. O'Brien suggests, without documentation, that the vortex shedding patterns may be sensititive to boundary proximity. If this is correct, it points out the need for test containers which are large compared to the largest dimension for greatly elongated bluff bodies. This need was also made clear for obtaining accurate viscous drag data as well. The reasons for the dual vortex patterns in the medium solidity range will no doubt be revealed in a stability analysis of coannular shear layers of opposite vorticity. To my knowledge, such a study has never been done.

Downloaded from http://solarenergyengineering.asmedigitalcollection.asme.org/fluidsengineering/article-pdf/105/3/320/5520360/318_1.pdf by guest on 18 July 2024