in Palmgren-Miner Damage Due to Random Vibration," Proceedings, Fourth U. S. National Congress of Applied Mechanics, ASME, 1962, pp. 119–126.

[6] S. H. Crandall and W. D. Mark, "Random Vibration in Mechanical Systems," Academic Press, New York, N. Y., 1963.

Symmetrical Bending of Circular Plates of Constant Radial Bending Stress[1]

C. W. BERT.[2] It is curious that, although the mathematical synthesis of an optimum thickness variation for a generalized-plane-stress problem (the de Laval uniform-stress rotating disk) was worked out over fifty years ago,[3] only recently have analogous syntheses of plate-bending problems been accomplished.

The author is to be congratulated for a careful analysis, including provision for a uniform-thickness ring at the outer support.

It is instructive to make a comparison between the author's results and those of another recent, yet independent, analysis.[4] In the latter work, an exact solution was obtained for a plate with a uniform in-plane shear stress, for the particular class of problems in which ν is 1/3 and there is no lateral pressure q. From consideration of the three-dimensional Mohr's circle, it is apparent that the in-plane shear stress is the maximum one only when the radial and tangential bending stresses are of opposite sign. For an annular plate with a clamped outer radius and a radial ring moment applied at the inner radius, which is taken to be $2/e$ times the outer radius (e is the base of natural logarithms), Tadjbakhsh[4] showed a 26 percent saving in plate material compared to that of a uniform-thickness plate. In this example, the bending stresses toward the outer edge are *not* of opposite sign, so that his synthesis was incorrect on the unconservative side.

The author obtained a synthesis in the form of a series solution for a plate with uniform radial bending stress. In the example considered by the author, the tangential bending stress was always smaller than the radial one *and of the same sign*. Thus the plate can be considered to have been designed for uniform *maximum* shear stress, and the author's synthesis is correct in this example. However, it would be worthwhile to determine whether this is true in general or whether it depends upon the particular case chosen.

[1] By J. P. Lee, published in the December, 1962, issue of the Journal of Applied Mechanics, vol. 29, Trans. ASME, vol. 84, Series E, pp. 696–700.

[2] Solid and Structural Mechanics Research, Battelle Memorial Institute, Columbus, Ohio. Assoc. Mem. ASME.

[3] A. Stodola, "Die Dampfturbinen und die Aussichten der Wärmekraftmaschinen," *Zeitschrift VDI*, vol. 47, 1903, pp. 51–52.

[4] I. G. Tadjbakhsh, "Elastic Optimum Design of Circular Plates," Developments in Mechanics, vol. 1, *Proceedings of the Seventh Midwestern Mechanics Conference*, 1961, Plenum Press, New York, N. Y., 1961, pp. 216–222.

Author's Closure

Dr. Bert is quite correct in his statement that the results obtained by Tadjbakhsh are incorrect on the unconservative side. For the particular case treated in my paper, the tangential bending stress is always smaller than the radial bending stress and of the same sign. Thus the plate can be considered to be uniform in strength if the strength of the material is estimated either by the maximum-stress theory or by the maximum-shear theory. Should other theories be required to determine the strength of the material, the plate is no longer uniform in strength. But the design is on the safe side.

It is difficult to present a general proof that this is also true for other cases. Investigation has been started for plates with different boundary and loading conditions. Because of the nonlinear nature of the governing differential equation, the power-series solution cannot be extended to other cases. Preliminary results obtained by a different method show that this is also true for the following four cases:[5]

1 Solid circular plate loaded at center and simply supported at outer boundary.

2 Ring plate loaded uniformly along outer boundary and built-in at inner boundary.

3 Ring plate subjected to uniform loading, built-in at inner boundary, and simply supported at outer boundary.

4 Same as case 3 except that the plate is free at the outer boundary.

These results will be presented in a forthcoming paper.

[5] Y. Kovach, "An Approximate Method for Symmetrical Bending of Circular Plates of Constant Strength," Master's Thesis, Department of Engineering Mechanics, Wayne State University, Detroit, Mich., 1962.