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A time-reversal mirror in a solid circular waveguide using a single, time-reversal element

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Abstract: The ability of a single acoustic element to produce a compact time domain signal from a multi-mode solid cylindrical waveguide using a time-reversal mirror (TRM) is considered. Two, single element, longitudinal contact transducers were used to excite and receive multiple longitudinal modes in a fused quartz waveguide in a TRM experiment. The TRM is demonstrated to be effective with the limited information from a single longitudinal transducer. Experimental results are presented along with a simple interpretation that shows how a TRM with only a single element can be used as a practical sensor.

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1. Background

Solid cylindrical waveguides have been used as buffer rods in a number of applications to isolate ultrasonic sensors from hostile environments [Jen *et al*., 1991; Jen *et al*., 1997; Peterson, 1994]. However, due to design constraints, it is often not possible to use a waveguide that is sufficiently thin to propagate only a single longitudinal mode. In sensor applications, a number of approaches have been taken to eliminate the propagation of multiple modes, including the bundling of thin waveguides, cladding of buffer rods, and introduction of surface roughness to eliminate spurious signals [e.g., Thurston, 1978; Jen *et al*., 1990]. However, in some cases, design constraints make the use of a multi-mode waveguide necessary [Peterson, 1994]. The propagation of multiple modes causes a signal that is compact in the time domain to have a large time signature after propagating through the waveguide, Fig 1. As a result, if the acoustic signal is propagated through a specimen, as well as a buffer rod, phase velocity and attenuation information about the specimen are difficult to extract. Although a number of approaches have been considered to solve this problem, the processing is highly complex [Peterson, 1999]. A time-reversal mirror is proposed to make simple, real-time processing possible by reducing the complexity of the received signal.

Time-reversal mirrors (TRM) have been developed based on the property of time-reversal invariance [Fink, 1997]. A time-reversal mirror experiment consists of three steps. In the case of a cylindrical rod, first, an acoustic signal is excited by a source at one end of the rod. The acoustic signal propagates through the rod, and the altered signal is recorded at the opposite end. Second, the recorded signal is reversed in time. Finally, the receiver is excited with the reversed signal. The reversed signal propagates through the rod, and a new signal is recorded at the source. If time invariance is satisfied, this new signal is the same as the original acoustic signal. This ability of the TRM can be used to produce a compact time signal from a dispersive system. This technique has been shown to be effective in eliminating the dispersion of Lamb waves for plate inspection [Ing and Fink, 1998].

Time reversal in a solid circular waveguide has been demonstrated recently in an application to concentrate acoustic energy at a point in a fluid [Montaldo *et al.*, 2001]. Multiple transducers on the end of a solid circular waveguide were excited by a 1-bit digitized time-reversed signal to create a high amplitude pulse in a fluid near the opposite end of the waveguide instead of the dispersed multi-mode signal. In this application and the applications mentioned previously, only the axially symmetric longitudinal modes are excited. Thus, at most, an annular array of transducers would appear to be required to reconstruct the general displacement field on the end of a cylinder. However, a single element, cylindrical transducer is most commonly used in sensor applications with cylindrical waveguides [Jen *et al*., 1991; Peterson, 1994]. The time reversal technique has been shown to be effective when only the first two longitudinal modes are excited in a solid circular waveguide using a single transducer [Puckett and Peterson, 2002]. However, the ability to extend time reversal to a cylindrical waveguide for which a large number of longitudinal modes propagate using only the information from a single transducer is of interest.

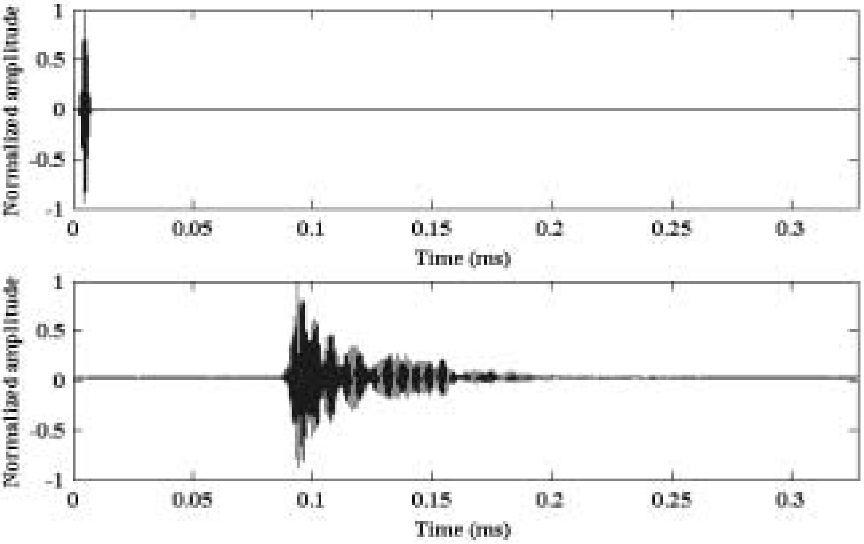

Fig. 1. Illustration of dispersion in a cylindrical waveguide. The top graph is the original signal with compact time domain. The bottom graph is the original signal after propagating through the cylindrical waveguide used in this research.

The stress and displacement of a longitudinal mode may be regarded as having two components. One component is the contribution from the superposition of plane dilatational waves. The second component is the contribution from the superposition of plane transverse waves [Redwood, 1960]. As the frequency increases, there are frequencies where both the dilatational and transverse components are strong. There are also frequencies where one component dominates, including frequencies where the mode is predominately the result of the superposition of plane transverse waves. These changes are exhibited in all of the longitudinal modes.

A single transducer is capable of exciting multiple longitudinal modes in a circular waveguide. For a transducer that is much larger than the waveguide (in this case about 4 times greater in diameter than the waveguide), the pressure distribution across the face of the waveguide can be considered constant with radius. Although the pressure is nearly constant with radius, all of the modes with cutoff frequencies within the spectrum of the signal will propagate. These real modes, along with some imaginary modes and an infinite number of attenuating complex modes are excited to satisfy the boundary conditions on the end of the waveguide [Zemanek, 1972]. The multiple propagating modes are evident in the large time signature in the bottom signal of Fig. 1. The frequency spectrum of the top signal in Fig. 1 and the dispersion curves of the waveguide appear in Fig. 2. From Fig. 1 and Fig. 2, it is evident that multiple dispersive modes are excited and propagated through the waveguide by a single transducer.

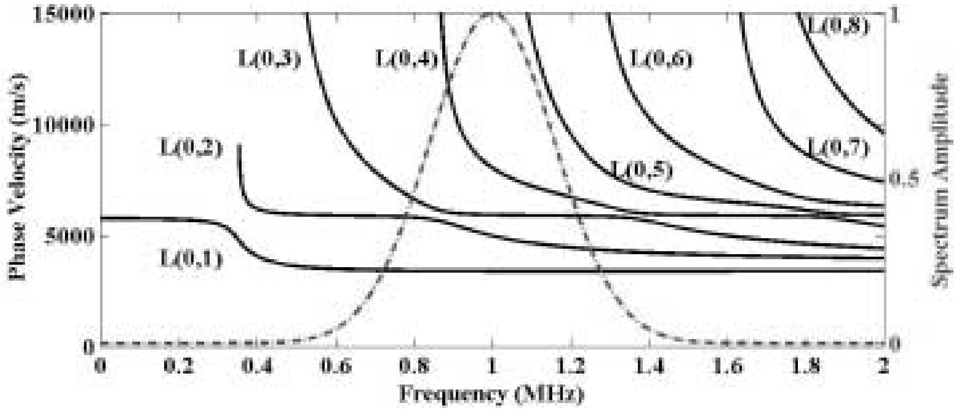

Fig. 2. Dispersion curves for the cylindrical waveguide used in the TRM experiments and the normalized frequency spectrum (dashed) of the signal used to excite the waveguide. The label L(0,N) represents the Nth longitudinal mode.

The signal from a single transducer should include sufficient information from a multi-mode signal to perform an accurate time reversal. As the signal propagates along the waveguide, the modes separate in time, and the complex modes attenuate to negligible amplitude. Thus, the pressure distribution on the receiving end of the waveguide is not constant with radius. The actual pressure distribution on the end of the waveguide at a particular time is the superposition of the normal stress of all of the modes and frequencies present. The transducer does not record the shear stress of any of the modes present. Additionally, the transducer has only the ability to measure the average pressure across the face. It is reasonable though, to assume that the most important information are the frequencies that are present in the received signal and the relative amplitudes of those frequencies. This is the information that is time-reversed and used to excite the transducer.

Thus, this work explores the ability of a TRM with a single transducer that is only capable of sensing the average normal stress even though the transducer can excite the modes that are associated with the superposition of plane transverse waves. A TRM experiment was conducted using single element, longitudinal contact transducers on either end of a solid fused quartz rod. The original excitation signal was compared to the final signal from the TRM experiment to determine the ability of the TRM to reconstruct the original input signal.

2. Methods

The configuration used for the experiments is shown in Fig. 3. The waveguide consisted of a 10 mm diameter, fused quartz cylindrical rod, 485 mm in length. An amorphous material was chosen for the waveguide because linear elastic and homogeneous assumptions are well satisfied. Fused quartz has a Young's modulus, E, of 73 GPa, a density, ρ, of 2200 kg/m^3, and a Poisson's ratio, ν, of 0.14.

Two transducers were used for the experiments. Both transducers were 38 mm diameter, 1 MHz broadband, longitudinal contact transducers [Panametrics, model V194, Waltham, MA]. The transducers had a bandwidth corresponding to a 6 dB drop in amplitude between 0.5 MHz to 1.5 MHz. A coupling fluid was used between the transducers and the waveguide [Sonotech, Inc. UT-30, State College, PA].

An arbitrary waveform generator [Agilent 33250A, Palo Alto, CA] produced the signal to drive the transducer. A radio frequency power amplifier [ENI A-300, Rochester, NY] with a gain of 55 dB was used to amplify the signal to the transducer. The received signal was recorded by a digital storage oscilloscope [Tektronix TDS 520A, Wilsonville, OR] after amplification of the signal by an ultrasonic pre-amplifier [Panametrics model 5660C, Waltham, MA] with a gain of 40 dB.

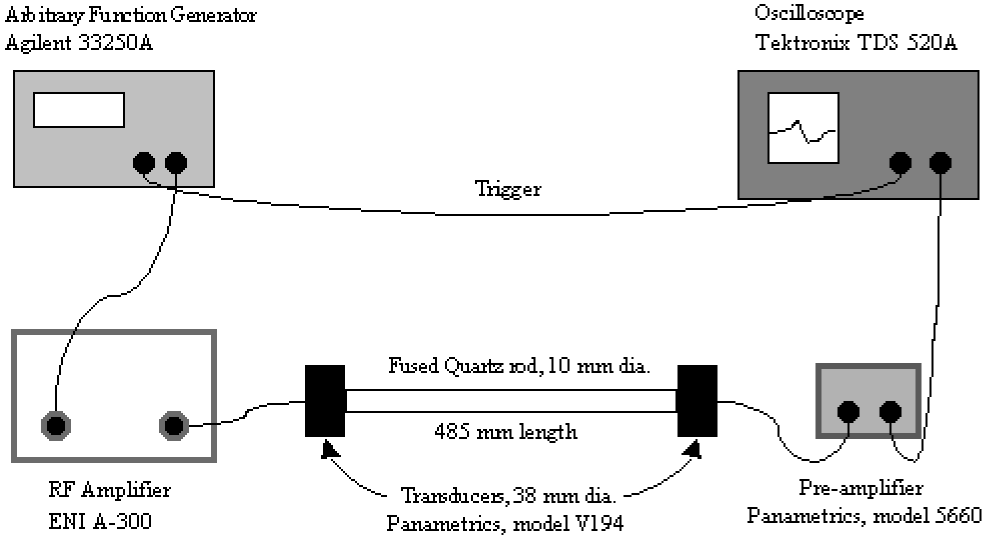

Fig. 3. Diagram of the experimental setup.

The acoustic signal used in the TRM experiments was a broadband signal. The signal had a frequency spectrum with a 40 dB drop in amplitude at 0.5 and 1.5 MHz and a central frequency of 1 MHz (Fig. 2). For the geometry of the waveguide and the frequency spectrum, six propagating longitudinal modes were excited in the waveguide, with a component of each mode being the superposition of plane transverse waves. Fig. 2 shows the dispersion curves calculated for the waveguide used in the experiments.

To ensure the correct signals were recorded, the time window was chosen to include only the initial propagated signal and no end reflections. The excitation signals were repeated at a frequency of 10 Hz to ensure that reflections from previous signals were sufficiently attenuated and were not included in the recorded signal. The recorded signals were averaged over 20 signals to remove noise. Finally, since the waveguide is symmetric about its length, the received signal that is reversed can be excited from the source transducer instead of the receiving transducer to produce the same results. So, for the experiments, all signals were sent from the same end of the waveguide using the same experimental set up.

It was necessary to include the experimental frequency response of the apparatus in the comparison of the original excitation signal to the final signal of the TRM, so the ability of the single element TRM in the waveguide could be determined more accurately. The frequency response includes an amplitude factor and a phase shift for each frequency. However, since the original excitation signal is compared to the final signal of the TRM the phase shift does not need to be known, due to the reversal of the signal in the second step of the TRM experiment. For example, if a signal that propagates through the system is altered by a phase shift of $\varphi(\omega)$, then the reversed signal will have a negative phase shift, $-\varphi(\omega)$. When the system is excited by the reversed signal, the phase shifts will cancel. Since the signal was always propagated from the same source for the TRM experiments, the phase shift was always the same. Therefore, only the amplitude of the frequency response was required to account for the equipment response.

The frequency response of each piece of equipment (RF amplifier, transducers, and ultrasonic pre-amplifier) was measured. The system response function is the convolution of the amplitude factors of each piece of equipment. The ability of the TRM in the waveguide is determined by the comparison of the final signal in the TRM experiment with the original excitation signal convolved with the system response function. For this convolution, the system response was squared because the original excitation signal was propagated through the experimental system twice before becoming the final signal.

3. Results and Conclusion

The signals from the TRM experiments are compared in Fig. 4. All of the signal amplitudes have been normalized, and the signals are plotted with the same time scale. The original excitation signal convolved with the system response function is shown as the top signal of Fig. 4. The bottom four signals in Fig. 4 are the signals from the TRM experiments in the waveguide. The second signal from the top in Fig. 4 is the dispersed signal recorded at the receiving transducer after the excitation signal has propagated through the waveguide. The dispersed signal was reversed in time, as shown in the third signal in Fig. 4, and was used to excite the ultrasonic transducer. The signal second to the bottom in Fig. 4 is the signal recorded at the receiving transducer after the reversed signal is propagated through the waveguide. The bottom signal in Fig. 4 is the previous signal reversed in time for comparison with the first signal. A closer comparison of these two signals appears in Fig. 5.

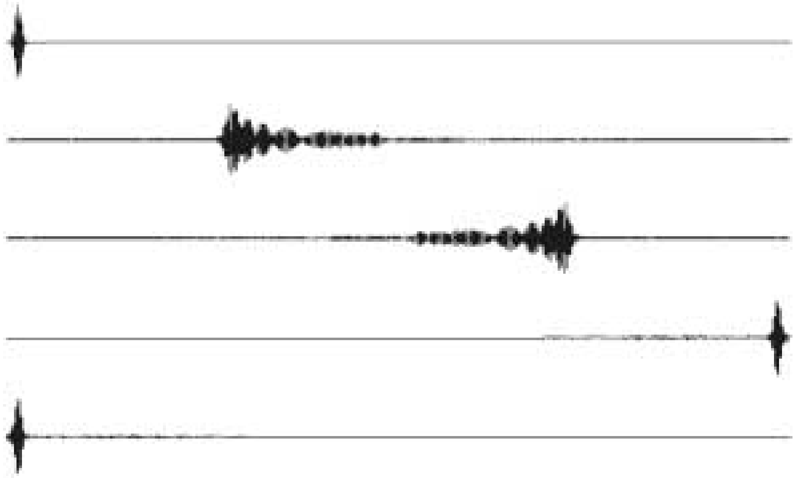

Fig. 4. The TRM experiment in a solid multi-mode waveguide. The signals are normalized and plotted on the same time scale. The signals are, from top to bottom, the original signal convolved with the system response, the dispersed signal, the reversed dispersed signal, the final signal created from the propagation of the reversed dispersed signal, and the final signal reversed in time.

The two signals in Fig. 5 are very similar, with additional noise evident in the experimental signal. The ability of a TRM to reconstruct the original excitation signal using the limited information of a single, longitudinal contact transducer appears to be very good. It was shown earlier that a single, longitudinal contact transducer excited multiple modes in a cylindrical waveguide, including the longitudinal modes that result from the superposition of plane transverse waves. The experimental signal in Fig. 5 implies that a single longitudinal contact transducer appears to be capable of reconstructing a compact time signal from a solid circular waveguide. Thus, the effects of the pressure distribution on the end of the waveguide and the lack of information about the shear stress appear to be minimal.

The most important characteristic of the resulting experimental signal in Fig. 5 is the compact time signature. By using the time-reversed signal as the excitation signal, the dispersive properties of the waveguide can be negated. This capability allows the use of a dispersive solid circular waveguide as a low cost sensor. The compact time domain signal greatly simplifies signal analysis that was previously used [Peterson, 1994].

For a practical application with a single waveguide, the signal that will cancel the dispersive effects of the waveguide is easily determined from the TRM experiment. For more complex configurations where significant changes with time are expected [Jen *et al*., 2001], either modeling or more extensive experiments are required. Future work remains to be done to show that measurements can be made in-situ and to develop appropriate models.

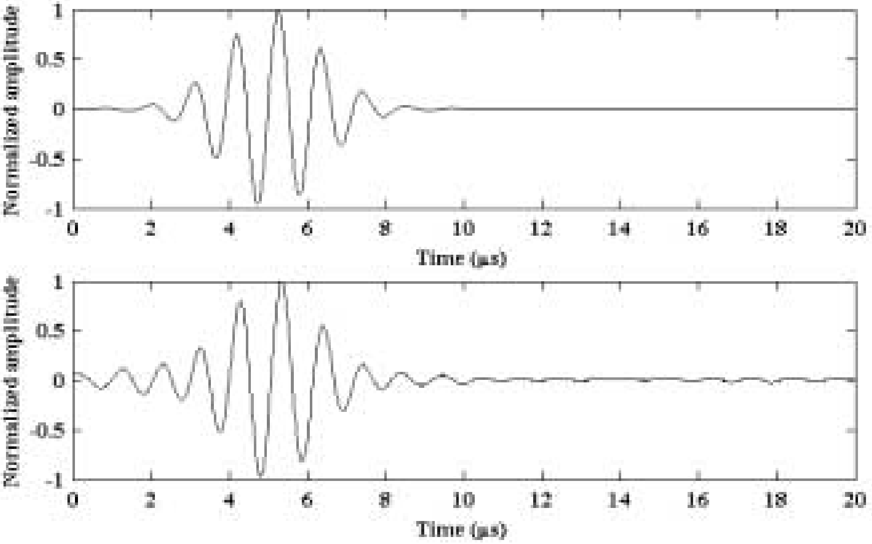

Fig. 5. Comparison of the original signal (top) to the final signal from the TRM experiment (bottom). The original signal has been convolved with system response function.

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