

Black hole demographics from the $M_{\bullet} - \sigma$ relation

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ABSTRACT

We analyse a sample of 32 galaxies for which a dynamical estimate of the mass of the hot stellar component, M_{bulge} , is available. For each of these galaxies, we calculate the mass of the central black hole, M_{\bullet} , using the tight empirical correlation between M_{\bullet} and bulge stellar velocity dispersion. The frequency function $N[\log(M_{\bullet}/M_{\text{bulge}})]$ is reasonably well described as a Gaussian with $\langle \log(M_{\bullet}/M_{\text{bulge}}) \rangle \approx -2.90$ and standard deviation ~ 0.45 ; the implied mean ratio of black hole mass to bulge mass is a factor of ~ 5 smaller than generally quoted in the literature. We present marginal evidence for a lower, average black hole mass fraction in more massive galaxies. The total mass density in black holes in the local Universe is estimated to be $\sim 5 \times 10^5 M_{\odot} \text{Mpc}^{-3}$, consistent with that inferred from high-redshift ($z \sim 2$) active galactic nuclei.

Key words: galaxies: active – galaxies: fundamental parameters – galaxies: kinematics and dynamics.

1 INTRODUCTION

With an ever-increasing number of secure detections, supermassive black holes (BHs) have evolved, in a ten-year span, from exotic curiosities to fundamental components of galaxies. It is now generally accepted that the formation and evolution of galaxies and supermassive BHs are tightly intertwined, from the early phases of protogalactic formation (Silk & Rees 1998), through hierarchical build-up in CDM-like cosmogonies (Efstathiou & Rees 1988; Haehnelt & Rees 1993; Haiman & Loeb 1998; Haehnelt, Natarajan & Rees 1998), to recent galaxy mergers (Merritt 2000). Studying the demographics of the local BH population might have a significant impact on models of galaxy evolution (e.g. Salucci et al. 1999; Cattaneo, Haehnelt & Rees 1999; Kauffmann & Haehnelt 2000).

Magorrian et al. (1998) presented the first, and to date only, demographic study of nuclear BHs. Ground-based kinematic data for 32 galaxies were combined with *Hubble Space Telescope* (*HST*) photometry to constrain dynamical models – based on the Jeans equation – under the assumptions of axisymmetry, velocity isotropy in the meridional plane and a spatially constant mass-to-light ratio for the stars. The mass of a putative nuclear BH was introduced as a free parameter, in addition to the stellar mass-to-light ratio and the galaxy inclination angle. In most of the galaxies, the addition of a central point mass improved the fit to the observed kinematics. Magorrian et al. concluded that most galaxies might contain central supermassive BHs with an average ratio of BH mass to spheroid mass of $M_{\bullet}/M_{\text{bulge}} \sim 10^{-2}$.

The Magorrian et al. study remains unique for targeting a large sample of galaxies, and for its coherent and homogeneous

treatment of the data. However, while the Magorrian et al. estimates of the bulge mass-to-light ratios are likely to be robust, a number of authors have noted that the inferred BH masses might be systematically too large. Van der Marel (1997) showed that the BH masses derived from well-resolved central kinematical data are a factor ~ 5 smaller than produced by the Magorrian et al. analysis; he suggested that the neglect of velocity anisotropy might have led to overestimates of the BH masses. Wandel (1999) compared BH masses derived from reverberation mapping studies of active galaxies with the Magorrian et al. estimates and found a discrepancy of a factor of ~ 20 in the BH-to-bulge mass ratio at a fixed luminosity. He noted the difficulty of resolving low-mass BHs in distant galaxies and suggested a distance-dependent bias in the estimates.

An independent argument along the same lines was presented by Ferrarese & Merritt (2000, hereafter referred to as Paper I). Using the tight empirical correlation between M_{\bullet} and σ , the velocity dispersion of the stellar bulge, for the 12 galaxies with the best-determined BH masses, Paper I showed that the Magorrian et al. masses are systematically high, some by as much as two orders of magnitude.

At the present time, the $M_{\bullet} - \sigma$ relation is probably our best guide to BH demographics. Ferrarese & Merritt (2000) found that the relation has a scatter no larger than that expected on the basis of measurement errors alone. The relation is apparently so tight that it surpasses in predictive accuracy what can be achieved from detailed dynamical modelling of stellar kinematical data in most galaxies. By combining the bulge stellar masses derived by Magorrian et al. with BH masses inferred from the $M_{\bullet} - \sigma$ relation, we are in a position to compute the most robust estimate to date of the BH mass distribution in nearby galaxies.

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Table 1. Physical parameters for the 32 galaxies in the Magorrian et al. sample.

Galaxy	Distance	L_{bulge}	M_{bulge}	σ_c	M_{fit}	$\log x_{\text{fit}}$	M_{\bullet}	$\log x$
N221	0.8 ± 0.1	0.0373	0.00807	76 ± 10	0.0228	-2.53	0.0135	-2.78
N224	0.8 ± 0.0	0.724	0.350	112 ± 15	0.598	-2.75	0.0842	-3.62
N821	24.7 ± 2.5	2.47	1.61	196 ± 26	2.48	-2.81	1.18	-3.13
N1399	20.5 ± 1.6	5.42	3.60	312 ± 41	59.7	-1.79	10.60	-2.53
N1600	68.5 ± 6.6	19.14	16.90	307 ± 40	159	-2.05	9.83	-3.24
N2300	27.0 ± 2.6	3.30	3.40	269 ± 35	23.3	-2.16	5.27	-2.81
N2778	23.3 ± 3.4	0.557	0.359	171 ± 22			0.621	-2.76
N2832	96.8 ± 9.4	14.9	10.56	349 ± 45	123	-1.94	18.00	-2.77
N3115	9.8 ± 0.6	2.31	1.59	278 ± 36	4.74	-2.53	6.15	-2.41
N3377	11.6 ± 0.6	0.891	0.216	131 ± 17	0.713	-2.47	0.176	-3.09
N3379	10.8 ± 0.7	1.69	0.822	201 ± 26	4.29	-2.29	1.33	-2.79
N3608	23.6 ± 1.5	2.51	1.27	206 ± 27	2.87	-2.64	1.49	-2.93
N4168	31.7 ± 6.2	3.28	2.22	185 ± 24	10.4	-2.36	0.900	-3.39
N4278	15.3 ± 1.7	1.90	1.25	270 ± 35	13.6	-1.98	5.36	-2.37
N4291	26.9 ± 4.1	1.65	1.11	269 ± 35	17.5	-1.81	5.27	-2.33
N4467	16.7 ± 1.0	0.0667	0.0355	87 ± 11			0.0256	-3.14
N4472	16.7 ± 1.0	10.87	8.94	273 ± 36	28.5	-2.51	5.65	-3.20
N4473	16.1 ± 1.1	1.85	0.934	188 ± 25	3.48	-2.46	0.971	-2.98
N4486	16.7 ± 1.0	9.12	9.04	345 ± 45	38.6	-2.36	17.04	-2.72
N4486B	16.7 ± 1.0	0.109	0.0358	178 ± 23	10.0	-0.541	0.750	-1.68
N4552	15.7 ± 1.2	2.37	1.56	269 ± 35	4.79	-2.52	5.27	-2.47
N4564	14.9 ± 1.2	0.767	0.416	153 ± 20	2.48	-2.24	0.367	-3.05
N4594	9.9 ± 0.9	5.10	3.08	248 ± 32	7.39	-2.65	3.59	-2.93
N4621	18.6 ± 1.9	4.07	2.34	222 ± 29	3.39	-2.84	2.13	-3.04
N4636	15.0 ± 1.1	3.83	3.16	180 ± 23	2.22	-3.15	0.791	-3.60
N4649	17.3 ± 1.3	7.85	6.00	331 ± 43	44.3	-2.14	14.02	-2.63
N4660	13.2 ± 1.3	0.226	0.118	211 ± 28	2.41	-1.70	1.67	-1.85
N4874	100.9 ± 9.8	26.1	22.3	230 ± 30	225	-2.00	2.51	-3.95
N4889	91.6 ± 8.9	18.2	11.9	373 ± 49	265	-1.68	24.63	-2.68
N6166	131 ± 13	28.2	18.6	311 ± 41	330	-1.75	10.45	-3.25
N7332	23.5 ± 2.3	1.05	0.193	125 ± 16			0.141	-3.14
N7768	114 ± 11	15.62	9.79	224 ± 29	101	-2.03	2.22	-3.64

Note: distances in Mpc, L_{bulge} in $10^{10} L_{\odot}$, M_{bulge} in $10^{11} M_{\odot}$, σ_c in km s^{-1} , M_{fit} and M_{\bullet} in $10^8 M_{\odot}$.

2 DATA

Table 1 gives the relevant physical parameters for the 32 galaxies in the Magorrian et al. sample. All galaxies, with the exception of M31, are early-type. In what follows, we refer to the hot stellar component in these galaxies as the ‘bulge’; this is in fact the case for M31, although for the other objects the ‘bulge’ is the entire galaxy. Distances were re-derived as in Paper I; values for the bulge V-band luminosity (L_{bulge}), bulge mass (M_{bulge}) and BH mass (M_{fit}) are the same as in Magorrian et al. except for the (mostly small) corrections resulting from the new distances.

Central velocity dispersions σ_c were taken from the literature and corrected to a common aperture size of 1/8 of the effective radius, as in Paper I. We then computed BH masses, M_{\bullet} , using the $M_{\bullet} - \sigma_c$ relation in the form given by Merritt & Ferrarese (2001, hereafter Paper II):

$$M_{\bullet} = 1.30 \times 10^8 M_{\odot} (\sigma_c / 200 \text{ km s}^{-1})^{4.72}. \quad (1)$$

This expression was derived by fitting to the combined galaxy samples of Ferrarese & Merritt (2000) (12 galaxies) and Gebhardt et al. (2000a) (15 additional galaxies), plus seven active galaxies for which both σ_c and M_{\bullet} are available, the latter from reverberation mapping (Nelson & Whittle 1995; Di Nella et al. 1995; Smith, Heckman & Illingworth 1990). The slope in equation (1) is fairly uncertain; we explore below how changing the assumed slope affects our conclusions.

The correlations between M_{\bullet} and L_{bulge} , and between M_{\bullet} and M_{bulge} , are shown in Fig. 1. There is a rough proportionality of both L_{bulge} and M_{bulge} with M_{\bullet} , though the vertical scatter in both relations is much larger than in the $M_{\bullet} - \sigma$ relation (Paper I).

We defined the two mass ratios:

$$\begin{aligned} x_{\text{fit}} &\equiv M_{\text{fit}} / M_{\text{bulge}}, \\ x &\equiv M_{\bullet} / M_{\text{bulge}}, \end{aligned} \quad (2)$$

based respectively on the BH mass estimates from Magorrian et al. and from the $M_{\bullet} - \sigma$ relation. Values of $\log x_{\text{fit}}$ and $\log x$ are given in Table 1. BH masses derived from the $M_{\bullet} - \sigma$ relation yield the mean values $\langle x \rangle = 2.50 \times 10^{-3}$ and $\langle \log x \rangle = -2.90$. These are substantially smaller than the mean values computed from the Magorrian et al. BH masses: $\langle x_{\text{fit}} \rangle = 1.68 \times 10^{-2}$ and $\langle \log x_{\text{fit}} \rangle = -2.20$. We note that one galaxy, NGC 4486b, has $\log x_{\text{fit}} = -0.54$, making it an extreme outlier in the Magorrian et al. distribution. Removing this single galaxy from the sample gives $\langle x_{\text{fit}} \rangle = 7.2 \times 10^{-3}$ while leaving $\langle \log x_{\text{fit}} \rangle$ essentially unchanged.

Fig. 2 reveals a clear trend of $M_{\text{fit}} / M_{\bullet}$ with the apparent radius of influence of the central black hole, assuming the masses predicted by the $M_{\bullet} - \sigma$ relation are correct. A natural interpretation is that there is a resolution-dependent bias in the Magorrian et al. modelling (e.g. van der Marel 1997; Wandel 1999): the radius of influence of most of the Magorrian et al. galaxies is smaller than 1 arcsec, too small to have been clearly resolved from the ground.

3 ANALYSIS

We seek an estimate of the frequency function $N(y) = N(\log x)$. Following Merritt (1997), we define this estimate as $\hat{N}(y)$, the

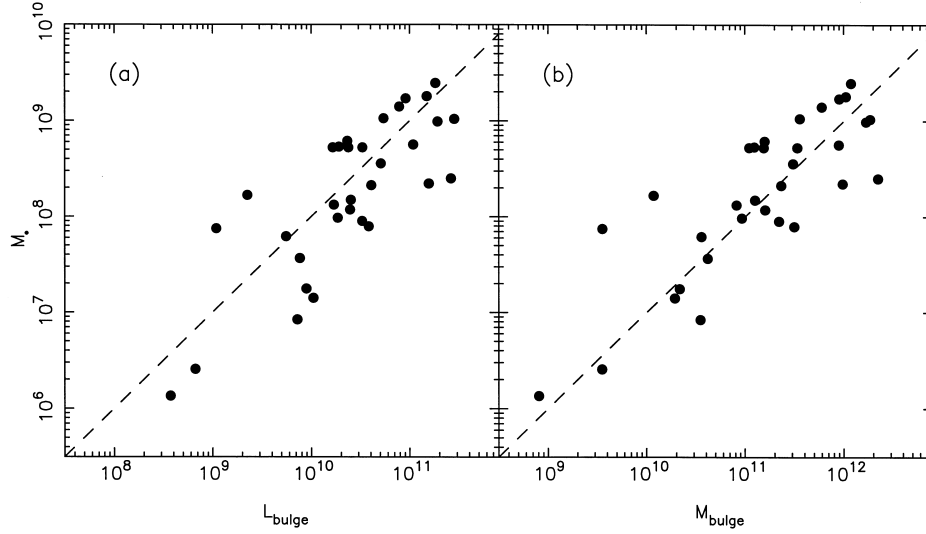


Figure 1. Correlations between black hole mass and (a) V-band bulge luminosity; (b) bulge mass. Masses are in units of solar masses and luminosities in solar luminosities. Dashed lines are $M_{\bullet}/M_{\odot} = 10^{-2} L_{\text{bulge}}/L_{\odot}$ (left panel) and $M_{\bullet}/M_{\odot} = 10^{-3} M_{\text{bulge}}/M_{\odot}$ (right panel).

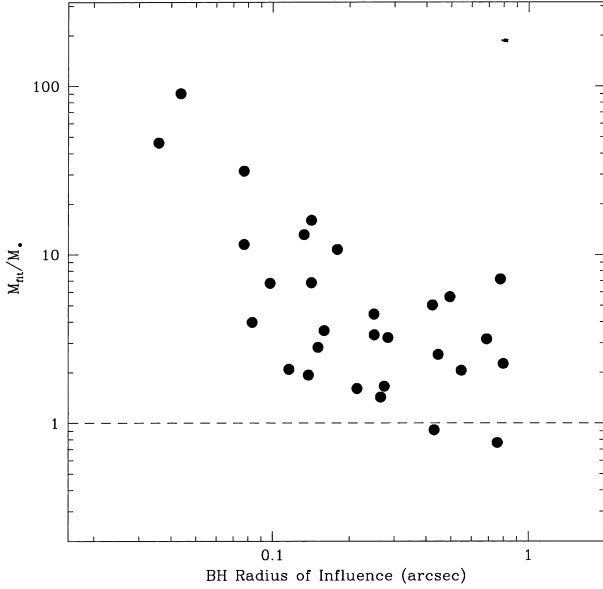


Figure 2. Ratio of black hole mass computed by Magorrian et al. (1998), M_{fit} , to black hole mass computed from the $M_{\bullet} - \sigma$ relation, M_{\bullet} , as a function of the radius of influence of the black hole.

function that maximizes the penalized log likelihood

$$\log \mathcal{L}_p = \sum_{i=1}^n \log(N \circ E)_i - \lambda P(N) \quad (3)$$

of the data y_i , $i = 1, \dots, n$, subject to the constraints

$$\int N(y) dy = 1, \quad N(y) \geq 0. \quad (4)$$

Here $N \circ E$ is the ‘observable’ function, i.e. the convolution of the true N with the error distribution of y . This error distribution is not well known; we assume that it is a Gaussian with some dispersion Δ_y . Failing to account for measurement errors in y would lead to a spuriously broad $\hat{N}(y)$.

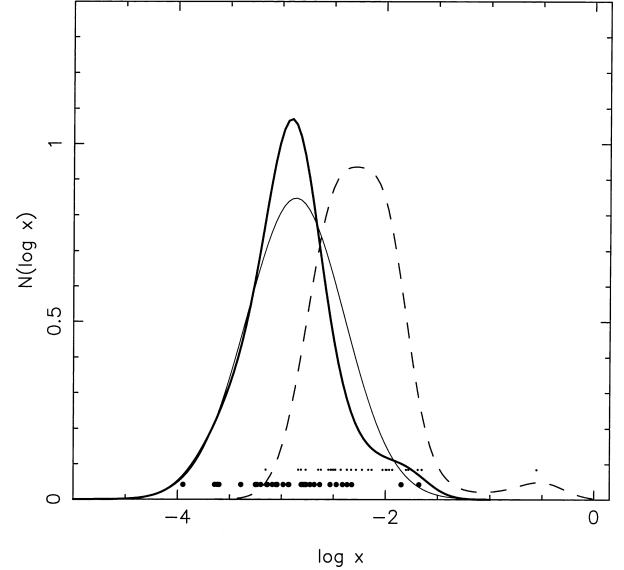


Figure 3. Frequency function of $\log x$ where $x = M_{\bullet}/M_{\text{bulge}}$. The heavy solid line was derived from black hole masses computed via the $M_{\bullet} - \sigma$ relation, equation (1); data are shown as the large dots. The dashed line was derived using the Magorrian et al. black hole masses; the data are shown as the small dots. The thin solid line is the best-fitting Gaussian approximation to $N(\log x)$. Each curve assumes a measurement uncertainty in $\log x$ of 0.15.

The natural penalty function to use is Silverman’s (1982):

$$P(N) = \int_{-\infty}^{+\infty} [(d/dy)^3 \log N(y)]^2 dy. \quad (5)$$

This function assigns zero penalty to any $N(y)$ that is Gaussian. In the limit of large λ , the estimate \hat{N} is driven toward the Gaussian function that is most consistent, in a maximum-likelihood sense, with the data; smaller values of λ return non-parametric estimates of $N(y)$.

The results are shown in Fig. 3, assuming $\Delta_y = 0.15$. $\hat{N}(y)$ is nicely symmetric and reasonably well described as a Gaussian, although with a narrower-than-Gaussian central peak.

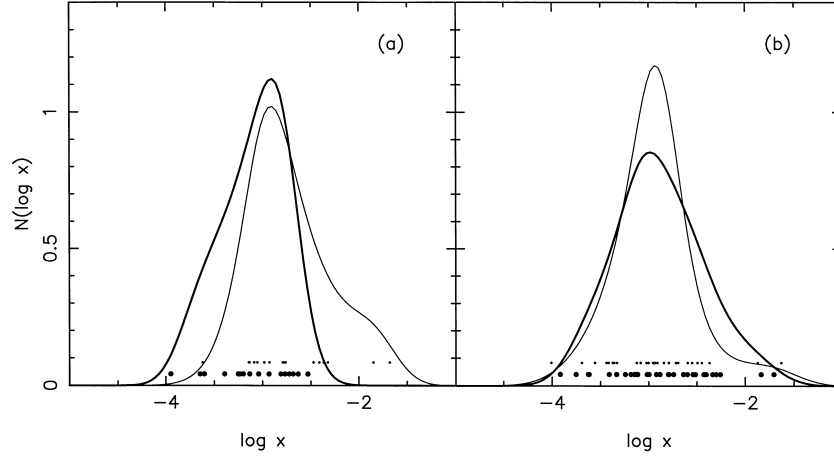


Figure 4. (a) $\hat{N}(\log x)$ computed separately for the high- M_{bulge} (thick line) and low- M_{bulge} (thin line) galaxies in Table 1. (b) Effect of varying the assumed slope of the $M_{\bullet} - \sigma$ relation. Thick line: $\alpha = 5.25$. Thin line: $\alpha = 3.75$. Each curve assumes a measurement uncertainty in $\log x$ of 0.15.

The best-fitting Gaussian has its mean at $y = \log x = -2.93$ and a standard deviation of 0.45.

By contrast, the Magorrian et al. masses define a more flat-topped distribution with one extreme outlier, NGC 4486b, at $\log x_{\text{fit}} = -0.54$. The Gaussian fit to the Magorrian et al. mass distribution has its mode at -2.25 and a standard deviation of 0.52.

The two galaxies with the largest BH mass ratios, NGC 4486b and NGC 4660, are both low-mass ellipticals. The smallest mass ratio, $\log x = -3.95$, is seen in a very massive galaxy, NGC 4874. It is therefore interesting to check whether low- and high-mass galaxies have different characteristic distributions of $\log x$. This hypothesis is tested in Fig. 4(a), which shows $\hat{N}(y)$ computed separately for the 16 galaxies from Table 1 with the lowest and highest values of M_{bulge} . There is in fact a slight difference between the two distributions: the high-mass galaxies have $\langle \log x \rangle = -3.10$ and $\sigma_{\log x} = 0.39$, while the low-mass galaxies have $\langle \log x \rangle = -2.71$ and $\sigma_{\log x} = 0.49$. However the offset in $\langle \log x \rangle$ is similar to the width of either distribution and may not be significant. We note that the massive galaxies define a narrower distribution.

Our conclusions about $N(y)$ might be substantially dependent on the assumed form of the $M_{\bullet} - \sigma$ relation, equation (1). The slope of that relation is fairly uncertain, $\alpha = 4.72 \pm 0.4$; however the normalization at $\sigma_c \approx 200 \text{ km s}^{-1}$ appears to be more robust (Papers I and II). We therefore set

$$M_{\bullet} = 1.30 \times 10^8 M_{\odot} (\sigma_c / 200 \text{ km s}^{-1})^{\alpha} \quad (6)$$

and investigated the effects of varying α . Fig. 4(b) shows that a larger slope implies a broader $N(y)$ because of the stronger implied dependence of M_{\bullet} on σ_c . However the mean value of $\log x$ is almost unchanged.

4 DISCUSSION

Our estimate of the mean BH-to-bulge mass ratio, $\langle \log x \rangle \approx -2.90$, falls squarely between the estimates of Magorrian et al. (1998) (~ -2.28), based on dynamical modelling of the same sample of galaxies used here, and Wandel (1999) (~ -3.50), based on BH masses computed from reverberation mapping in a sample of 18 active galaxies.

Bulge masses in the Wandel (1999) study were computed directly from bulge luminosities assuming a simple scaling law for the mass-to-light ratio, and not from dynamical modelling. There is reason to believe that these luminosities are systematically too large and therefore that the derived mass ratios $M_{\bullet}/M_{\text{bulge}}$ are too low. Gebhardt et al. (2000b) and Merritt & Ferrarese (2001) found that the reverberation mapping BH masses in seven galaxies were consistent with the $M_{\bullet} - \sigma$ relation even though they fall systematically below the $M_{\bullet} - L_{\text{bulge}}$ relation. A reasonable conclusion is that the true or derived luminosities of these active galaxies are systematically higher than those of normal galaxies with comparable velocity dispersions. A mean offset of a factor ~ 4 in the bulge luminosities would suffice to bring the average mass ratio for active galaxies in line with the value inferred here. Gebhardt et al. (2000b) discuss a number of possible reasons why an error of this sort is likely in the active galactic nucleus (AGN) bulge luminosities.

The discrepancy with the Magorrian et al. (1998) masses is perhaps unsurprising given past indications that these masses are systematically too large (van der Marel 1997; Ho 1999). The difference between $\langle \log x \rangle$ and $\langle \log x_{\text{fit}} \rangle$ corresponds to a factor of ~ 5 average error in the Magorrian et al. BH masses. One possible explanation is the neglect of anisotropy in the modelling (van der Marel 1997), but we emphasize that the errors in M_{fit} implied by Fig. 2 are enormous, of the order of 10–100, in many of the galaxies. If the BH masses predicted by the $M_{\bullet} - \sigma$ relation are correct, the kinematical data for these galaxies would not have contained any useful information about the mass of the BH. This conclusion, if correct, underscores the dangers of an ‘assembly-line’ approach to galaxy modelling.

We may crudely estimate the total mass density of BHs in the local universe by combining our result, $M_{\bullet}/M_{\text{bulge}} \sim 1.3 \times 10^{-3}$, with the mean mass density of spheroids, $\rho_{\text{bulge}} \sim 3.7 \times 10^8 M_{\odot} \text{ Mpc}^{-3}$ (Fukugita, Hogan & Peebles 1998, for $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$). This simple argument (first invoked by Haehnelt et al. 1998) gives $\rho_{\bullet,L} \sim 4.9 \times 10^5 M_{\odot} \text{ Mpc}^{-3}$. Salucci et al. (1999) presented a more sophisticated treatment based on convolution of the spheroid luminosity function with $N(\log x)$. They assumed a Gaussian distribution with $\langle \log x \rangle = -2.60$ and found $\rho_{\bullet,L} \sim 1.7 \times 10^6 M_{\odot} \text{ Mpc}^{-3}$. Correcting their value of $\langle \log x \rangle$ to our value of -2.90 implies a factor of ~ 2 decrease in $\rho_{\bullet,L}$, consistent with the result of our simpler calculation.

The total mass density of BHs at large redshifts can be estimated using an argument first suggested by Soltan (1982). Requiring the optical QSO luminosity function to be reproduced purely by accretion on to nuclear BHs, and assuming an accretion efficiency of 10 per cent, leads to $\rho_{\bullet,z} \sim 2 \times 10^5 M_{\odot} \text{Mpc}^{-3}$ (Chokshi & Turner 1992; Salucci et al. 1999). While independent of the cosmological model, this result is subject to uncertainties in the bolometric corrections applied to the QSO magnitudes (e.g. Salucci et al. 1999); furthermore, concerns have been raised about the completeness of the QSO luminosity function (e.g. Goldschmidt & Miller 1998; Graham, Clowes & Campusano 1999). A similar argument, based on the hard X-ray background, gives $\rho_{\bullet,z} \sim 3\text{--}4 \times 10^5 M_{\odot} \text{Mpc}^{-3}$ at $z \sim 1.5$ (Fabian & Iwasawa 1999; Salucci et al. 1999; Barger et al. 2001). These numbers are consistent with our estimate of $\rho_{\bullet,L}$.

By contrast, $\rho_{\bullet,z}$ differs from the local BH mass density implied by the Magorrian et al. relation by over an order of magnitude, assuming a canonical 10 per cent accretion efficiency on to the central black hole in high-redshift AGNs. Haehnelt et al. (1998) and Barger et al. (2001) point out that if the remnants of the QSOs are to be identified with the BHs in present-day galaxies, the Magorrian et al. mass distribution requires either that a large fraction of BHs reside within high redshift sources that are too obscured (both in the optical and the X-rays) to be observed, or else that a significant amount of accretion (with low radiative efficiency) proceeds to the present epoch. The need for these alternative explanations is largely removed when the more robust estimate of $\rho_{\bullet,L}$ presented in this paper is adopted.

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