Gamma-ray pulsars: the pulse profiles and phase-resolved spectra of Geminga

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ABSTRACT

We present a calculation of a three-dimensional pulsar magnetosphere model to explain high-energy emission from the Geminga pulsar with a thick outer gap. High-energy γ-rays are produced by primary accelerated particles with a power-law energy distribution through curvature radiation inside the outer gap. We also calculate the emission pattern, pulse profile and phase-resolved spectra of high-energy γ-rays of the Geminga pulsar, and find that its pulse profile is consistent with the observed one if the magnetic inclination and viewing angle are ~50° and ~86° respectively. We describe the relative phases among soft (thermal) X-rays, hard (non-thermal) X-rays, and γ-rays. Our results indicate that X-ray and γ-ray emission from the Geminga pulsar may be explained by the single thick outer gap model. Finally, we discuss the implications of the radio and optical emission of the Geminga pulsar.

Key words: radiation mechanisms: non-thermal – pulsars: general – pulsars: individual: Geminga – gamma-rays: theory.

1 INTRODUCTION

It is generally believed that high-energy γ-ray pulsars have complex pulse profiles. Using COS-B data, the spectral properties of the pulse component of the Crab and Vela pulsars have been analysed (Clear et al. 1987; Grenier, Hermsen & Clear 1988). Up to now, the Compton Gamma Ray Observatory (CGRO) telescopes have detected pulsed γ-ray radiation from at least seven spin-powered pulsars: Crab (PSR B0531+21), Vela (PSR B0833−45), Geminga (PSR J0633+1746), PSR B1509−58, PSR B1706−44, PSR B1951+32 and PSR B1055−52 (for a summary of observational results for pulsars, see, e.g., Thompson et al. 1996, 1999). The observed pulse profiles of high-energy γ-ray pulsars are different. Crab, Vela, Geminga and PSR B1951+32 pulse profiles are all characterized by two narrow pulses separated by 0.4−0.5 in phase. PSR B1055−52 has two broader pulses with a phase separation of about 0.2 (Thompson et al. 1999), similar to PSR B1706−44 (Thompson et al. 1996). Unfortunately, only three γ-ray pulsars, Crab, Vela and Geminga, have sufficient photon counting to perform a meaningful phase-resolved study. Recently, Fierro et al. (1998) analysed the phase-resolved emission characteristics of these three γ-ray pulsars using EGRET observations. They have obtained detailed pulse profiles and phase-resolved spectra of these γ-ray pulsars. Theoretically, there are two kinds of γ-ray pulsar models. One is the polar gap models (e.g. Harding 1981, Dermer & Sturmer 1994, Sturmer & Dermer 1994, Daugherty & Harding 1996 and Zhang & Harding 2000), and the other is the outer gap models (e.g. Cheng, Ho & Ruderman 1986a,b, hereafter CHR I and CHR II; Ho 1989; Chiang & Romani 1992, 1994; Romani & Yadigaroglu 1995, hereafter RY; Romani 1996; Zhang & Cheng 1997, hereafter ZC; Zhang & Cheng 1998). Here we will focus on the outer gap models.

In order to explain the observed pulse profiles and phase-resolved spectra of γ-ray pulsars, three-dimensional outer gap models have been proposed. There are two versions of these models. One is given by Romani and his co-workers (Chiang & Romani 1992, 1994; RY; Romani 1996). Chiang & Romani (1992, 1994) explored and modified a CHR-type model for a three-dimensional magnetosphere. They assumed that the outer gaps exist along all field lines on the boundary of the closed region instead of just on the bundle of field lines lying in the plane of the rotation and magnetic dipole axes. In the static magnetic field approximation, they showed that a single pole connected only the outgoing ‘curvature radiation’ particle flow, which is the same on each field line, through the null surface will produce a broad, irregularly shaped beam of emission which is particularly dense near the edge. Then two γ-ray peaks will be observed when the line of sight from the Earth crosses these enhanced regions of the γ-ray beam, while the inner region of the beam provides a significant amount of emission between the peaks. A wide range of peak phase separations can be accommodated with a proper choice of the observer co-latitude. Further, RY developed a model for the beaming of high-energy γ-ray emission from a single outer gap of a rotating inclined dipole. They assumed that very little emission is beamed inwards along the field lines, and that the field lines are those which would exist if there were no particles present. Although no detailed spectra or luminosities have been calculated for this model, their studies help in modelling the phase...
emission features. Romani (1996) has successfully applied this model to explain the pulse profile and phased-resolved spectra of the Vela pulsar. The other version of the three-dimensional outer gap model is given by Cheng, Ruderman & Zhang (2000). In their model, the three-dimensional geometry of the outer gap is determined by various physical processes (including pair production, which depends sensitively on the local electric field and the local radius of curvature, surface field structure, reflection of $e^\pm$ pairs because of mirroring and resonant scattering). They have shown that two outer gaps and both outgoing and incoming currents are in principle allowed, but it turns out that outgoing currents dominate the emitted radiation intensities. The observed currents are in principle allowed, but it turns out that outgoing currents dominate the emitted radiation intensities. The observed features of the Crab pulsar can be well explained by this model.

ZC proposed a self-sustained outer gap model of $\gamma$-ray emission from rotation-powered pulsars. In their model, the fractional height of the outer gap ($f^0_0$) in ($\Omega$, $\mu$) plane is limited by the pair production between the thermal X-rays with energy $E_X(f^0_0)$ from the stellar surface and the $\gamma$-ray photons with energy $E_\gamma(f^0_0)$ emitted by the primary electrons/positrons accelerated in the outer gap. Therefore the fractional size can be estimated for given pulsar parameters (period and surface magnetic field). For the Geminga pulsar, the fractional size of the outer gap is ‘thick’ ($f^0_0 \approx 0.7$), so the $\gamma$-rays are produced inside the outer gap. This model can explain the phase-averaged spectra of known $\gamma$-ray pulsars except for the Crab pulsar. Combining this model and the three-dimensional outer gap model given by Cheng et al. (2000), the present work is a detailed study of the light curve and phase-resolved spectra of the Geminga pulsar. In Section 2 we describe the three-dimensional model. In Section 3 we applied this model to the Geminga pulsar. Finally, a brief discussion is given in Section 4.

2 THE OUTER GAP MODEL

The outer gaps, powerful acceleration regions, can form in the vicinity of ‘null charge surface’ ($\Omega \cdot B = 0$) (Holloway 1973; Cheng, Ruderman & Sutherland 1976), because the charged carriers on either side of the null charge surface have opposite charges. In fact, the charge density of the magnetosphere in the corotating frame of a neutron star is (Goldreich & Julian 1969)

$$p^0_0 \approx (\Omega \cdot B) / 2mc,$$

where $B$ and $\Omega$ are the magnetic field and angular velocity of the neutron star. The charge density will change sign when a current flows through the null surface where $\Omega \cdot B = 0$. As a result, a charge-deficient region ($p^0_0 = 0$) in the outer magnetosphere near the null surface will be formed. Any deviation of the charge density from $p^0_0$ results in an electric field along $B$. This electric field can become strong enough to accelerate $e^\pm$ pairs to ultrarelativistic energies. These $e^\pm$ pairs could radiate $\gamma$-rays tangential to the curved $B$ field lines. These ‘curvature $\gamma$-rays’ are further converted into $e^\pm$ pairs via $\gamma + \gamma \rightarrow e^+ + e^-$. Therefore, in order to keep a steady state current flow and the charge density $p^0_0$ in the regions outside the gap, the gap will grow until it is large enough and the electric field is strong enough to maintain a copious supply of charges to the rest of the open field line region. If the gap ends in a region $p^0_0 \neq 0$, charges from the surrounding region will flow in through the end. If both ends are located on the null surface, any $e^\pm$ pairs produced in the gap will act to replace the charge deficiency inside the gap, and finally the gap will be filled up. However, if a vacuum gap extends to the light cylinder, charged particles created in the gaps will escape from the magnetosphere, so the gap will not be quenched.

Hence, stable outer gaps (if they exist) are those from the null surface to the light cylinder along the last closed field lines. In each outer gap, the inner boundary of the outer gap lies near the intersection of the null surface where $p = 0$ and the boundary of the closed field lines of the star on which the magnetosphere current does not flow. The thickness of the outer gap is bounded from above by a layer of electric current which contributes a surface charge density (CHR I).

According to CHR I, four outer gaps exist in the open zone in the plane of ($\Omega$, $\mu$) (two of them are topologically connected in three-dimensional space), but only two longer outer gaps should give the observed fan beams. They argued that these two longer outer gaps may create enough $\gamma$-rays and $e^\pm$ pairs to quench the two shorter, less powerful ones. In the CHR model, the $\gamma$-ray emission is approximated to occur only along the last closed field line in the plane of the dipole and rotation axes. Because the charged particles of both positive and negative charges are accelerated in the gap, the emissions should beam both towards and away from the pulsar. Therefore the observed fan beams consist of those coming from different gaps, and the measured phase separation between the two peaks is determined by the time travel difference between these two outer gaps, the relativistic aberration of emission, and the bending of the magnetic field lines near the light cylinder. The emission from each peak is highly cusped because of the relativistic aberration, so there will be some bridge emission but very little other off-pulse emission. Obviously, the pulse profiles of CHR model are not consistent with those observed. So the three-dimensional description of the outer gaps is necessary.

Cheng et al. (2000) have shown that, in principle, there can exist two outer gaps in the open volume of the pulsar magnetosphere. However, it may be common that there is a single outer gap in the pulsar magnetosphere, as pointed out by Cheng et al. (2000). In fact, if the surface magnetic field of some pulsars has a sunspot-like geometry as suggested by Ruderman (1991), then an off-centre dipole implies that one of the outer gaps should be de-activated by polar gap activity, because some of the high-energy photons emitted by polar gap primary charged particles will be deflected by the gravitational field of the star and be converted to pairs on the magnetic field lines which are connected with one of the two outer gaps. This effect is not limited to an off-centre, sunspot-like dipolar field geometry which results in light deflection by gravity. For example, strong multipole fields can give surface magnetic field lines bending downward. Some photons emitted tangentially to the field lines in the polar gap accelerator will then make $e^\pm$ pairs flowing away on field lines leading to the outer gap accelerator and quench it (Jones 1986; Ruderman & Cheng 1988). Therefore, in general, a single outer gap accelerator, or even none, could be a common structure in the outer magnetospheres of pulsars whose $\Omega$ and $\mu$ are sufficient to sustain outer-gap accelerators. Below we consider only the high-energy emission produced in a single outer gap. Fig. 1 shows a schematic description of the single outer gap model.

In order to determine three-dimensional outer gaps in the outer magnetosphere of a pulsar with a thick outer gap, we have considered, following RY, a rotating magnetic dipole where the spin axis of the pulsar is on the $z$-axis and the magnetic axis is in the ($x$, $z$) plane. High-energy photons are assumed to be emitted along the magnetic field, and aberration and travel time effects are taken into account for the photon emission (details of the three magnetospheric structures can be found in Cheng et al. 2000).

Moreover, we use the self-consistent pair-production scheme
long as the centre of mass energy of the X-ray and curvature photons is higher than the threshold energy for electron/positron pair production, i.e., $E_X E_X^\gamma (f_0) = (m_e c^2)^2$. Because of this photon–photon pair production, the size of the outer gap limited by the soft thermal X-rays from the neutron star surface is determined as

$$ f_0 \approx 5.5 P^{26/21} B_1^{4/7} \xi^{1/7}, $$

where $f_0 = h_0 / R_L$, $h_0$ is the average vertical extension (perpendicular to the magnetic field) of the outer gap, and $R_L$ is the light cylinder radius of the pulsar. In the three-dimensional case, $h_0$ is the average separation between the two equipotential surfaces which define the boundary of the outer gap. In the ZC model, $\xi$ was chosen to be unity. We consider equations (1) and (2) as a zero-order estimate of the structure of an outer gap. It should be pointed out that ZC have discussed a natural source of soft X-rays produced by a neutron star cooling mechanism. According to the standard cooling models without rapid cooling mechanisms, e.g., kaon condensation or the direct Urca process (e.g. Tsuruta 1992), neutron stars with age $\sim 10^6$ yr should have a surface temperature $\sim 10^8$ K, which is comparable to that produced by polar cap heating soft X-rays. Therefore, either $f_0$ or $f_1$ will give a similar gap size. On the other hand, when the age of the star is greater than $10^6$ yr, the stellar temperature will drop to $10^6$ K. In this case, the surface temperature of a neutron star without any heating mechanism will be lower than that given by equation (1). However, because of various possible heating mechanisms, e.g., superfluid–normal crust frictional heating (Alpar et al. 1984; Shibazaki & Lamb 1989), crustal cracking (Cheng et al. 1992), etc., the surface temperature of the neutron star may be comparable to that given by equation (1). However, the exact stellar surface temperature due to the heating mechanisms is unknown, because it depends on various internal parameters, e.g., the critical lag between superfluid and normal matter, the critical strain angle, the mass of the inner crust, etc., which are poorly known. Therefore ZC assumed that if soft X-rays are dominated by neutron star cooling, the size of the outer gap is $f_c = 4.5 P^{17/8} B_1^{2/3} T_c^{-2/3}$, where $T_c = 10^6 T_{\text{co}}$ K is the surface temperature of the star determined by the cooling model. Though lacking decisive evidence for determining the origin of the soft X-ray from older pulsars (e.g., Geminga), different models will predict a rather similar surface temperature, which affects the size of the outer gap by only a factor of $(T_c/T_d)^{2/3}$ and does not alter the qualitative picture of our model.

Inside a thick outer gap, according to ZC, local electric field along the magnetic field lines is approximated as $E_{\perp} = f_2^\gamma (r/R_L)^{1/2} B_{\perp}$. Therefore the primary $e^\pm$ pairs in the steady state have a power-law distribution with a spectral index 16/3, because the Lorentz factor of the accelerated particles is $\gamma_e \propto (r/R_L)^{-1/3}$. These accelerated particles emit high-energy gamma-rays through curvature radiation inside the outer gap. ZC have shown that this model can explain observed phase-averaged spectra of known gamma-ray pulsars except for the Crab pulsar.

We now consider the extensions of the outer gap along radial and azimuthal directions. According to ZC model, in the $(\Omega, \mu)$ plane, $E_{\gamma} \propto r^{-1/3}$. Such a strong dependence of $E_{\gamma}$ on $r$ will make the cross-section for pair production decrease rapidly. Since pair production must take place at the beginning of the outer gap (otherwise there is no current passing through the outer gap), we argue that the radial extension of the outer gap must be

$$ r_{\text{lim}} = f_4 R_L. $$

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**Figure 1.** Schematic description of the single-pole outer gap model. One outer gap exists in the outer magnetosphere because of an off-centre sunspot-like dipole field which results in light deflection by gravity, or a strong multipole field near the star surface. $\gamma$-rays are emitted both outward and inward from an outer gap, hard non-thermal X-rays are emitted inwards through synchrotron radiation near the star surface, and soft X-rays are emitted outward from the star surface.

**2.1 The structure of an outer gap**

In order to develop the three-dimensional outer gap model of gamma-ray pulsars with thick outer gaps, we introduce briefly the basic features of the ZC model first. In the ZC model, the $e^\pm$ pairs needed to control the height of the outer gap are produced by photon–photon pair production resulting from collisions between the curvature photons from the outer gap and the thermal X-rays from the neutron star surface. These thermal X-rays are created by the collision of the backflowing current with the neutron star surface. The return particle flux can be approximated by $N_{\text{rec}} = \frac{1}{2} f_\gamma N_{\text{GJ}}$, where $N_{\text{GJ}}$ is the Goldreich–Julian particle flux (Goldreich & Julian 1969). Although most of the energy of the primary particles will be lost on the way to the star via curvature radiation, about 10.6$p^{1/2}$ erg per particle will still remain and finally deposit on the stellar surface. This energy will be emitted as X-rays from the stellar surface (Halpern & Ruderman 1993). The characteristic energy of X-rays is $E_{X} = 3kT = 1.2 \times 10^3 P^{-1/6} B_{12}^{1/4}$ eV. The keV X-rays from a hot polar cap will be reflected back to the stellar surface by cyclotron resonance scattering if there is large density of $e^\pm$ pairs near the neutron star surface (Halpern & Ruderman 1993; Zhu & Rudner 1997), and eventually be re-emitted as soft thermal X-rays with characteristic temperature

$$ T_s \approx 3.8 \times 10^5 f_0^{0.4} P^{-5/12} B_{12}^{1/4} \xi^{0.4} K, $$

where $\xi = \Delta \Phi / 2\pi$ is the azimuthal width of the three-dimensional outer gap, and $\Delta \Phi$ is the transverse extension of the outer gap.

Every pair produced by X-ray and curvature photons collision can emit $10^3$ photons in the outer gap. Such a huge multiplicity can produce sufficient number of $e^\pm$ pairs to sustain the gap as

$\epsilon_{\gamma} = \frac{\alpha_{\text{pair}} \cdot \alpha_{\text{sc}}}{1 + \alpha_{\text{pair}} \cdot \alpha_{\text{sc}}} \approx 1$. 

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In other words, the radial range of the pair production region is limited to a certain region \((r_{\text{min}} - r_{\text{max}})\) of the outer gap region, where \(r_{\text{min}}\) is the distance of the null surface which is a function of \(\phi\), since \(r_{\text{min}} - r_{\text{max}}(\phi) \equiv 0\). This gives the extent of the outer gap along azimuthal direction, \(\Delta \phi \approx 180^\circ\) for the Geminga pulsar.

Therefore, for the structure of the thick outer gap in the open field line volume of the pulsar magnetosphere, the gap height is determined by \(f_0\), the azimuthal extension of the outer gap is \(\approx 180^\circ\), and the extension along the magnetic field is from \(r_{\text{min}}\) to \(R_L\). However, both outgoing and incoming e\(^+\) flows along the magnetic field lines occur in the region \(\Delta \phi \approx 180^\circ\), and only outgoing flows in region \(R_L\). So, the radius of inner boundary of the outer gap, \(r_{\text{in}}\), is estimated by \(\Omega \cdot B = 0\) and \(r_{\text{in}} = (4/9)R_L/\tan^2 \alpha\) if the inclination angle \(\alpha\) is large.

2.2 Photon emission morphology and the pulse profile

In order to show the high-energy photon morphology inside an outer gap, we need a polar cap shape for a given magnetic inclination. The polar cap shape defines the boundary of the open volume at the stellar surface. Because the outer gap is within the open volume, we divide the open volume into many parts, in which the shape of each part at the stellar surface is the same as the polar cap shape, but the size is smaller. First, we determine the coordinate values \((x_{\text{0}}, y_{\text{0}}, z_{\text{0}})\) of the last closed field lines at the stellar surface. Then the coordinate values \((x', y', z')\) for different parts can be determined by using \(x_{\text{0}} = a_{x}x_{\text{0}}, y_{\text{0}} = a_{y}y_{\text{0}}\) and \(z'_{\text{0}} = [1 - (x'^2_{\text{0}} + y'^2_{\text{0}})]^{1/2}\), and by changing \(a_{x}\).

Once the polar cap shape is defined, we find the last closed field lines using Runge-Kutta integration: (i) take a set of initial values at the stellar surface; (ii) use the Runge-Kutta integration to follow a particular field line in space and determine whether this line closes inside the light cylinder or crosses it, and (iii) iterate on the initial value until the field line is just tangential to the light cylinder. Furthermore, we assume that relativistic charged particles in the open zone radiate in their direction of propagation, i.e., along the magnetic field lines in the corotating frame. For each location within the open zone the direction of emission expressed as \((\zeta, \Phi)\) is calculated, where \(\zeta\) is the polar angle from the rotation axis, and \(\Phi\) is the phase of rotation of the star. In these calculations, the effects of travel time and aberration are taken into account. A photon with velocity \(\mathbf{u} = (u_x, u_y, u_z)\) along a magnetic field line with a relativistic addition of velocity along the azimuthal angle gives an aberrated emission direction \(\mathbf{u}' = (u'_x, u'_y, u'_z)\). The travel time gives a change of the phase of the radiation in the \((x, y)\) plane from the centre of the star, \(\zeta\) and \(\Phi\) are given by (Yadigaroglu 1997)

\[
\cos \zeta = u'_x, \quad (4)
\]

\[
\Phi = -\Phi_0 - r \cdot \mathbf{u}' ,
\]

where \(\Phi_0\) is the azimuthal angle of \(\mathbf{u}'\), and \(r\) is the emitting location in units of \(R_L\). We project photon emissions on to the \((\zeta, \Phi)\) plane and observe the emission patterns on the sky. In the \((\zeta, \Phi)\) plane, the null surface can be determined exactly, because it consists of the points at which magnetic field lines are perpendicular to the rotation axis. For a field line, the null charge crossing is the location which the projected line crosses the equatorial line (\(\zeta = 90^\circ\)).

2.3 High-energy \(\gamma\)-ray emission from the thick outer gap

Because the outer gap is thick, high-energy emission is produced mainly in the outer gap. ZC used the synchro-curvature mechanism (Cheng & Zhang 1996) to describe the high-energy emission inside the outer gap. Here, for simplicity, we use the curvature mechanism. We assume that the gap height is almost constant (i.e., \(f_0\) is constant). The curvature spectrum for a single electron is

\[
F_{\text{cur}}(E_\gamma) = \frac{\sqrt{3}e^2}{h} \frac{\gamma}{s} \frac{F(y)}{E_\gamma}, \quad (5)
\]

where \(E_\gamma\) is photon energy, \(e\) is the electron charge, \(h\) is Planck’s constant, \(s\) is the radius of curvature, \(F(y) = \int_0^y K_{5/3}(z)\; dz\) with \(y = E_\gamma/E_{\text{cut}}\), and \(K_{5/3}\) is the modified function of order \(5/3\). According to ZC, the primary \(e^+\) pairs in the steady state have a power-law distribution with a spectral index of 16/3, i.e., \(dN_\gamma/dE_\gamma \propto E^{-16/3}\), and the Lorentz factor of accelerated particles is a function of radius (i.e., \(\gamma_c \propto r^{-3/8}\)). The \(\gamma\)-rays are produced by the primary accelerated particles through curvature radiation, which depends sensitively on the local curvature. Assuming that \(x = s/R_L = (r/R_L)^{1/2}\), the distribution of the accelerated particles at radius \(r_c\) satisfies

\[
dN_\gamma = \frac{N_0}{s^{9/4}} \delta(x - x_c). \quad (6)
\]

Therefore the curvature spectrum for the accelerated particles with a power-law distribution at radius \(r_c\) is

\[
F_{\text{cur}}(E_\gamma, x_c) = F_0 \xi^{-5/3}E_\gamma^{-5/3}F(y_c), \quad (7)
\]

where \(F_0\) is a constant, and \(y_c = E_\gamma/E_{\text{cut}}(r_c)\). From equation (7), we can calculate the \(\gamma\)-ray spectrum at any radial distance of the outer gap. For given magnetic inclination angle and viewing angle, we calculate the radial distances for different \(a_\phi\) values, and then calculate the spectrum at each radial distance. Finally, the spectrum of high-energy \(\gamma\)-rays from a given emission region is the sum of the spectra of the \(\gamma\)-rays produced in each radial distances of the emission region.

3 APPLICATION TO GEMINGA PULSAR

We now apply this model to explain the pulse profile and phase-resolved spectra of the Geminga pulsar. Geminga (PSR J0633+1746) is a pulsar with a period of 237 ms and a period derivative of \(-1.1 \times 10^{-14}\) s s\(^{-1}\). Its period was first determined by using ROSAT observations (Halpern & Holt 1992) and EGRET observations (Bertsch et al. 1992). Using measured period and period derivative, Geminga has a characteristic age of \(\sim 3.4 \times 10^5\) yr and its spin-down power is \(\sim 3.3 \times 10^{32}\) erg s\(^{-1}\). Recently, Fierro et al. (1998) have presented the observed light curve and phase-resolved spectra of high-energy \(\gamma\)-rays of Geminga pulsar detected by EGRET. The observed pulse profile by EGRET indicates that the phase separation is 0.49 ± 0.05 (Fierro et al. 1998). In order to obtain the observed phase-resolved spectra, Fierro et al. divided Geminga pulsar phase into eight parts: leading wing 1 (LW1), peak 1 (P1), trailing wing 1 (TW1), bridge, leading wing 2 (LW2), peak 2 (P2), trailing wing 2 (TW2) and offpulse (OP). The phase intervals of all these parts are 0.11, 0.09, 0.11, 0.15, 0.13, 0.13, 0.08 and 0.21 respectively. They have obtained the spectra for these different phases and shown that changes of spectral indexes for different profile components.

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According to the model described in Section 2, the fractional height is $f_0 \sim 0.7$, so the $r_{\text{lim}} \sim 0.7R_L$ and $\Delta \Phi \sim 180^\circ$. Based on the ZC model, Cheng & Zhang (1999) proposed a model of X-ray emission from rotation-powered pulsars. They applied this model to the Geminga pulsar and found that the magnetic inclination angle is $\sim 50^\circ$. Here, we use this value of the magnetic inclination angle. Then, assuming an uniform emissivity of $\gamma$-rays, we solve the emission pattern by following the method in Section 2.2. In panel A of Fig. 2, the $\gamma$-ray emission from a single outer gap in $(\xi, \Phi)$ plane is shown. Because azimuthal and radial limits of the outer gap by photon–photon pair production, the $\gamma$-rays emitted outwards (solid curves) from the outer gap are mainly above $\xi = 90^\circ$ in the $(\xi, \Phi)$ plane, but the $\gamma$-rays emitted inwards (dotted curves) are mainly below $\xi = 90^\circ$ [N.B., the inward $\gamma$-rays are emitted in the range ($r_{\text{lim}} - r_{\text{in}}$) of the outer gap]. We also show polar caps and the Earth line of sight (dashed line) in panel A. The pulse profile of $\gamma$-rays (histogram) corresponding to the emission pattern of panel A of Fig. 2 is shown in panel B of Fig. 2. From this figure, it seems that the pulse profile of $\gamma$-rays from the Geminga pulsar can be explained by our model. The observed $\gamma$-ray emission comes from the high-energy photons emitted outwards in the thick outer gap.

From the emission morphology we see that photons emitted into a given phase come from different positions of the outer gap. For a thick outer gap, the high-energy emission inside the outer gap can occupy a finite thickness. Since we have assumed that high-energy photons are emitted tangentially to the local magnetic field, if the viewing angle to the Geminga pulsar is known, for a single $a_1$ surface, the emission regions are functions of $\phi$ and $r$ on the $a_1$ surface. For a given viewing angle $(\approx 86^\circ)$, Fig. 3 shows the trajectories of the emission regions for different $a_1$ values. Once the radial distances are determined, the spectrum of photon emission can be calculated for a given radial distance $r$. In order to compare with the observed phase-resolved spectra of the Geminga pulsar, we calculate the radial distances in the emission region from $a_1 = 0.6$ to $0.98$ for the viewing angle of $86^\circ$, and then divide the calculated pulse phases into different parts according to the division given for the observed phases (Fierro et al. 1998). Therefore, using the possible values of the radial distances for each part of the phase and equation (7), we calculate the phase-resolved spectra for the phases labelled peak 1, trailing wing 1, bridge, leading wing 2 and peak 2. We also calculate the phase-averaged spectrum of the Geminga pulsar.

We compare the model results of pulse profile components (P1, TW1, Bridge, LW2, P2 and phase-averaged) with the observed data for the Geminga pulsar in Fig. 4. It can be seen that our model results are consistent with the observed data, although the components have been individually normalized. Peaks 1 and 2 are weak by factors of 3 and 2 respectively. It should be pointed out that we have used constant gap height in our calculations. In fact,

![Figure 2](https://academic.oup.com/mnras/article-abstract/320/4/477/977023/fig2)

**Figure 2.** Emission projection on to the $(\xi, \Phi)$ plane, and the pulse profile for Geminga parameter $a \approx 50^\circ$. Panel A: Photon emission from the pulsar as a function of phase; the solid and dashed curves indicate inward and outward photon emission from a single outer gap respectively. The observed lines of sight at $\xi = 86^\circ$ and $90^\circ$ are shown (dashed and solid lines), and both polar caps (contours) are also shown. Panel B: Pulse profile for Geminga. The model $\gamma$-ray pulse at $\xi = 86^\circ$ is shown, along with schematic pulses of the hard and soft X-ray emission.

![Figure 3](https://academic.oup.com/mnras/article-abstract/320/4/477/977023/fig3)

**Figure 3.** The variation of radial distance with the pulse phase for different $a$ surfaces of the Geminga pulsar. The inclination angle is $50^\circ$. Five regions for different pulse phases which are the same as those observed are indicated.

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4 SUMMARY AND DISCUSSION

We described a three-dimensional magnetospheric model to explain the observed pulse profile and the phase-resolved spectra of the $\gamma$-ray pulsars with thick outer gaps. In this model, the local photon–photon pair production in the outer gaps limits the extension of the outer gaps along the azimuthal direction. We find that double-peaked pulse profiles with varying phase separation, depending on the magnetic inclination ($\alpha$) and the viewing angle ($\zeta$), and strong bridge emission occur naturally. Although pair production inside the outer gap is limited only to a certain region, high-energy $\gamma$-rays are mainly produced by the primary accelerated particles through the curvature radiation inside the outer gap. We have applied our model to explain the Geminga pulsar’s pulse profile and phase-resolved spectra. In our calculation, we have chosen $\alpha = 50^\circ$ and $\zeta = 86^\circ$ in order to explain the phase separation of $\sim 0.5$ for Geminga pulsar. The emission pattern and pulse profile are shown in Fig. 2. Because there is no information about the magnetic inclination angle from other wavebands, the $\alpha$-value is uncertain. Using the single-pole outer gap model proposed by Romani & Yadigaroglu (1995), Yadigaroglu & Romani (1995) estimate that the magnetic inclination angle of Geminga pulsar is about $40^\circ$, which is not inconsistent with the value which we used here. Furthermore, we calculated the phase-resolved spectra and phase-averaged spectrum of the Geminga pulsar, and the model results are consistent with the observations (see Fig. 4).

Measurements of the Geminga pulsar at X-ray energies by ROSAT and ASCA can be divided into soft and hard spectral components, as described by Halpern & Ruderman (1993) and Halpern & Wang (1997). Halpern & Wang have made a comprehensive study of the Geminga pulsar at energies 0.1–10 keV using data from ASCA, ROSAT and EUVE. They found that soft X-ray flux can be fitted by a blackbody of temperature $T = (5.6 \pm 0.6) \times 10^3$ K, and the hard X-ray spectrum can be explained as a power law of energy index $1.0 \pm 0.5$. The pulse profile of observed X-rays by ROSAT indicates a single broad peak which is energy-dependent, while the pulse profile of hard X-rays with an energy range from 1 to 4 keV observed by ASCA has a strong main peak and a weaker secondary peak. According to the ZC model (see also Wang et al. 1998 and Cheng & Zhang 1999), the soft X-rays would be interpreted as emission from nearly the
full surface of the neutron star, while the hard X-ray pulse could arise from synchrotron radiation of $e^\pm$ pairs produced in the strong magnetic field near the star surface by curvature photons emitted by charged particles on their way from the outer gap to the star surface. As was pointed out by Wang et al. (1998), because copious $e^\pm$ pairs may be produced on closed field lines, a cyclotron resonant blanket could form around the hot polar cap. X-rays escape a blanketed neutron star, either through the hole of the blanket on their open field lines or by diffusing through the blanket itself. Therefore there is a modulation of soft X-rays, in which a strongly modulated part of the pulse profile could come from hole emission and a relatively non-varying part through the blanket. In our three-dimensional outer gap model, the peak of soft X-rays emitted outward from the star surface is near peak 2 of the high-energy $\gamma$-ray pulse profile. For hard X-rays from the Geminga pulsar, they are produced inwards through synchrotron radiation near the star surface, so the main peak of hard X-rays is near peak 1 of the high-energy $\gamma$-ray pulse profile. For the weaker secondary peak of the hard X-rays in the 1−4 keV energy range, if it actually exists, we expect that it comes mainly from the synchrotron radiation outwards near the null surface because of $e^\pm$ pairs with significant pitch angle produced there, as described by Romani (1996). In panel B of Fig. 2, the schematic pulses showing the phase of the hard and soft X-ray emission is shown. Therefore the relative phase offsets between soft X-rays and hard X-rays, as well as between X-rays and high-energy $\gamma$-rays, may be qualitatively explained by this model.

The optical observations of the Geminga pulsar provide important information on the Geminga pulsar. For example, the Geminga pulsar was identified using proper motion (Bignami, Caraveo & Mereghetti 1993; Bignami & Caraveo 1996), and the distance to this pulsar was estimated using Hubble Space Telescope (HST) observations (Caraveo et al. 1996); estimates of radius of the emitting region (Mignami, Caraveo & Bignami 1998) and the precise position needed for the timing (Mattox, Halpern & Caraveo 1998) have been given. Recently, Shearer et al. (1998) have tentatively detected pulsations in the B band, and the pulsed signal at just 3.5$\sigma$ level was found during only one of the three nights devoted to the project. They showed that the similarity between the optical pulse shape and the $\gamma$-ray light curve, and pointed out that a large fraction of the optical emission is non-thermal in origin. However, it is possible that the optical emission can be interpreted in terms of an emission line superimposed on the thermal continuum best fitting the extreme UV/soft X-ray data (Mignami et al. 1998; Jacchia et al. 1999). In our model, the non-thermal optical emission is the synchrotron tail of the hard X-rays. Since local electron cyclotron energy is $\sim 0.1$ keV (Wang et al. 1998; Cheng & Zhang 1999), the synchrotron spectrum must break below this energy to a spectrum which is proportional to $\nu^{1/3}$ at low frequencies. We can estimate the optical luminosity by $L_\lambda (\nu_{opt}/\nu_\lambda)^{1/3} \sim 10^{39}(1$ eV $0.1$ keV)$^{1/3}$ erg s$^{-1}$ $\sim 2 \times 10^{27}$ erg s$^{-1}$. Here, we have assumed that the distance to the Geminga pulsar is $\sim 150$ pc. However, in order to obtain the best fit for the ion emission line, the inclination angle and the viewing angle are 90° and 20° respectively (Jacchia et al. 1999). These values seem to contradict the values obtained by fitting the phase-resolved spectra of $\gamma$-rays. We believe that the viewing angle $\sim 90^\circ$ and the inclination angle $\sim 20^\circ$ should also provide a good fit to the ion emission line because the difference between these two angles is still $\sim 70^\circ$, which gives same linewidth and intensity modulation. Then the discrepancy between these two inclination angles is $\sim 30^\circ$. Furthermore, the ion line is emitted from a region 100 m from stellar surface (Jacchia et al. 1999), where the local field can easily dominate the dipolar field. The inclination angle for $\gamma$-rays is related to the dipole field, but the inclination angle for fitting the ion line is related to the surface local field. In our model, the optical emission should have the same phase as that of hard X-ray emission. It should be pointed out that (i) the optical pulsation given by Shearer et al. has not yet been confirmed, (ii) the relative phase offset between optical and $\gamma$-ray emission is not clear, and (iii) our model does not exclude other possibilities for the optical emission from the Geminga pulsar. In order to disentangle thermal and non-thermal emission in the optical band from the Geminga pulsar, more refined observations are required. Finally, we wish to point out that Geminga is a peculiar radio isolated neutron star only for the polar gap model, in which both radio and $\gamma$-rays are emitted near the polar cap. In the outer gap model, radio and $\gamma$-rays come from different regions. Because the $\gamma$-ray beam is wider than the radio beam, we have predicted that there are more radio-quiet pulsars than radio-loud pulsars. Detail studies have been published in Cheng, Gil & Zhang (1998) and Zhang, Zhang & Cheng (2000).

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REFERENCES

Bertsch D. L. et al., 1992, Nat, 357, 306

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