Binary formation in stellar clusters

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ABSTRACT

We consider how the tidal potential of a stellar cluster or a dense molecular cloud affects the fragmentation of gravitationally unstable molecular cloud cores. We find that molecular cloud cores which would collapse to form a single star in the absence of tidal shear, can be forced to fragment if they are subjected to tides. This may enhance the frequency of binaries in star-forming regions such as Ophiuchus and the frequency of binaries with separations $\lesssim 100$ au in the Orion Trapezium Cluster. We also find that clouds which collapse to form binary systems in the absence of a tidal potential will form bound binary systems if exposed to weak tidal shear. However, if the tidal shear is sufficiently strong, even though the cloud still collapses to form two fragments, the fragments are pulled apart while they are forming by the tidal shear and two single stars are formed. This sets an upper limit for the separation of binaries that form near dense molecular clouds or in stellar clusters.

Key words: hydrodynamics – binaries: general – stars: formation – open clusters and associations: general.

1 INTRODUCTION

Recent observations, combined with the results from past surveys, suggest that the frequency of wide binary stellar systems (separations $\gtrsim 50$ au) depends on the stellar density where they formed (e.g. Duchêne 1999). Binarity is common amongst low-mass main-sequence stars: about 60 per cent of G-type stars, 45 per cent of K-dwarfs, and 42 per cent of M-dwarfs are members of multiple systems in the solar neighbourhood (Duquennoy & Mayor 1991; Mayor et al. 1992; Fischer & Marcy 1992). Over the past decade, much effort has been directed toward finding out what processes determine binary frequency and the time-scale over which they act.

Early surveys concentrated on the nearest star-forming regions such as Taurus–Auriga, Ophiuchus (Leinert et al. 1993; Ghez, Neugebauer & Matthews 1993; Simon et al. 1995), Chameleon, Lupus, and Corona Australis (Reipurth & Zinnecker 1993; Ghez et al. 1997; Brandner et al. 1996). All of these regions display an excess of companions when compared to field stars, with varying degrees of significance (Duchêne 1999). They are also regions of low-mass star formation, low stellar density, and are all only a few Myr old.

More recent surveys of binarity have concentrated on stellar clusters that have much higher stellar densities. Several clusters in Orion (Prosser et al. 1994; Petr et al. 1998; Padgett, Strom & Ghez 1997; Simon, Close & Beck 1999) and the IC 348 cluster (Duchêne, Bouvier & Simon 1999) have been surveyed. These have ages of a few Myr. In addition, the Pleiades (Bouvier, Rigaut & Nadeau 1997) and Hyades (Patience et al. 1998) open clusters have been studied. These two clusters are much older: 120 and 600 Myr, respectively. None of the clusters display a significant binary excess over that of the field.

Thus, both young and old clusters have a similar binary frequency to field stars, while low-density star-forming regions tend to have an excess of binaries. The implication is that the binary frequency in the ranges surveyed ($\gtrsim 50$ au) is primarily dependent on stellar density and is determined on a timescale of $\lesssim 1$ Myr (Duchêne et al. 1999).

Several mechanisms have been proposed which could produce such a dependence on stellar density. The first is that the temperature of the star-forming region determines binary frequency, with cooler clouds being more prone to fragmentation (Durisen & Sterzik 1994). The gas in low-density star-forming regions is observed to be cooler than that in clusters due to the absence of high-mass stars.

Another possibility is simply that binaries form with the same frequency everywhere, but that the widest are disrupted by stellar encounters in regions with high stellar density (e.g. Kroupa 1995a,b,c, 2000; Kroupa, Petr & McCaughrean 1999). This idea is supported by searches for very wide binaries in the Orion Trapezium Cluster. Bate, Clarke & McCaughrean (1998) analysed the mean surface density of companions in the centre of the Trapezium Cluster and found weak evidence for a deficit of binaries with separations $\gtrsim 500$ au. Scally, Clarke & McCaughrean (1999) used proper motion data to search for binaries in the Trapezium Cluster with separations in the range 1000–5000 au and obtained results consistent with there being no binaries in this separation range.

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However, while the idea of binary disruption might be appealing, it neglects the effect of the environment of the cluster on the initial fragmentation process. Field stars have companions out to separations of \( \approx 10^4 \) au (Duquennoy & Mayor 1991) while the stellar density in the centre of the Orion Trapezium Cluster is \( \approx 2 \times 10^7 \) pc\(^{-3} \) (Hillenbrand & Hartmann 1998) which gives a mean separation between stars of \( \approx 8000 \) au. Thus, it is inconceivable that very wide binaries could have formed in the cluster and later have been disrupted.

One way in which the environment of a stellar cluster may affect the fragmentation of molecular cloud cores and the formation of binaries is through the tidal field of the cluster. Boss (1981) studied the collapse and fragmentation of tidally perturbed clouds. He found that a uniform cloud with a low thermal-energy content which would otherwise collapse to form a single object could be made to fragment if it was tidally perturbed. A relatively large tidal force was required to prevent the collapse entirely. He concluded that tidal perturbation of molecular cloud cores was a natural way to induce binary formation.

In this paper, we study how the tidal field of a cluster or a dense molecular cloud can affect fragmentation by modelling the gravitational collapse of molecular cloud cores which are subjected to a tidal field. We are interested both in whether the tidal field can induce fragmentation of cores which would otherwise collapse to form single stars, and in whether the tidal potential can inhibit the formation of wide binaries. The numerical method is described in Section 2. We present our results in Section 3, and discuss their implications for the formation of binaries in Section 4. Section 5 contains our conclusions.

2 Calculations

The calculations presented here were performed using a three-dimensional, smoothed particle hydrodynamics (SPH) code. The SPH code is based on a version originally developed by Benz (Benz 1990; Benz, Bowers & Cameron 1990). The smoothing lengths of particles are variable in time and space, subject to the constraint that the number of neighbours for each particle must remain approximately constant at \( N_{\text{neigh}} = 50 \). The SPH equations are integrated using a second-order Runge–Kutta–Fehlberg integrator with individual time-steps for each particle (Bate, Bonnell & Price 1995). Gravitational forces between particles and a particle’s nearest neighbours are found by using a binary tree. We use the standard form of artificial viscosity (Monaghan & Gingold 1983; Monaghan 1992) with strength parameters \( \alpha_v = 1 \) and \( \beta_v = 2 \).

2.1 Molecular cloud cores

We performed calculations with molecular cloud cores of one solar mass and initial temperatures of 10 K. Two types of molecular cloud core were modelled: spherical and cylindrical with an axial ratio of 2:1. For each type of cloud, three values of the ratio of thermal energy to the magnitude of the gravitational energy were used: \( \alpha_t = 0.3, 0.5, 0.8 \). The different values of \( \alpha_t \) were achieved by varying the initial size (and density) of the clouds. For example, a spherical cloud with \( \alpha_t = 0.5 \) had a diameter of 0.042 pc. The clouds range from 0.025 to 0.085 pc in size. None of the clouds had any initial rotation. The clouds initially had uniform densities, \( \rho_0 \).

The equation of state of the gas is given by

\[
P = K \rho^\gamma,
\]

where \( P \) is the pressure, and \( K \) is a constant that depends on the entropy of the gas. The ratio of specific heats, \( \gamma \), varies with density as

\[
\gamma = \begin{cases} 
1.0, & \rho \leq 10^{-14} \text{ g cm}^{-3}, \\
1.4, & \rho > 10^{-14} \text{ g cm}^{-3}.
\end{cases}
\]

The value of \( K \) is defined such that when the gas is isothermal \( K = c_s^2 \), with \( c_s = 2.0 \times 10^4 \) cm s\(^{-1} \) for molecular hydrogen at 10 K, and the pressure is continuous when the value of \( \gamma \) changes. This equation of state gives an isothermal collapse initially, but allows heating when the gas reaches high densities.

All calculations were performed with \( 2 \times 10^5 \) SPH particles which is sufficient to ensure that the local Jeans mass is always resolved (Bate & Burkert 1997). The particles were positioned on a regular lattice with random offsets of half the lattice spacing to model uniform-density gas with small-scale, low-amplitude noise.

2.2 Tidal forces

The molecular cloud cores are subject to an external tidal potential of the form

\[
\phi_{\text{tid}}(\mathbf{r}) = -\frac{GM_{\text{tid}}}{|\mathbf{r} - \mathbf{r}_{\text{tid}}|},
\]

where \( M_{\text{tid}} \) is the mass of a point mass at a distance \( r_{\text{tid}} \) from the centre of mass of the cloud, and \( G \) is the gravitational constant. The tidal shear across the cloud is given by the spatial derivative of the tidal force \( (\approx M_{\text{tid}}/r_{\text{tid}}^3) \) so that, for large distances, the degree of tidal shear is proportional to \( M_{\text{tid}}/r_{\text{tid}}^3 \). Thus, the strength of the tidal shear can be expressed as a density, and/or as some fraction, \( \epsilon \) of the density of the molecular cloud core, \( \rho_0 \):

\[
\frac{3M_{\text{tid}}}{4\pi r_{\text{tid}}^3} = \epsilon \rho_0.
\]

The magnitude of the tidal forces were quantified by Boss (1981) using a parameter

\[
\delta = \frac{5}{12} \left( \frac{M_{\text{tid}}}{M_c} \right) \left( \frac{2R_c}{r_{\text{tid}}} \right)^{2},
\]

where \( M_c \) and \( R_c \) are the mass and radius of the molecular cloud core, respectively. This parameter measures the energy of the tidal potential relative to the gravitational energy of the molecular cloud core, rather than the degree of tidal shearing. For comparison, \( \epsilon \) is related to \( \delta \) by

\[
\delta = \frac{5}{6} \epsilon \left( \frac{r_{\text{tid}}}{R_c} \right)^2 = \frac{10}{3} \epsilon,
\]

since Boss used \( r_{\text{tid}} = 2R_c \) for his calculations.

As pointed out by Boss (1981), if an initially stationary cloud is subject to the tidal force given by equation (3) it will fall towards the point mass. This will result in an increase of the tidal shearing with time. The distance moved by the cloud in time, \( t \), is given by (see Boss 1981)

\[
d = \frac{3}{2} r_{\text{tid}} \left( \frac{t}{t_{\text{ff}}} \right)^{2},
\]

where \( t_{\text{ff}} = [3\pi/(32G\rho_0)]^{1/2} \) is the free-fall time of the cloud. Since the cloud will collapse in approximately one free-fall time, it will move only a small fraction of the distance between the cloud and the point mass during the calculation for small \( \epsilon \). The
effective value of \( e \) at the end of the calculation will be within a factor of 2 of that at the start of the calculation if \( e \leq 0.4 \).

Furthermore, if the cloud is rotating around the point mass, the distance it falls toward the point mass will be even less. Thus, for simplicity, we wish to fix the value of \( r_{\text{tide}} \) (and \( e \)) during a calculation. Boss (1981) also made this simplification. This requires us to eliminate the centre-of-mass motion of the cloud. The simplest way to do this with SPH is to move the centre of mass of the cloud back to the origin of the calculation volume and zero the centre of mass velocity after each time-step. We use constant volume boundary conditions.

In summary, for each calculation, there are three parameters: cloud shape, \( \alpha_i \), and \( e \).

## 3 Results

### 3.1 Spherical clouds

We performed calculations which follow the collapse of spherical, uniform-density clouds with \( \alpha_i = 0.3, 0.5 \) or 0.8. In the absence of tidal shear, these clouds collapse to form single objects. None of the calculations with \( \alpha_i = 0.8 \) result in fragmentation, even though tidal shearing up to \( e = 1 \) is applied. Only single objects are formed.

When tidal shear is applied to clouds with \( \alpha_i = 0.3 \) or 0.5 they collapse to form a bar that is orientated in the direction of the tidal perturber (Fig. 1). This occurs because collapse parallel to the direction of the perturber is delayed by the tidal shear. The length of the bar increases and the cloud goes from producing a single object to multiple fragments (which can be seen in the plots of density along the \( z \)-axis). For \( e \approx 0.75 \), the ends of the bar are being pulled apart by the tidal shear. This sets an upper limit to the length of the bar. The clouds have diameters of 8500 au.

Figure 1. The fragmentation of tidally perturbed spherical clouds with \( \alpha_i = 0.5 \). The clouds are identical initially, but the tidal shears have values of \( e = 0.25 \) (left), 0.55 (centre), 0.75 (right). Density in a slice along the shearing \( z \)-axis is shown as a grey-scale running from \( 10^{-15} - 10^{-13} \) g cm\(^{-3} \). Also shown is the velocity and density along the \( z \)-axis (through the centre of the gaseous bar and the fragments). As the tidal shear increases, the length of the bar increases and the cloud goes from producing a single object to multiple fragments (which can be seen in the plots of density along the \( z \)-axis). For \( e \approx 0.75 \), the ends of the bar are being pulled apart by the tidal shear. This sets an upper limit to the length of the bar. The clouds have diameters of 8500 au.

![Figure 1](https://www.astro.leeds.ac.uk/~rc313/graphics/figure1.png)

Figure 2. The length of the gaseous bar formed during the collapse of tidally perturbed spherical clouds with \( \alpha_i = 0.3 \) (solid line) and \( \alpha_i = 0.5 \) (dashed line). Fragmentation of the bar occurs for values of \( e \) where the lines are thick. Clouds with \( \alpha_i = 0.8 \) do not produce bars or fragment. The bars are shorter with larger \( \alpha_i \). The length of the bar is given in units of the diameter of the clouds.

Figure 3. The fragmentation of tidally perturbed cylindrical clouds with $\alpha_i = 0.5$. The clouds are identical initially, but the tidal shear has values of $\epsilon = 0$ (top left), 0.15 (top right), 0.25 (bottom left), or 0.35 (bottom right). Density in a slice along the long $z$-axis is shown as a grey-scale running from $10^{-18}$–$10^{-14}$ g cm$^{-3}$. Also shown is the velocity along the $z$-axis (through the centre of the gaseous bar and the two fragments), with the velocities of the fragments marked with points. As the tidal shear increases, the separation of the binary increases and the relative velocity of the fragments changes from being toward each other (forming a bound binary) to away from each other (disrupting the binary as it forms). The change from a bound to an unbound system occurs with a tidal shear of $\epsilon = 0.25$. The clouds are 11,000 au long.
of the bar increases with the strength of the tidal shear (Figs 1 and 2). The length of the bar also depends on the value of $\alpha_1$ (Fig. 2). Larger values of $\alpha_1$ result in shorter bars because the greater degree of thermal support slows the collapse perpendicular to the bar and, thus, makes the collapse more spherical. The clouds with $\alpha_1 = 0.8$ do not form well-defined bars even with large values of $\epsilon$, although there is a concentration of gas along the axis between the clump and the tidal perturber.

A gravitationally unstable bar of gas collapses more rapidly perpendicular to its major axis than along its major axis. If the bar remains isothermal during its collapse, the end state of the collapse is an infinitely-dense spindle (Inutsuka & Miyama 1992, 1997; Truelove et al. 1998). However, in reality, the gas begins to heat up when the rate of energy liberation due to compression exceeds the cooling rate (e.g. Larson 1969; Masunaga, Miyama & Inutsuka 1998). This occurs at approximately the same time as when the gas becomes optically thick to infrared radiation. The extra thermal support perpendicular to the major axis slows the collapse of the bar along its minor axes. If the bar has sufficient mass per unit length, the slowing of the collapse allows time for the bar to fragment into multiple objects along its major axis (Inutsuka & Miyama 1992, 1997). In the calculations presented here, the heating of the gas at high densities is handled using the equation of state described in Section 2.1. Due to this heating of the high-density gas, fragmentation occurs in calculations which form sufficiently long bars (e.g. Fig. 1, $\epsilon = 0.55$ and 0.75). The fragmentation occurs in the central part of the bar and does not reach all the way to the ends. Typically, about half of the length of the bar produces fragments and the fragments are initially separated by $\sim 100$ au (e.g. Fig. 1). Fragmentation occurs for $\epsilon \geq 0.25$ for clouds with $\alpha_i = 0.3$ and $\epsilon \geq 0.45$ for clouds with $\alpha_i = 0.5$. These regions of parameter space are represented in Fig. 2 by the thick lines.

For very large tidal shear ($\epsilon \geq 0.75$ for both $\alpha_i = 0.3$ and 0.5), a bar forms and fragments in the centre of the cloud while the outer parts of the bar are pulled apart by the tidal shear (Fig. 1). This sets an upper limit to the length of the bar for large values of $\epsilon$ which can be seen in Fig. 2.

We now compare our results with those of Boss (1981) who also followed the collapse of tidally perturbed spherical clouds with uniform initial density. The main differences from the calculations presented here are that Boss used $\alpha_i = 0.25$ or 0.63 and the molecular cloud cores were rotating. Thus, whereas we obtain either single objects or bars, Boss obtained either rings or bars. Qualitatively, Boss finds the same trend as we do: increasing the tidal shear increases the length of the bar. Unfortunately, Boss does not quote the lengths of the bars which form in his calculations. Only the distances of the fragments from the origin are quoted. However, by comparing his figures to the tabulated values, it appears that his bars extend approximately twice as far from the origin as the fragments. Using this factor of two, Boss finds that a cloud with $\alpha_i = 0.25$ forms a bar with a length of $\sim 1/4$ of the cloud’s diameter with $\delta = 1.3$. This corresponds to $\epsilon \approx 0.4$ and as can be seen in Fig. 2 our results for $\alpha_i = 0.3$ are in fair agreement. Boss found that the tidal shear was sufficient to start disrupting the cloud somewhere in the range $2.5 < \delta < 4.2$. This corresponds to $0.75 \approx \epsilon \approx 1.3$. We find that the outer parts of the bar are pulled apart with $\epsilon \approx 0.75$ which is also in reasonable agreement. We note that the bars produced in Boss’s calculations are much thicker than in our calculations and, therefore, only produce two or three fragments whereas ours produce many ($\sim 10$). This is due to the lower spatial resolution of Boss’s calculations which stops the bars from collapsing as far as in our calculations.

### 3.2 Cylindrical clouds

We have seen that tidal shear can induce the fragmentation of spherical clouds. What effect does tidal shear have on clouds which would otherwise fragment? Cylindrical clouds typically collapse to form binary systems (e.g. Bonnell et al. 1991, 1992). Thus, modelling the collapse of cylindrical clouds with tidal shearing allows us to study how tidal shearing may inhibit the fragmentation process.

We performed calculations of cylindrical clouds with axis ratios of 2:1 and $\alpha_i = 0.3$, 0.5 or 0.8. All of the calculations with $\alpha_i = 0.8$ produced single objects. Some calculations were performed with tidal shear perpendicular to the major axis of the cloud. All those with $\alpha_i = 0.3$ or 0.5 produced bound binary systems. For the rest of this section, we discuss calculations with $\alpha_i = 0.3$ or 0.5 where the tidal shear acts along the major axis of the clouds.

Isolated clouds with $\alpha_i = 0.3$ or 0.5 produce two fragments which are connected by a low-density bar of gas (Fig. 3). As the magnitude of $\epsilon$ is increased, the separation of the binary, and the length of the bar, increases (Figs 3 and 4). The separation of the binary also depends on the value of $\alpha_i$ (Fig. 4). For $\epsilon \approx 0.25$, larger values of $\alpha_i$ result in smaller separations, with the extreme case being when $\alpha_i = 0.8$ and there is no fragmentation (i.e. the separation is zero).

Considering the velocity structure along the major axis of clouds with $\alpha_i = 0.5$ (Fig. 3), we find that when $\epsilon \approx 0.25$, even though two fragments are formed, their relative motion is away from each other. Thus, the binary is pulled apart by the tidal shear even as it is being formed. The value of $\epsilon$ for which this occurs has a weak dependence on $\alpha_i$: for $\alpha_i = 0.3$, disruption of the binary occurs when $\epsilon \approx 0.35$. Disruption requires a larger value of $\epsilon$ for clouds with smaller $\alpha_i$ because of two effects. First, the pressure.

![Figure 4](https://academic.oup.com/mnras/article-abstract/321/3/585/1101299/0.2.png)
gradients along the major axis of the cloud are weaker for clouds with lower \( \alpha_1 \) (the thermal energy is lower) and therefore the pressure gradients exert a weaker outward acceleration on the gas. Secondly, clouds with lower \( \alpha_1 \) collapse in fewer free-fall times than warmer clouds which gives less time for the outward acceleration of the tidal shear to act. The disruption sets an upper limit for the semi-major axis of a binary which can be expressed as

\[
a_{\text{max}} \approx 1.1r_{\text{ide}} \left( \frac{0.3M_\star}{M_{\text{ide}}} \right)^{1/3},
\]

where \( M_\star \) is the mass of the molecular cloud core and we have taken \( \epsilon = 0.3 \) as the value required for disruption, which is the mean of the values required for \( \alpha_1 = 0.3 \) and 0.5. This equation is derived from equation (4) by setting \( \rho_0 = M_\star/(4\pi r_\star^3) \) (where \( r_\star \) is the radius of a molecular cloud core with 2:1 axial ratio), rearranging to solve for \( r_\star \), and then taking the distance between the two fragments (which is twice their semi-major axis separation) at \( \epsilon = 0.3 \) from Fig. 4 to be \( \approx 0.8 \) of the length of the molecular cloud core, \( 4r_\star \).

We note that equation (8) always gives the maximum distance between bound fragments (\( 2a_{\text{max}} \)) to be less than the lengths of the bars formed from the tidally perturbed spherical clouds of the previous section (cf. Fig. 2). For \( \alpha_1 = 0.3 \), \( 2a_{\text{max}} = 0.85 \) of the cloud’s diameter whereas the bars range in length from 0–0.75. For \( \alpha_1 = 0.5 \), the maximum distance is \( 2a_{\text{max}} = 0.81 \) and the bars range in length from 0–0.27 of the cloud’s diameter. Thus, the fragmentation of spherical clouds to form multiple bound fragments which was found in Section 3.1 and the limit on the maximum binary separation from this section are consistent.

4 DISCUSSION

4.1 Enhanced fragmentation

We find that tidal shear of \( \epsilon \approx 0.25–0.45 \), depending on \( \alpha_1 \), can induce fragmentation of spherical, uniform-density molecular cloud cores. This tidal shear is quite strong. For example, for a one solar mass cloud of radius 0.05 pc, \( \epsilon = 0.25 \) corresponds to a tidal perturber of mean density 500 \( M_\odot \) pc\(^{-3} \). This could be provided by 250 \( M_\odot \) at a distance of 0.5 pc, 16,000 \( M_\odot \) at a distance of 2 pc, or 2 \times 10^6 \( M_\odot \) at a distance of 10 pc. Such tidal shearing is found in the Orion Trapezium Cluster, from near the centre out to a radius of about 1 pc, and is also present in parts of the Ophiuchus star-forming region (see Fig. 5). Thus, we might expect that binary formation was enhanced in these star-forming regions. However, a significant fraction of the wide binaries which originally formed in the Orion Trapezium Cluster will have been disrupted by encounters with other stars by now (see Section 5). Indeed, as discussed in the introduction, the frequency of binaries with separations \( \geq 50 \) au in the Trapezium Cluster is actually lower than in low-density star-forming regions such as Taurus. The only way enhanced binary formation due to tidal shear might be detected in the Trapezium Cluster would be to determine the frequency of binaries with separations small enough that dynamical encounters could not have significantly depleted them (i.e. separations \( \leq 100 \) au).

The initial conditions used for the calculations of Section 3.1 have no large-scale initial density perturbations. If, for example, the clouds were initially slightly prolate, we would expect weaker tidal shear to induce fragmentation since less action would be required by the tidal shear in order to form a bar. This increases the likelihood that tidal shear enhances binary formation. On the other hand, molecular cloud cores are generally observed to be somewhat centrally condensed whereas we have only considered uniform-density cores. Centrally condensed cores are known to be less prone to fragmentation (Boss 1987; Myhill & Kaula 1992). Other factors which are beyond the scope of this study, but which may also alter the likelihood of fragmentation, include the presence of turbulent motions and magnetic fields. Future studies of how these factors affect tidally induced fragmentation would be worthwhile.

4.2 The maximum binary separation

If we know the mass, \( M_{\text{ide}} \), and distance, \( r_{\text{ide}} \), to a tidal perturber, we can use equation (8) to determine the widest binary separation, \( a_{\text{max}} \), which is permitted by the tidal shear as a function of the mean mass density of the tidal perturber [i.e. \( M_{\text{ide}}/(4\pi r_{\text{ide}}^3/3) \)]. This is given in Fig. 5. On the figure we plot points for various star-forming regions. We can see that binary separations are limited to be less than a few tens of thousand AU by distant giant molecular clouds (GMCs), \( \sim 10^4 \) au in regions like Ophiuchus, and as small as a few thousand AU in the centres of clusters such as the Orion Trapezium and NGC 2023. Thus, tidal shear during fragmentation may explain why the widest binaries observed in the field have their separations limited to \( \sim 10^4 \) au (Duquennoy & Mayor 1991).

For the Orion Trapezium Cluster, which has been studied in more detail than any other young cluster, we can use equation (8) to determine the maximum binary separation that is permitted by the tidal shear of the cluster as a function of radius from its centre. Hillenbrand & Hartmann (1998) fit the structure of the Trapezium Cluster using a King model with a core radius of 0.16 pc and a central stellar density of \( 1.7 \times 10^5 \) pc\(^{-3} \). By assuming virial
equilibrium, they found that the cluster should contain 4500 \(M_\odot\) within a radius of 2.06 pc. The total stellar mass within this radius is observed to be \(\approx 1800 M_\odot\), not including unresolved multiple systems. If the cluster is in virial equilibrium, the missing mass could be in the form of gas. In any case, for the rest of the discussion we will assume that the cluster contains the above virial mass. Using the King model to calculate the enclosed mass as a function of radius from the centre of the cluster and applying equation (8), we can calculate the maximum binary separation as a function of radius in the cluster. This is plotted in Fig. 6 for molecular cloud cores of one solar mass (there is a weak dependence on the mass of the cores). Also plotted is the mean separation of stellar systems as a function of radius, and the typical radius for the disruption of binaries by passing stars which we calculate as the radius which would give a cross-section large enough that a star is expected to have had one encounter within the age of the cluster (see Binney & Tremaine 1987)

\[
r_{\text{dis}} = (16\sqrt{\pi n \sigma^2 r})^{-1/2}
\]

where \(n\) is the local stellar density, \(\sigma\) is the observed one-dimensional velocity dispersion (2.3 km/s: Jones & Walker 1988; Tian et al. 1996), and \(t\) is the age of the cluster. The mean age of the stars in the Trapezium Cluster is thought to be \(\approx 1\) Myr (Hillenbrand & Hartmann 1998). Equation (9) is a crude estimate of the disruption radius of binaries by stellar encounters since it does not take account of stellar motions within the cluster. In reality, stars which are currently in the outer parts of the cluster may have spent time in denser regions, and stars near the centre may have recently fallen in from the outside. However, this simple model has been compared to the results from more detailed \(N\)-body calculations (Kroupa et al. 1999; Scally & Clarke, in preparation) and does give a surprisingly accurate representation of the typical separation as a function of radius.

Fig. 6 shows that the tidal shear of the Trapezium Cluster limits binaries to have separations which are significantly smaller than the local mean separation between stellar systems. However, the stellar densities are so large in the centre of the cluster that the radius of disruption of binaries by stellar encounters, even after 0.1 Myr, is significantly smaller. The collapse and fragmentation of a typical molecular cloud takes \(\approx 0.1\) Myr. Thus, in the centre of the Trapezium Cluster, the separations of binaries would have been limited by interactions between other molecular cloud cores and protostars even as fragmentation was taking place. Binaries with separations \(\geq 1000\) au probably never existed in the centre of Orion. Only in the outer parts of the Trapezium Cluster, \(\geq 1\) pc from the centre, is the stellar density low enough (\(\leq 100\) pc\(^{-3}\)) that tidal shear during fragmentation is likely to have determined the maximum binary separation. At these distances, binary separations would have initially been limited by tidal shear. Only later, over the subsequent 1 Myr, would the widest of these binaries have been disrupted by stellar encounters. The line showing the maximum binary separation due to stellar encounters at 1 Myr demonstrates that disruption by encounters can explain the observation that there are no binaries with separations in the range 1000–5000 au (Scally et al. 1999), at least out to distances of \(\approx 1\) pc.

In summary, tidal shear during fragmentation can determine the initial maximum binary separation in low-density star-forming regions (\(\leq 100\) pc\(^{-3}\)) which are near to GMCs, dense molecular clouds, or in the outer parts of large stellar clusters. If these star-forming regions disperse quickly (\(\leq 1\) Myr) and/or have very low stellar density, this mechanism can explain the final upper limit to the separations of binaries. However, in dense or long-lived star-forming regions, the maximum binary separation will eventually be limited by dynamical encounters with other stars.

5 CONCLUSIONS

We have studied the effect of a tidal potential on the fragmentation of gravitationally unstable molecular cloud cores. In agreement with Boss (1981), we find that cores which would collapse to single objects in the absence of tidal shear can be made to fragment by sufficiently strong tides. This may enhance the frequency of binaries in star-forming regions that are subjected to large tidal shear, such as the inner 1 pc of the Orion Trapezium Cluster and parts of the Ophiuchus star-forming region. However, in order to search for this effect in clusters such as the Trapezium, small binary separations \(\leq 50\) au need to be surveyed which have not been significantly depleted by disruptive stellar encounters within the current age of the cluster.

We also study the collapse of molecular cloud cores which would fragment to form binaries in the absence of tidal shear. We find two regimes. If the tidal shear is weak, the cloud collapses and fragments to form a bound binary system. However, if the tidal shear exceeds a critical value (\(\approx 0.3\rho_0\)), even though the cloud still collapses to form two fragments, the resulting fragments are pulled apart by the tidal shear and two single stars are formed. Thus, tidal shear can set an upper limit to the separation of binary stellar systems. In Ophiuchus, tidal shear limits the separations of binaries to be \(\leq 10^4\) au. This provides an explanation for the observed maximum separation of field binary stars (Duquennoy & Mayor 1991). In the Orion Trapezium Cluster, the maximum binary separation due to the tidal potential of the cluster varies from \(\approx 4000\) au in the centre of the cluster to \(\approx 20000\) au at 2 pc from the centre of the cluster. This is significantly smaller than the local mean separation between stellar systems, but wider than the typical separation for which


Figure 6. For the Orion Trapezium Cluster, as functions of radius from the centre of the cluster, we plot: the mean separation of stellar systems (dotted line); the maximum binary separation, beyond which tidal shear disrupts binaries as they are forming (solid line); and the typical disruption radius of binaries due to encounters with other stars when the cluster is 0.1 Myr (short-dashed line) and 1.0 Myr (long-dashed line) old.
binaries would be expected to have been disrupted by encounters with other stars within the current age of the cluster. Thus, tidal shear may set the final upper limit to binary separations in star-forming regions with low stellar density or which disperse rapidly, while in dense or long-lived star-forming regions the final maximum separation will be set by dynamical encounters with other stars.

ACKNOWLEDGMENTS

We thank Aylwyn Scally for useful discussions. IAB acknowledges support from a PPARC advanced fellowship.

REFERENCES


This paper has been typeset from a TeX/LaTeX file prepared by the author.