Adaptive control of algae detachment in regulated canal networks

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ABSTRACT

Open-channel distribution networks are subject to algal developments that can induce major disturbances such as clogging of hydraulic devices (pipes, weirs, filters, flow meters). Flushes can be used as a strategy to manage these algae developments. A flush is carried out by increasing the hydraulic shear conditions using hydraulic control structures of the canal network. In response to the shear stress increase, a part of the fixed algae is detached, then re-suspended into the water column, and finally transported downstream. This leads to a peak of turbidity that has to be controlled. In this paper, we develop a distributed linear model of the turbidity dynamics that is used for real-time adaptive control of the flushes. Simulations show the effectiveness of the adaptive controller, which can, at the same time, estimate the gain of the system, linked to the amount of initial fixed biomass, and perform a flush without exceeding the turbidity limit.

Key words | adaptive control, algae, hydraulic flush, linear model, open channels, turbidity

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INTRODUCTION

A large number of optimal control strategies have been developed for open-channel network management for hydraulic performance criteria (Malaterre 1998; Rogers & Goussard 1998; Clemmens *et al.* 2005). In comparison, there are fewer studies that are interested in water quality control. Whitehead (1978) applied real-time control to dissolved oxygen concentration in a river downstream of a sewage outfall using mechanical aerators. Most applications of real-time control to quality issues are related to sewage or wastewater management and focus on minimizing overflow volumes and frequencies (e.g. see Schutze *et al.* 2004). Several authors directly control a quality parameter measured on-line as dissolved oxygen (Rauch & Harremoes 1999) or ammonium concentration (Vanrolleghem *et al.* 2005) using tanks and/or pumps. Other studies use hydraulic regulation to control both the hydraulic variables (discharge and water level) and a quality parameter: the salinity. For example, Hof & Schuurmans (2000) and Xu *et al.* (2010) designed a proportional integral controller to control the quantity and quality of water in Dutch canals, and Augustijn *et al.* (2011) investigated Dutch rivers. Recently, Alvisi *et al.* (2012) proposed an optimal control process selection based on a genetic algorithm to limit the consumed volume of water if contamination occurs in the network. In all these cases, the controller acts on pumps or gates either for storage or dilution.

Algae development induce other types of disturbances in open-channel networks, such as clogging of hydraulic devices (flowmeters, pipes, weirs, filters, etc.) and possible water quality depreciation. Most current management strategies of algae development consist of using chemicals or mechanically removing fixed vegetation. Algaecides depreciate the water quality and their use is not possible in canals that supply drinking water plants. Mechanical operations are costly and time consuming, and should also be avoided. Regular flushing flows can be used as a strategy to manage fixed algae developments in open channels (Fovet *et al.* 2012a). They consist of increasing the hydraulic shear stress

doi: 10.2166/hydro.2013.166

in order to detach part of the algal biomass. Carrying out a flush every 2 to 3 weeks maintains an acceptable biomass fixed on the canal banks. The shear stress increase is obtained by releasing a supplementary volume of water at the canal head for a few hours, so that the mean bottom shear stress is increased along the canal. The detached algae and other particles attached to the biofilm are re-suspended in the water column, causing a turbidity plume. The canal managers define a threshold of maximal turbidity for not disturbing the end users and the hydraulic devices. The performance of the flushes must integrate two conflicting objectives: detaching enough biomass while not exceeding a maximal turbidity. In this paper, we propose to develop control methods applied to a non-hydraulic variable: the turbidity.

The problem of turbidity is somewhat different from salinity and solutes, since the detachment process caused by the flush propagation must be considered. The dilution of re-suspended algae is not possible here because increasing the volume of incoming water at the canal head induces an increase in the amount of detached algae. Intermediate storage of turbid water is not possible: in the best-case scenario, only the storage capacity is a downstream tank. Fovet *et al.* (2012b) proposed a quasi-linear model to design the flushing flow under turbidity constraints using an open-loop controller. This quasi-linear model requires the calibration of four parameters related to the hydraulic characteristics of the reach, and to the biomass characteristics: initial biomass, length of filaments and their resistance to detachment. The most sensitive parameter is the attached biomass. However, if it is under-estimated, this leads to design a flush that will detach too much biomass, and which will subsequently exceed the maximal turbidity objective. To avoid this, it is possible to compensate for the discrepancies between the model and reality by adding a closed-loop controller. This controller should adjust the amount of water released, in real time, based on the difference between actual and expected turbidity.

The problems are then to take account of: (i) the propagation delays, which are much larger for the particle transfer than for the wave propagation; (ii) the large uncertainty of the amount of biomass before the flush, and, to some extent, on the link between detached algae concentration and turbidity; and (iii) the decreasing stock of biomass in the canal due to detachment during the flush. The issue of the delay must be addressed by positioning the turbidity sensors at an appropriate distance from the canal head, not too close in order to get a clear turbidity response, and nor too far in order to have a particle travel time much shorter than the flush duration. The quasi-linear model we propose in this paper is distributed and simulates the turbidity dynamics during the flush at each section of the canal. There is a need to have a distributed model that is able to represent the turbidity dynamics at every point of the canal.

Adaptive control strategy is a way to address points (ii) and (iii). Adaptive control was developed in the 1950s as a way to design controllers that can adapt to process with unknown parameters (Sastry & Bodson 1989; Ioannou & Sun 1996). It has been applied to the quantitative control of water flow (Begovich & Ortega 1989) (local downstream adaptive control of an open channel), but rarely for quality control. Wang *et al.* (2006) designed a control method of chlorine residual in a drinking water network.

This article is an application of adaptive control to real-time canal management, with an objective to control turbidity during a flush. The non-linear comprehensive model of algal detachment described in Fovet *et al.* (2012a) is based on an accurate representation of the hydraulic dynamics: the Saint-Venant equations. However, these two non-linear partial differential equations are very complex from the control point of view, and very few methods can use them directly. Most control design methods rather use a simpler description of the system: a linear time invariant linearization around an equilibrium regime (see review in Malaterre *et al.* (1998)), as even if hydraulic processes are non-linear, the hydraulic regimes change relatively slowly. This approach (called 'gain scheduling') is also widely used in, for example, aerospace applications (Leith & Leithead 2000). It enables well-established linear design methods to be applied, and ensures global stability under some technical conditions. These conditions are that the variation of the operating points in time remains sufficiently slow, and are difficult to guarantee theoretically in the case of open channels that are boundary controlled. The analysis of stability is therefore often reduced to intensive non-linear simulation *a posteriori*, but this approach works well in practice. State-space and frequency approaches are two main approaches for designing control methods of linear time-invariant systems. We use the frequency or 'input–output' approach, which is based on a transfer function, as it is more appropriate to frame the control problem and to represent uncertainties and the robustness problem (Litrico & Fromion 2009).

Therefore, we first derive from the model developed in Fovet et al. (2012a) a quasi-linear model that simulates turbidity dynamics during a flush. Compared with the linear model derived by Fovet et al. (2012b) for open-loop control, it is more physically based, and it is distributed, which means that turbidity is given at any location along the channel.

We first performed a simulation study of the adaptive controller to illustrate the design of a flush, the calibration of the closed-loop controller, and its advantage for estimation of initial biomass. Following this theoretical study, the controller was implemented in the field, on a 32 km long branch of a real canal located in southern France, the Canal de Provence.

A QUASI-LINEAR DISTRIBUTED (QLD) MODEL OF THE TURBIDITY RESPONSE TO A FLUSH

Linearized Saint-Venant equations and bottom average shear stress

The flow is described using the Saint-Venant one-dimensional equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\frac{\partial Y}{\partial x} + gA(S_f - S_b) = 0 \tag{2}$$

with x the abscissa (m), t the time (s), A the wetted area (m^2), Q the mean discharge (m^3 s^{-1}), g the gravitational acceleration (m s^{-2}), Y the water level (m), S_f the friction slope and S_b the bed slope. A, Q, Y and S_f depend on x and t. We first consider a trapezoidal channel and a steady uniform flow, referred to as the reference regime, where discharge is Q_0 and water depth is Y_0. Taking $q(x, t) = Q(x, t) - Q_0$ and $y(x, t) = Y(x, t) - Y_0$ as small variations around the reference regime, we obtain the linearized Saint-Venant equations (Litrico & Fromion 2009, pp. 64–66):

$$T_0\frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{3}$$

$$\frac{\partial q}{\partial t} + (k_1 - k_2)\frac{\partial q}{\partial x} + k_3 q + k_1 k_2 T_0 \frac{\partial y}{\partial x} - k_4 y = 0 \tag{4}$$

with T_0 the top width of the water surface, c_0 the wave celerity, U_0 the mean value between maximal and minimal velocity values during the flush, $k_1 = c_0 + U_0$, $k_2 = c_0 - U_0$,

$$k_3 = \frac{2g}{U_0}\left(S_b - \frac{\partial Y_0}{\partial x}\right) \text{ and}$$

$$k_4 = U_0\frac{\partial T_0}{\partial x} + gT_0\left((\kappa+1)S_b - (1+\kappa-(\kappa-2)Fr_0^2)\frac{\partial Y_0}{\partial x}\right),\ \kappa$$

is a shape factor and Fr_0 is the Froude number of the reference state.

The system is rewritten as:

$$\frac{\partial \xi}{\partial t} + \mathcal{A}\frac{\partial \xi}{\partial x} + \mathcal{B}\xi = 0$$

where

$$\xi(x, t) = \begin{pmatrix} T_0 y(x, t) \\ q(x, t) \end{pmatrix},\ \mathcal{A} = \begin{pmatrix} 0 & 1 \\ k_1 k_2 & k_1 - k_2 \end{pmatrix},$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 \\ -k_4 & k_3 \end{pmatrix}$$

Using the Laplace transform $\mathcal{L}(f(t)) = \int_0^{+\infty} f(t)e^{-st}dt$, and owing to the property $\mathcal{L}(df/dt) = s\mathcal{L}(f(t)) - f(0)$, the system can be expressed as an ordinary differential equation:

$$\frac{\partial \hat{\xi}(x, s)}{\partial x} = -\mathcal{A}^{-1}(sI + \mathcal{B})\hat{\xi}(x, s) + \mathcal{A}^{-1}\xi(x, 0) \tag{5}$$

The matrix $\mathcal{A}^{-1}(sI + \mathcal{B})$ is diagonalizable; we denote $\lambda_1(s)$ and $\lambda_2(s)$ its eigenvalues. The diagonalization provides the solution $\xi(x, s)$ of the system, expressed as functions of these eigenvalues. After some manipulations, and neglecting the influence of downstream flow variations, we obtain:

$$\hat{q}(x, s) = e^{\lambda_1(s)x}\hat{q}_0(s) \tag{6}$$

$$\hat{y}(x,s) = \frac{\lambda_1(s)}{T_0 s} e^{\lambda_1(s)x} \hat{q}_0(s) \tag{7}$$

$$\lambda_1(s) = \frac{(k_1 - k_2)s + k_4 - \sqrt{d(s)}}{2k_1k_2} \tag{8}$$

$$d(s) = (k_1 + k_2)^2 s^2 + 2((k_1 - k_2)k_4 + 2k_1k_2k_3)s + k_4^2 \tag{9}$$

with $\hat{q}_0(s) = \hat{q}(0, s)$. This is the downstream response to an upstream discharge variation. Using a low-frequency approximation and the moment matching method, Munier *et al.* (2008) approached the discharge transfer function (Equation (6)) by a first-order with delay linear model:

$$\hat{q}(x,s) = \frac{e^{-\tau(x)s}}{1 + K(x)s} \hat{q}_0(s) \tag{10}$$

in which $\tau(x)$ and $K(x)$ are functions of the initial model parameters:

$$\tau(x) = \frac{2x}{U_0(1+\kappa)} - K(x) \tag{11}$$

$$K(x) = \sqrt{\frac{2x\left(4 - (\kappa - 1)^2 Fr_0^2\right)}{gS_b(1+\kappa)^3 Fr_0^2}} \tag{12}$$

To express the detachment of algae, we need to introduce the mean bottom shear stress $\sigma_0(x, t)$ (Fovet *et al.* 2012a), which can also be obtained by linearization. Using Manning's equation, $\sigma_0(x, t)$ is given by:

$$\sigma_0(x,t) = \rho g \frac{n^2 Q^2(x,t)}{A^2(x,t) R_h(x,t)^{1/3}} \tag{13}$$

with ρ the water density, n the Manning roughness coefficient and R_h the hydraulic radius. Linearization around the reference regime $\sigma_{0,0}$ provides:

$$\sigma(x,t) = \frac{2\sigma_{0,0}}{Q_0} q(x,t) - \sigma_{0,0}\left(\frac{7}{3}T_0 - \frac{2}{3}R_{h0}\right) y(x,t) \tag{14}$$

Denoting $\mathcal{H}_0 = \sigma_{0,0}\left(\frac{7}{3}T_0 - \frac{2}{3}R_{h0}\right)$, and applying the Laplace transform to $\sigma(x, t)$, we get:

$$\hat{\sigma}(x,s) = \left(\frac{2\sigma_{0,0}}{Q_0} - \frac{\mathcal{H}_0 \lambda_1(s)}{T_0 s}\right) e^{\lambda_1(s)x} \hat{q}_0(s) \tag{15}$$

The Taylor series expansion at the first order gives $\lambda_1(s) \approx (-2s/U_0(1+\kappa)) + o(s)$ leading to the following expression of shear stress in the frequency domain:

$$\hat{\sigma}(x,s) \approx \left(\frac{2\sigma_{0,0}}{Q_0} + \frac{2\mathcal{H}_0}{U_0 T_0 (1+\kappa)}\right) \frac{e^{-\tau(x)s}}{1 + K(x)s} \hat{q}_0(s) \tag{16}$$

Modelling the detachment–transport of algae and turbidity

A non-linear model (simulation of irrigation canals (SIC) software) was developed to simulate algae detachment and transport in response to flushing flows (Fovet *et al.* 2012a). Flow dynamics are given by solving the one-dimensional Saint-Venant equations. The algae detachment rate is a non-linear process written as a function of the mean bottom shear stress increase. Algae transport is simulated by solving the advection–dispersion equation:

$$\frac{\partial AC}{\partial t} + \frac{\partial QC}{\partial x} = \frac{\partial}{\partial x}\left(AD\frac{\partial C}{\partial x}\right) - \frac{\partial B}{\partial t} \tag{17}$$

$$\frac{\partial B}{\partial t} = -\frac{1}{\delta}\left(\frac{\sigma_0 - \sigma_{0,cr}}{\sigma_{0,cr}} - s_B\right)^{\eta} B \text{ if } \sigma_0 \geq \sigma_{0,cr}(1 + s_B) \tag{18}$$

$$\frac{\partial B}{\partial t} = 0 \text{ otherwise} \tag{19}$$

with C the drift algae concentration, B the fixed algae biomass and D the diffusion–dispersion coefficient. These equations are linearized around reference values C_0 and B_0, considering small variations of $c(x, t) = C(x, t) - C_0$ and $b(x, t) = B(x, t) - B_0$. The Laplace transform is applied to the linearized equation, so that Equation (17) reduces to an ordinary differential equation:

$$\frac{d^2\hat{c}(x,s)}{dx^2} - \frac{U_0 d\hat{c}(x,s)}{D_0\, dx} - \frac{s}{D_0}\hat{c}(x,s) = -\frac{s}{A_0 D_0}\hat{b}(x,s) \tag{20}$$

The detachment rate expression is linearized around a reference regime, in which the mean bottom shear stress $\sigma_{0,0}$ is chosen greater than $\sigma_{0,cr}(1 + s_B)$

(Equations (18) and (19)), leading to:

$$s\hat{b}(s) = -\frac{1}{\delta}\left(\frac{\sigma_{0,0}-\sigma_{0,cr}}{\sigma_{0,cr}} - s_B\right)^{\eta}\hat{\sigma}(s) - \frac{B_0\eta}{\delta\sigma_{0,cr}}\left(\frac{\sigma_{0,0}-\sigma_{0,cr}}{\sigma_{0,cr}} - s_B\right)^{\eta-1}\hat{b}(s) \tag{21}$$

and then

$$\hat{b}(x, s) = -\frac{G_t}{1+K_d s}e^{\lambda_1(s)x}\hat{q}_0(s) \tag{22}$$

$$G_t = G_{\det}\left(\frac{2\sigma_{0,0}}{Q_0} + \frac{2\mathcal{H}_0}{U_0T_0(1+\kappa)}\right) \tag{23}$$

The solution of Equation (20), which is similar to the Hayami transfer function, can be solved analytically (see appendix, available online at http://www.iwaponline.com/jh/015/166.pdf). During a flush, the relationship between drift algae concentration and turbidity was very well approached by a linear relationship (Fovet *et al.* 2012a). The turbidity Tb increases due to discharge release and is then given by:

$$T\hat{b}(x, s) = \frac{G_d}{1+K_d s}\left(\frac{e^{-\tau(x)s}}{1+K(x)s} - \frac{e^{-\tau H(x)s}}{1+K_H(x)s}\right)\hat{q}_0(s) \tag{24}$$

in which:

$$\tau_H(x) = \frac{x}{U_0} - K_H(x) \tag{25}$$

$$K_H(x) = \sqrt{\frac{2xD_0}{U_0^3}} \tag{26}$$

- $\tau_H(x)$ is the delay due to advection;
- $K_H(x)$ is the time constant due to dispersion;
- G_d is a gain due to the detachment process (conversion from discharge increase to biomass detachment) and the relationship between turbidity and algal concentration;
- K_d is the time constant linked to the detachment process;
- $\tau(x)$ and $K(x)$ are linked to propagation of hydraulic waves.

Details of the calculations are given in the appendix (http://www.iwaponline.com/jh/015/166.pdf). Physical characteristics can give a first guess of these parameters. However, they can be easily calibrated from field records of discharge and turbidity.

Comparison between the QLD and non-linear models

The flush is simulated with both models on a uniform rectangular canal 4 km long, 2.6 m wide, with a bed slope of 0.001 and a Manning roughness coefficient of 0.0192. The initial discharge and water level in the canal are 1.50 m^3 s^{-1} and 0.62 m, respectively. The flush is 2.5 h long, and the maximum discharge (2.5 m^3 s^{-1}) is reached in 1 h. The hydraulic delay τ and the attenuation parameter K are computed using Equations (11) and (12), which leads to $\tau = 1522$ s and $K = 715$ s at the downstream end ($x = 4$ km). The discharge is calculated with both linear (first order with delay linear solution) and non-linear (full Saint-Venant solution) models, at 2 and 4 km downstream from the flow release (Figure 1(a)).

Then, we calibrate the quasi-linear turbidity (QLT) model on the non-linear model simulation results. First, the delay and attenuation of particles transport, τ_H and K_H, are computed using analytic expressions (Equations (25) and (26)). Then, the gain and the detachment attenuation constant, G_d and K_d, are calibrated by optimizing the Nash criterion, using the non-linear response (with SIC) as a reference:

$$\text{Nash} = 1 - \frac{\sum_{i=1}^{n}\left(Tb_{\text{QLDM},i} - Tb_{\text{SIC},i}\right)^2}{\sum_{i=1}^{n}\left(Tb_{\text{QLDM},i} - Tb_{\text{QLDM,mean}}\right)^2} \tag{27}$$

with $Tb_{QLDM,i}$ the turbidity simulated by the quasi-linear model at time i, $Tb_{\text{SIC},i}$, the turbidity simulated by the non-linear model at time i, $Tb_{\text{QLDM,mean}}$ the mean value of turbidity simulated by the quasi-linear model and n the number of time steps.

Figure 1(b) shows that the amplitude and the phasing of the turbidity peak simulated with both models are very close. The parameters obtained using each calibration procedure and corresponding Nash criteria are reported in Table 1. The calibrated values of G_d and K_d are quite close between the two stations (2 and 4 km). It is also interesting to compare the simulated turbidities at other stations than those used for calibration. Therefore, we simulated the flush using the values calibrated on the intermediate

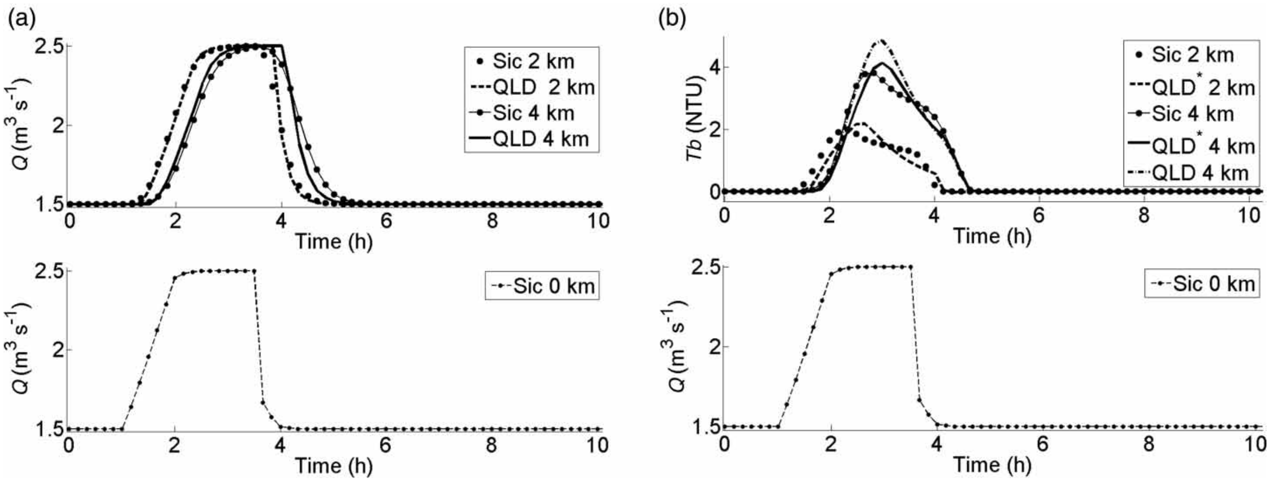

Figure 1 | Comparison of simulations of the quasi-linear distributed (QLD) model and the non-linear (SIC) model: (a) discharge simulation (b) turbidity simulation. QLD* are simulations where G_d and K_d are calibrated on turbidity simulations, for the curve denoted QLD 4 km, the detachment parameters are fixed to the values calibrated at $x = 2$ km.

Table 1 | Parameter values of the transfer functions at $x = 2$ and 4 km. The values denoted * are obtained by calibration, the others are computed using analytical expressions

Parameter	Unit	2 km	4 km	4 km (calibration on $x = 2$ km)
τ	s	613	1,522	1,522
K	s	505	715	715
$Nash_Q$	(–)	0.99	0.98	0.98
τ_H	s	1,377	2,898	2,898
K_H(s)	s	245	347	347
G_d	(–)	29*	28.5*	29
K_d	s	3,496*	4,332*	3,496
$Nash_{Tb}$	(–)	0.93	0.98	0.95

station (2 km). It does not seem to affect the quality of the simulation results at the downstream station (4 km), since we still have a Nash value of 0.95.

We conclude that the QLD model is able to reproduce the turbidity dynamics during a flush at each station of the reach by calibrating only two parameters: the time constant K_d and the gain G_d.G_d largely depends on the initial biomass (Equation (21) and the appendix, available online at http://www.iwaponline.com/jh/015/166.pdf).

Closed-loop adaptive control of the flush

The initial amount of algae in the canal B_0 is an important parameter of the model of turbidity response, as the gain of the transfer function (Equation (24)), G_d, is directly proportional to B_0 (see appendix, http://www.iwaponline.com/jh/015/166.pdf), and then B_0 determines the amplitude of the turbidity peak. Unfortunately, this amount is not known *a priori*. There is indeed a high variability of biomass distribution in the canal, and determining this distribution would require a large number of field samples, which is not feasible in a real-time management context. If we design an open-loop flush (e.g. by inverting the transfer function as proposed in Fovet *et al.* (2012b)), and the amount of algae in the canal is twice as large as what is assumed in the model, then the resulting downstream turbidity will be twice as large as the one initially designed. This should be avoided, since high turbidity is a problem for treatment processes and for end users. A closed-loop control strategy may be implemented in order to adapt the flush inflow so that a maximum turbidity is not exceeded. Since it is difficult to know *a priori* the amount of algae in the canal, we design an adaptive control strategy in order to address the issue of the poorly known parameter G_d of the transfer function.

General setup for adaptive control

Adaptive control was developed in order to adapt the control to a system that is slowly time varying or uncertain. For example, as an aircraft flies, its mass will slowly decrease as a result of fuel consumption; an adaptive control law would be able to adapt itself to such changing conditions. The general setup for adaptive control is depicted in Figure 2(a). The process is here defined by the expression $kG(s)$ that, according to (Equation (24)), corresponds to:

$$G(s) = \frac{1}{1+K_d(s)}\left(\frac{e^{-\tau(x)s}}{1+K(x)s} - \frac{e^{-\tau_H(x)s}}{1+K_H(x)s}\right) \tag{28}$$

$$k = G_d \tag{29}$$

The adaptive control action is defined by $u(t) = \theta(t)u_r(t)$.

Model reference adaptive control (MRAC)

MRAC was developed as a means to control a system with unknown parameters, such as the gain or a time constant (Ioannou & Sun 1996). We use in the following a classical scheme of MRAC for systems where the gain is unknown and needs to be estimated on-line. Ioannou & Sun (1996) also propose a method in order to normalize the tuning parameters of the adaptive controller using a reference signal.

The system we have to control is uncertain in the sense that it is very difficult to precisely estimate the initial amount of algae fixed to the bank of the canal B_0. This implies that the gain of the system is not known precisely. We describe below a model reference adaptive scheme that can estimate the gain of the system in an adaptive way.

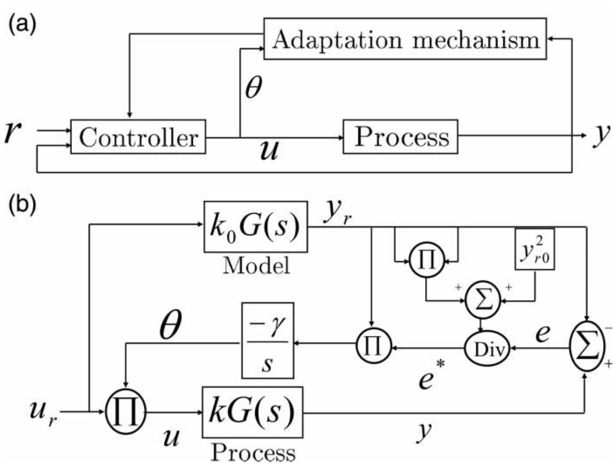

Figure 2 | (a) General setup for adaptive control. r is the pre-designed command, u is the adapted command, y the system response and θ the adaptation parameter. (b) Model reference adaptive control scheme. u_r is the open-loop command, u the adapted command, y_r the model response to u_r, y the effective response of the system, $\theta = k_0/k$, $e = y - y_r$ the error and $e^* = e/y_{r0}^2 + y_r^2$ is normalized using the reference signal y_{r0}.

Let us denote $kG(s)$ the system, and assume that k is unknown. Let k_0 be our initial guess, and let us denote $\theta = k_0/k$. We use the so-called Massachusetts Institute of Technology (MIT) rule for adjusting the parameter θ when k is unknown. The error is given by:

$$e = y - y_r = k\theta G(s)u_r - k_0 G(s)u_r$$

where u_r is the reference command signal (obtained, for example, using the open-loop method based on the simplified model proposed by Fovet et al. (2012b)), y_r the model output to the reference input and y the output (here, the turbidity measured in the canal). The MIT rule gives the following adaptation law (Ioannou & Sun 1996):

$$\frac{d\theta(t)}{dt} = -\gamma y_r(t)e(t)$$

where γ is a tuning parameter that defines the speed of the adaptation. When y and y_r are not guaranteed to be bounded, the problem of error minimization is ill-posed. In this case, Ioannou & Sun (1996) propose to divide each input and output signal by a normalizing signal $m > 0$, which can be written as $m^2 = n_{s,0}^2 + n_s^2$, where n_s is a reference signal and $n_{s,0}$ a non-zero reference value for n_s. Choosing $n_s = y_r$ and $n_{s0} = y_{r0}$, the normalized signals are then written:

$$e^* = \frac{e}{m} = \frac{y}{m} - \frac{y_r}{m} = y^* - y_r^* \tag{30}$$

The adaptive law becomes:

$$\frac{d\theta(t)}{dt} = -\gamma\frac{y_r(t)e(t)}{m^2} = -\gamma y_r(t)\frac{e(t)}{y_{r0}^2 + y_r^2} \tag{31}$$

This leads to the MRAC scheme depicted in Figure 2(b). The controller adaptation is parametrized by the adaptation speed γ and the initial assumption on $\theta(0) = \theta_0$.

Simulation results

Open-loop design

The reference input u_r is designed using the method proposed by Fovet *et al.* (2012b). The reference input is then replicated with a given periodicity, such that the adaptive system is able to adapt to changes.

Adaptive control with the quasi-linear model

We first focus on the case where the model is assumed to be known perfectly, except its gain, which is linked to the amount of algae fixed to the banks of the canal (quasi-linear model test).

We consider the rectangular canal used in the subsection on comparison between the quasi-linear distributed and non-linear models. The initial flow is $Q_0 = 1.5\,\mathrm{m^3/s}$ and the water depth is $Y = 0.6$ m. In order to obtain a reaction time for the turbidity response much shorter than the flush duration, the controller is applied to an intermediate sensor. This sensor must also be located not too close to the canal head in order to obtain a clear turbidity response. Therefore, the sensor is located at the middle of the canal, 2,000 m from the upstream end.

The reference signal y_{r0} is set to the mean value of y_r. Considering that the flush constraint is not to exceed a maximal turbidity value, fixed here to 20 nephelometric turbidity units (NTU), we fix $\theta(0) = 0.3$, which supposes that the unknown parameter has been 70% under-estimated. Parameter γ is related to the adaptation speed; therefore, a too low value of γ will limit the adaptation, and too high values will decrease the robustness of the controller. In order to obtain an estimation of γ, let us assume that $y_r(t) = \varsigma y_{r0}$ and $e(t) = \varsigma' y_{r0}$ in Equation (31). Then, we obtain the following estimation:

$$\gamma \approx \frac{(1+\varsigma)|\Delta\theta|}{\varsigma\varsigma'|\Delta t|} \tag{32}$$

In the simulated flushes, $\Delta\theta = (k_0/k) = 0.5$, $\Delta t = 21600$s, $\varsigma \approx 0.5$ to 2 and $\varsigma' \approx 0.5$, leading to $\gamma = 0.0001$. The simulation results are given in Figure 3. y_c is the desired turbidity evolution, y_r is the turbidity evolution using a single open-loop controller (response to reference input u_r) and y the evolution using the adaptive controller (response to input u corrected with feedback). We assume that the gain is $k = 2$, while the initial guess is $k_0 = 1$. In this case, using a purely open-loop strategy would lead to a downstream concentration twice as large as the set-point. We observe that the adaptive controller adapts the upstream discharge so that the downstream turbidity deviation is decreased.

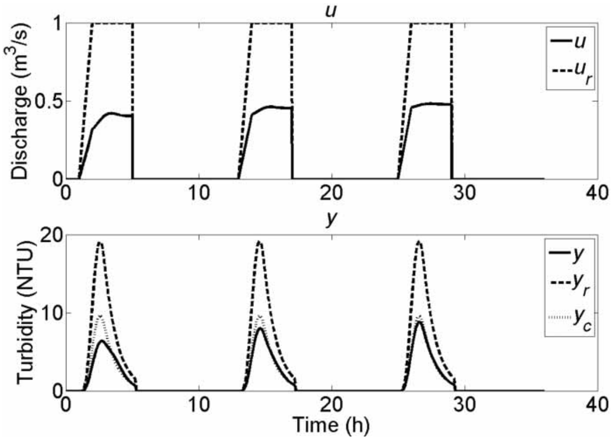

Figure 3 | Adaptive control with $k = 2$ and $k_0 = 1$ on the quasi-linear model.

The evolution of the parameter θ estimated using this scheme is depicted in Figure 4. We observe that θ converges towards $k_0/k = 0.5$ in a few cycles. This means that the adaptive controller is able to improve the

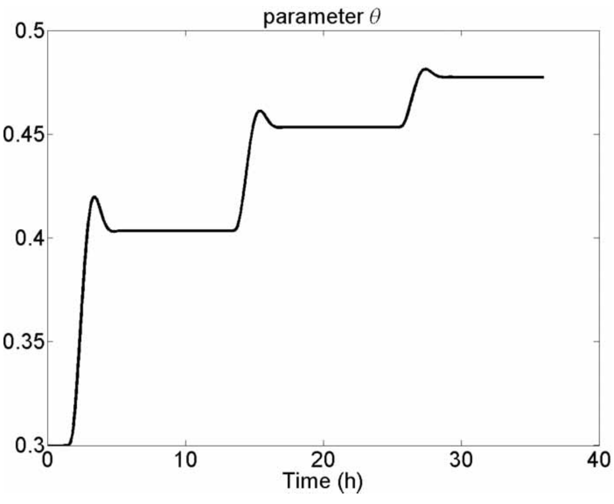

Figure 4 | Evolution of parameter θ with $k = 2$ and $k_0 = 1$ on the quasi-linear model.

estimation of the gain of the system while controlling its output.

FIELD IMPLEMENTATION

Strategy of flushing flows in the Canal de Provence

The Canal de Provence, located in southern France, is fed by the Verdon River, an alpine river characterized by a very low turbidity level. The canal comprises hard-surfaced open channels and galleries. Most of the sections are trapezoidal and open. The hydraulic management of the Canal de Provence is a fully automatic system, based on a control system called 'dynamic regulation' (Plantey & Molle 2003). The branch of Marseilles North (BMN) is 31.8 km long, with 20.6 km of open channel. The BMN is fed by the Bimont reservoir and ends in a tank called Vallon Dol with a 3.5 $m^3 s^{-1}$ downstream discharge capacity. It provides water for irrigation, industries and water treatment plants, and it provides 30% of the water consumption of the city of Marseilles. Some stations are equipped with continuous sensors; they are located at the main uptake or control points: Meyreuil, G1, G5, G7 and Vallon Dol (VD), respectively, 7, 12, 19, 21 and 32 km downstream from the Bimont dam. Hydraulic conditions (water level, discharge, gate opening) and quality parameters (turbidity and temperature) are recorded every 15 min at these stations. A sketch of the branch and the monitoring station locations are depicted in Figure 5.

Since 2004, flushing flows have been experimented on the branch in order to remove fixed algae. Different datasets (flow measurement, turbidity) have been used to calibrate the aforementioned transfer functions (Fovet *et al.* 2012a). Since 2011, closed-loop adaptive control has been implemented during the flushes: discharge is corrected in real time in order to maintain the turbidity under a maximum value, fixed to 20 NTU at Vallon Dol station. Indeed, high concentration of suspended material is a problem for the filtering systems, which are necessary to protect the pumping stations and the water treatment process. Records are available for three flushes in 2011, performed on 20 June, 6 July and 26 July.

Controller design

Performance of transfer models

To implement the control strategy, two models have been used: the QLT model derived for open-loop (Fovet *et al.* 2012b), which can be easily inverted, and the distributed model (QLD) presented here. The distributed model is also quasi-linear, but its inversion is not easy since the transfer function is the sum of two exponential functions. The validity of each model is first analyzed for the flushes performed on 20 June, 6 July and 26 July. Calibrations are carried out for the reach delimited by stations Bimont and G5. For the QLT model, the calibration gives Nash values of 0.38, 0.88 and 0.66, respectively. Except for the first one, which remains acceptable, the performance is similar

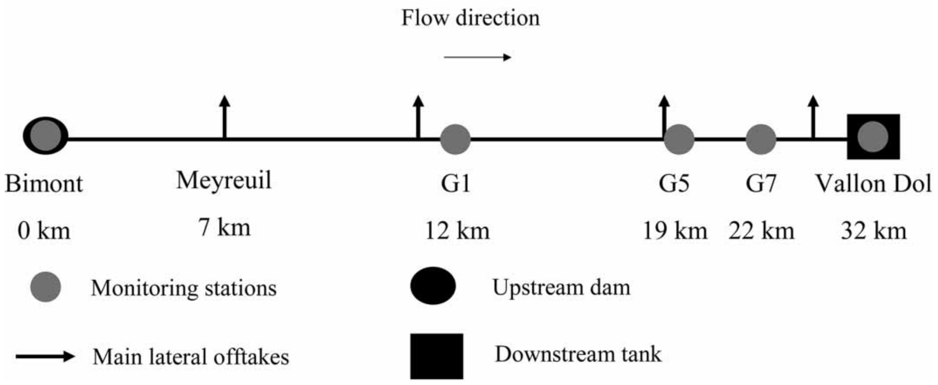

Figure 5 | Sketch of the branch of Marseilles North and the locations of monitoring stations.

to the calibrations obtained for previous periods (Fovet *et al.* 2012b). Using the distributed model (QLD), the Nash criteria are 0.77, 0.71 and 0.54, respectively. Simulations are presented in Figures 6 and 7. The simulation of the first flush is improved, whereas the performance is slightly decreased for the two others. Yet, the performance of both models is correct, as they both capture the evolution of turbidity during the flushes.

The interest of the non-distributed model for flush design *a priori* has been emphasized and discussed in Fovet *et al.* (2012b). The interest of the distributed model is to make the link between the turbidities at the different stations. Therefore, G5 is used for real-time control, whereas the objective is to maintain the turbidity under a maximum value throughout the canal. For example, Figure 7(c) shows the simulated turbidity at station G7, using parameters calibrated on the reach Bimont–G5. This property is interesting for feedback control because the location of the sensor used for feedback control can be located at a different station from the one where the maximum turbidity must be controlled. This point is discussed later (see the section on implications for sensor positioning).

Application to controller design

Flushes are initially designed using an open-loop controller based on inversion of the QLT model (Fovet *et al.* 2012b). The model was calibrated using a previous flush, performed 1 month previously. As deviations always remain between prediction (by simulation) and real-time field data, due to

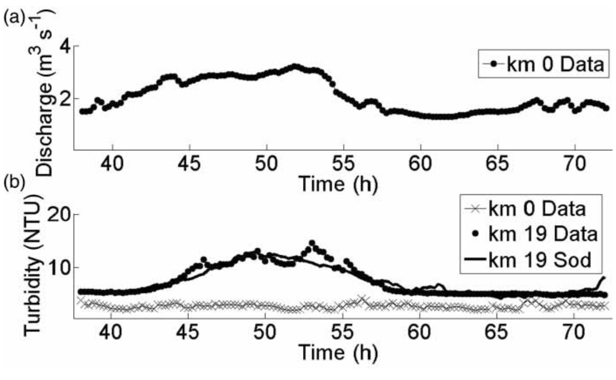

Figure 6 | Measured and simulated discharge (a) and turbidity (b) with the quasi-linear model at station G5 on 6 July 2011. The quasi-linear model simulation is denoted 'Sod' for second-order delay. It is not distributed.

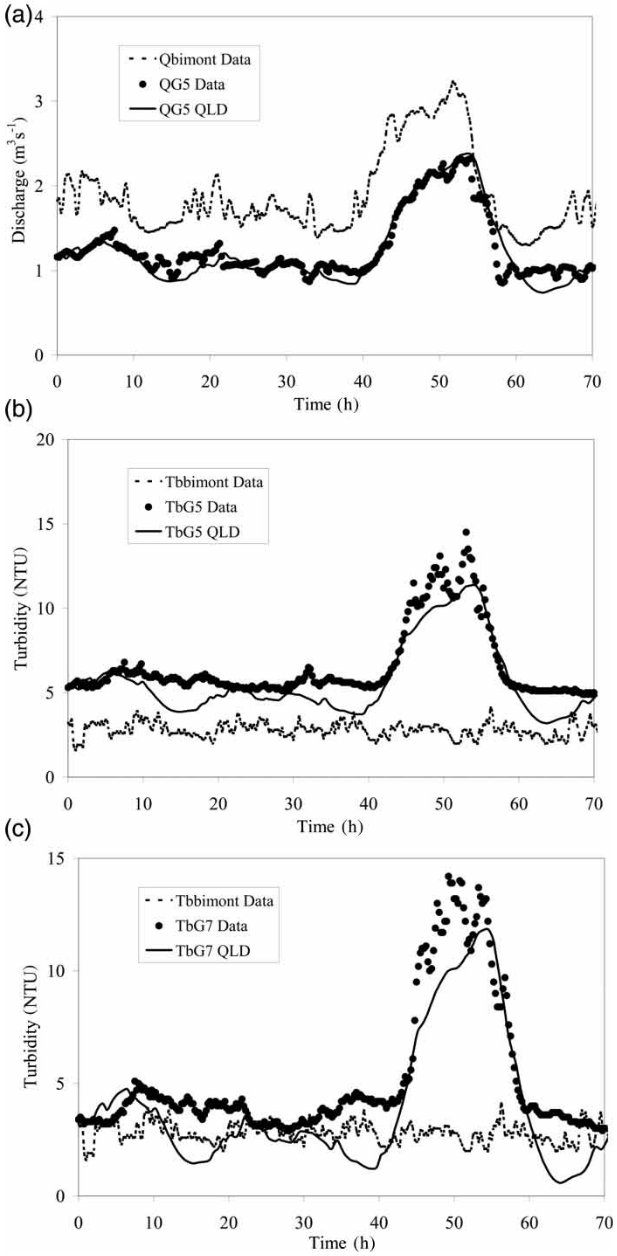

Figure 7 | Measured and simulated discharge (a) and turbidity (b) with the quasi-linear distributed model at station G5, and measured and simulated turbidity (c) at station G7, on 6 July 2011.

modelling or measurement errors, it is important to add a feedback. The closed-loop controller uses the intermediate station (G5), which is not too far from the canal head (Bimont dam), where the released discharge can be adjusted.

We expect two things from the controller: to maintain the turbidity under a maximum value throughout the canal, and to estimate the gain of the turbidity transfer function after the flush. The controller was initially designed with an adaptation speed fixed to 0.2, and the initial adaptation parameter θ_0 fixed to 1.

Discussion

Using turbidity for real-time management

Figure 8 shows the turbidity measured at the different monitoring stations, during each of the three tests and during the last flush performed in June 2012. During the flush, there is a clear increase of turbidity. This confirms that the flow release is able to detach a part of the material attached to the banks. The propagation dynamics from upstream to downstream are also visible. For most stations, the turbidity is under the desired threshold (about 20 NTU), except for one test at the downstream station.

The evolution of the controller is illustrated by Figures 9 to 12. In the first test (Figure 9), the actual flow is identical to the open-loop designed flow during the first 5 h. Actually, the controller was blocked by another control operation, which delayed the response of the adaptive turbidity control by about 1 h. In the second test (Figure 10), the increase is stopped at 1.7 $m^3 s^{-1}$ because of control bounding in order to limit the amount of water during the flush. In each case, the controller reacts well to the deviation observed between measurement and objective of turbidity: when the measured turbidity is lower than the predicted turbidity (region 1 for Figures 9 to 11 and region 3 in Figure 11),

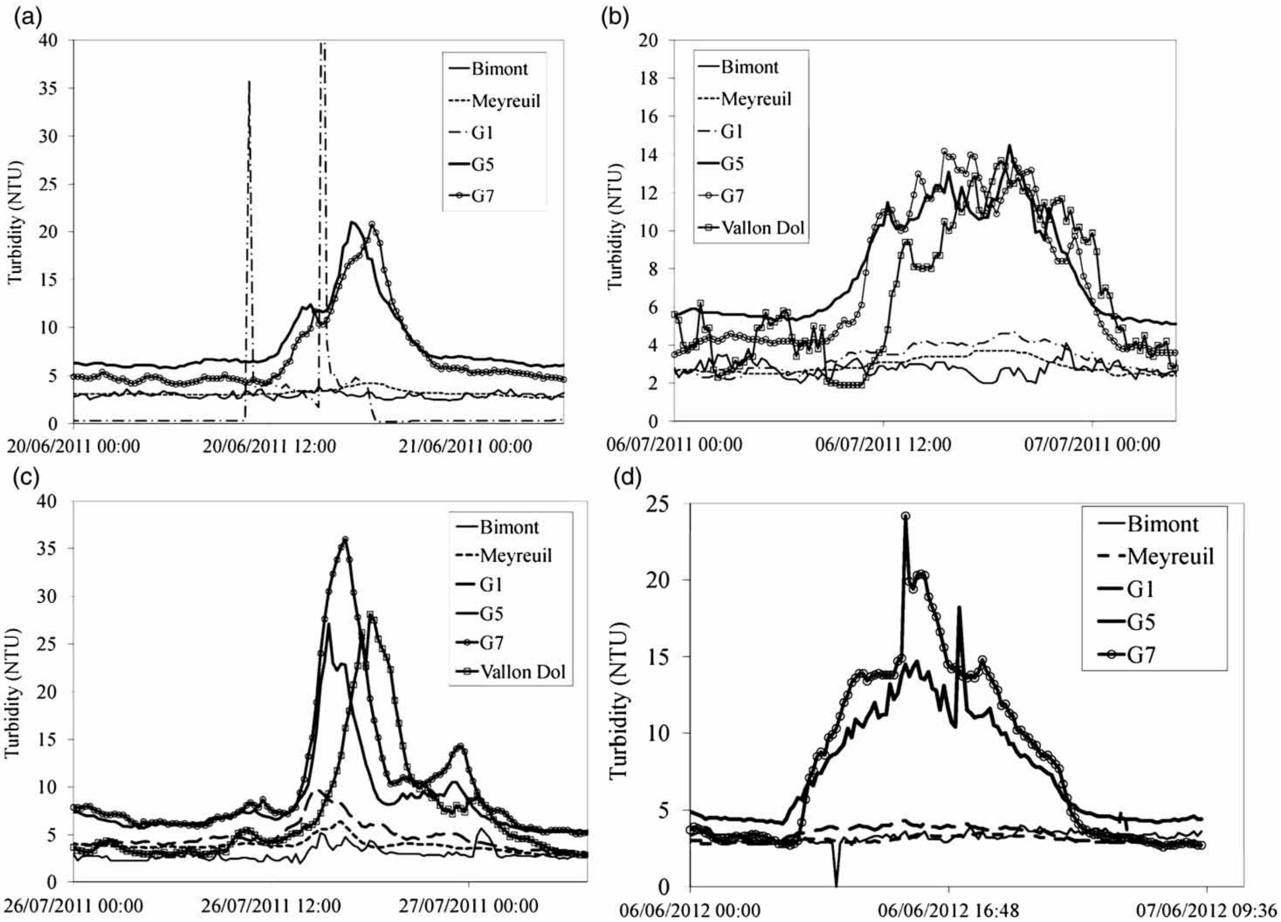

Figure 8 | Turbidity evolution at the monitoring stations in the branch of Marseilles North during adaptive control tests in: (a) 20 June 2011 (the Vallon Dol sensor was out of order), (b) 6 July 2011, (c) 26 July 2011 and (d) 6 June 2012 (the Vallon Dol sensor was also out of order).

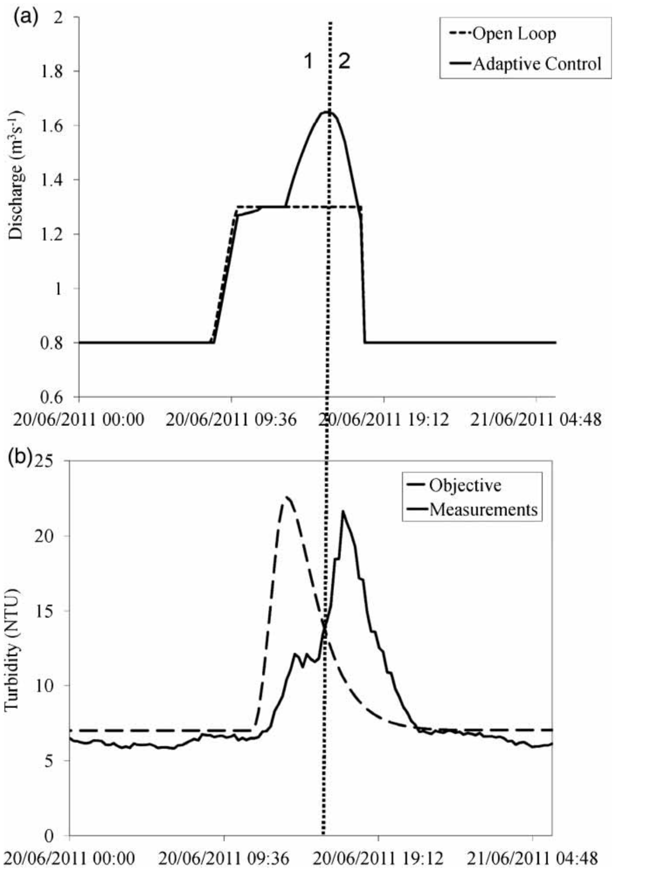

Figure 9 | (a) Discharge and (b) turbidity evolution at station G5 during adaptive control tests on 20 June 2011.

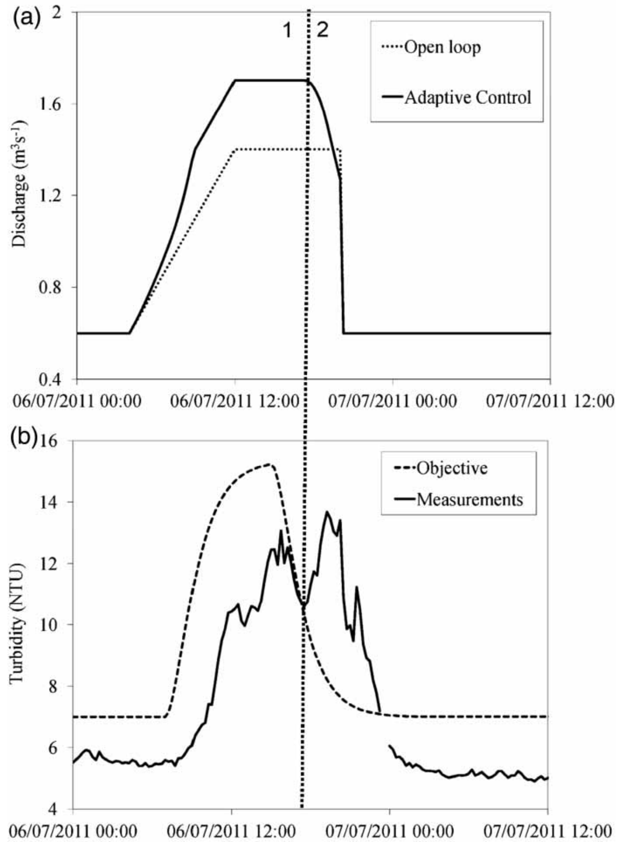

Figure 10 | (a) Discharge and (b) turbidity evolution at station G5 during adaptive control tests on 6 July 2011.

the discharge is increased. On the contrary, when the measurement exceeds the objective (region 2 for Figures 9 to 12), the discharge is decreased. The speed adaptation allows the controller to react fast enough despite the mis-estimations of the model parameters. When the gain is correctly estimated prior to the flush, as was the case in June 2012 (Figure 8(d)), a limited correction is necessary, since the observed response is very close to the measured one (region 1, Figure 12). This flush also confirms the reliability of the QLT model.

Importance of time delay

We observed in Figure 10 that the increase of the upstream discharge is faster than the predicted one. The delay of turbidity response was over-estimated because the delays were calibrated for an initial discharge of $1.5\,m^3\,s^{-1}$, whereas the initial discharge was less than $1\,m^3\,s^{-1}$.

For the last test (Figure 11), the delay of turbidity response was under-estimated. Moreover, there was an initial deviation between the measured and predicted initial turbidity levels. Therefore, the controller slowed the discharge increase during the flow rise, which slowed the turbidity increase. Then, the controller induced a large increase of discharge in order to reach the maximum turbidity. This resulted in a too large turbidity at the downstream station.

These first application tests highlight the importance of time delay estimation. This parameter is calibrated, but the delay varies with the regime. Since then, several calibrations have been performed on the Canal de Provence in order to provide different sets of parameters corresponding to different initial regimes.

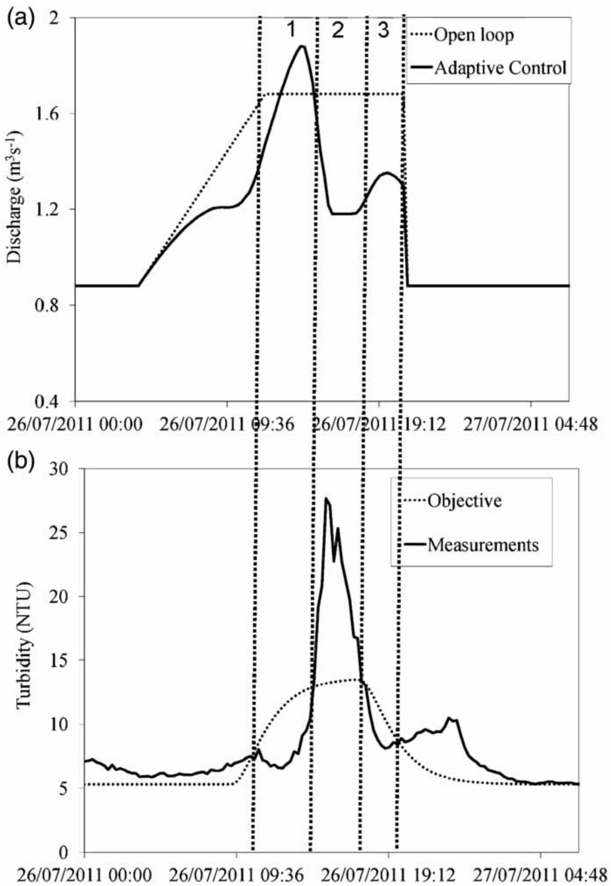

Figure 11 | (a) Discharge and (b) turbidity evolution at station G5 during adaptive control tests on 26 July 2011.

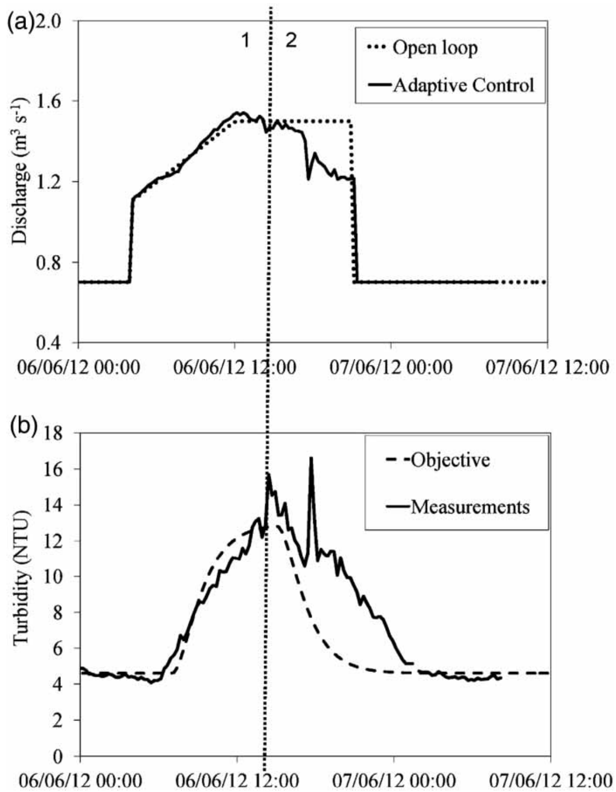

Figure 12 | (a) Discharge and (b) turbidity evolution at station G5 during adaptive control tests on 6 June 2012.

Implications for sensor positioning

As shown previously, the quasi-linear distributed model provides simulated turbidity for different abscissas. This is illustrated by Figure 8, in which we can see the propagation of the turbidity plume from upstream to downstream. This is precious information for feedback control. Indeed, the choice of sensor location for feedback control must be a compromise between the time delay for the response, which must be short enough to react quickly, and the amplitude of the turbidity response, which must be clear enough. For instance, we can see that station G1, which is located 12 km downstream from the branch head and is also equipped with a turbidity sensor, is not a relevant location for control. The turbidity at G1 is not very sensitive to the perturbation, for all the flushes, whereas the increase of turbidity is much clearer at the next station (G5).

CONCLUSION

We proposed a simplified linear model to simulate the impact of a flush on fixed algae detachment and transport in regulated open channels. The model is based on a linearization of the partial differential equations representing open-channel flow, algal detachment and transport. The solution is expressed in the frequency domain by using a Laplace transform. The resulting infinite-dimensional transfer function can be approximated using the moment matching technique, leading to a rational model with delay. The gain of the transfer function is proportional to the amount of fixed algae, a quantity that can vary according to the period of the year and even during a flush.

Based on this model, a closed-loop control strategy is designed to adjust the flow release, in real time, with the objective to control the resulting turbidity at different locations of the canal. The controller uses a MRAC scheme, which can also estimate the gain of the system.

The strategy is illustrated with different simulation results. Then, it has been implemented in an operational context. The model can reproduce the observed turbidity dynamics, and, since it is distributed, it also simulates the turbidity at any location in the canal. The field tests show the ability of the control scheme to regulate the water quality during a flush, but also highlight the importance of the sensor location for the design of the controller.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the contribution of ANR (French National Research Agency), who supported the research project ALGEQUEAU (ANR-06-ECOT-001).

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First received 28 October 2011; accepted in revised form 23 October 2012. Available online 2 January 2013