

**In-Plane Vibrations of a Thin Rotating Disk (Deshpande, M., and Mote, C.D., Jr., 2003, ASME J. Vibr. Acoust., 125, No. 1, pp. 68–72)**

**Jörg Wauer**

Professor

e-mail: wauer@itm.uni-karlsruhe.de

[DOI: 10.1115/1.1691073]

In a recent paper by Deshpande and Mote Jr., the stiffening effect for a thin disk due to its rotation on the eigenfrequencies of the in-plane vibrations was discussed. The paper is interesting and correct in all details but the reference list is not complete so that it seems to be useful to add some original papers on this topic not only for disks but also for other rotating structural members.

Mostly considered is this stiffening effect on the eigenfrequencies of the bending vibrations of rotating beams, e.g., slender turbine blades. Although even Biezeno and Grammel in their famous book [1] recognized this effect and Weidenhammer [2], for example, analyzed the natural frequencies of the bending vibrations of slender turbine blades starting with a more complete nonlinear formulation, some irritations arose in the late 1970s and early 1980s when several FEM computations using the classical geometrically linear deformation theory of beams predicted incorrect results calculating the eigenfrequencies: They found eigenfrequencies decreasing with increasing speed and instabilities (in the case of vanishing eigenfrequencies). It took a while to find the errors where the original work of Hodges and other co-authors (see [3], for example, as one of their concluding papers) clarified the discrepancies within a general nonlinear framework. Today, see contributions by Naguleswaran [4] and Pesheck et al. [5], for instance, the stiffening effect by centrifugal forces discussing bending vibrations of rotating beams is well-understood and discussed in detail.

But this stiffening effect due to rotation is typical not only for bending vibrations of beams but also for (in-plane) vibrations of rotating rings and disks and even for rotating cylindrical shells and circular cylinders. For such structures as mentioned for the case of disks by the authors of the discussed article, some papers once again formulated the governing boundary value problem within a completely linear deformation theory and therefore, found for the corresponding natural frequencies incorrect results. It was recognized in the 1980s where several original papers by different authors re-considered such problems (for rings and disks). In-plane flexural vibrations of thin rings were examined by

Endo et al. [6] and Wauer [7], the corresponding problem of disks was treated in three papers by Seemann and Wauer [8–10]. The analysis was continued by Seemann [11] extending the considerations to rotating, hollow and solid circular cylinders. For all types of such structural members, the centrifugal pre-deformation is essential and leads under normal circumstances (stress-free outside boundary of a disk or a cylinder, no radial constraint of a ring, for example) to a stiffening effect with increasing eigenfrequencies for higher speeds. Due to the more complex geometry so that radial and circumferential stresses interact even for the non-rotating case, the eigenfrequencies of planar vibrations of rings, disks and cylinders in lower speed ranges may decrease with increasing speed rate, finally for sufficient high speed they all monotonically increase. Only if the centrifugal force acts as a compressive load on the structure (rotating beam with a inner free and an outside fixed end, for example), the corresponding eigenfrequencies may globally decrease with increasing speed where finally, the structure buckles when the speed is sufficiently high.

## References

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## Response

The authors would like to thank Prof. Wauer for his interest in our publication. The references to the ongoing work in Germany are valuable in the context of this discussion and will certainly benefit many of our readers.