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NONLINEAR RESPONSE OF THICK LAMINATED COMPOSITE PLATES

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EXTENDED ABSTRACT

The computer algebra system MACSYMA is used to derive the nonlinear equations of motion of composite plate structures undergoing large deformations by using a higher-order shear-deformation theory. It is based on the assumptions that there is a cubic variation of the in-plane displacements through the plate thickness and that the transverse shear strains vanish at the free surfaces of the plate. The plates under investigation are made of linearly elastic anisotropic layers, and the von Karman strains are used in the derivation of the equations of motion. The equations of motion and boundary conditions are derived by using Hamilton's principle.

Linear analysis is performed of the free-vibrations of laminated plates. The analysis is carried out by using MACSYMA. The case of antisymmetric cross-ply laminates is considered. Uniform transverse harmonic loads are considered. The plate is simply-supported. Solutions are obtained for the case of plate strips in cylindrical bending. The free-vibration problem is solved by a Navier-type method, and the natural undamped frequencies and mode shapes are calculated. The state-space concept is used to solve exactly the linear dynamic equations for the case of cylindrical bending. The linear analysis is the first step in studying the nonlinear vibrations of the structures.

An analytical technique for the analysis of the nonlinear response of a wide variety of physical systems that exhibit quadratic and cubic nonlinearities is described. The technique is a perturbation method, such as the method of multiple scales or the method of averaging, that attacks the Lagrangian of the system rather than the equations of motion and boundary conditions. Writing down the Euler-Lagrange equations of the averaged Lagrangian yields ordinary-differential equations that govern the modulation or evolution of the amplitudes and phases of the response. The technique produces accurate second-order, uniformly valid, approximate solutions in the neighborhoods of different resonances that can arise in different physical systems.

The time-averaged-Lagrangian technique is implemented in a MACSYMA code that produces second-order perturbation solutions of the nonlinear equations of composite-plate structures. The effects of the quadratic nonlinearities are incorporated into the solution. First-order differential equations are derived for the evolution of the amplitudes and phases for the following resonances: primary resonance, subharmonic resonance of order one-half, and superharmonic resonance of order two. The evolution equations are used to determine the frequency-response equations, which are used in turn to obtain representative frequency-response and force-response curves for each case. The local stability of these solutions is investigated. Stable and unstable solutions may coexist. Multi-valued solutions are possible, and in these cases the initial conditions determine which solution describes the response. Moreover, the multi-valuedness of the solutions is responsible for the jump phenomenon. The results show that subharmonic resonances of order one-half cannot be activated unless the excitation amplitude exceeds a threshold value.