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Nested Threshold Autoregressive (NeTAR) Models for Studying Sources of Nonlinearity in Streamflows

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Historical development of nonlinear stochastic modeling of streamflows is discussed. Physical considerations and graphical investigations of daily streamflows revealed that air temperature and state of basin storage are the most important sources of nonlinearity in catchments with seasonal snow accumulation. The average temperature for the preceding three days, and the flow one or two days earlier were adequate proxies for the temperature and storage conditions of a catchment. The Nested Threshold Autoregressive (NeTAR) model, which considers these sources of nonlinearity simultaneously, was applied to two years of daily flows of the Oldman River near Brocket in Alberta, Canada. A third year of daily data was used for validating the model. The final NeTAR model provided useful insights into the dynamics of this streamflow system.

Introduction

Applications of time series analysis methodologies in hydrology, and particularly in streamflow, are rooted to the demand for models that can generate flow sequences that are statistically indistinguishable from the relevant historical series. Lawrance and Kottegoda (1977) review applications of traditional time series methods to streamflows and their hydrological adaptations; they point out the distinctive non-Gaussian and periodic features of typical streamflow time series and suggest that nonlinear approaches would be more appropriate than linear models such as the ARMA (Autoregressive Moving Average) and the TF (Transfer Function).

Hydrologists have long recognized the nonlinear interrelationships between hydrological and meteorological variables associated with streamflows. It is well known that in drainage systems where precipitation can occur as either rain or snow, air *temperature* plays an important nonlinear role during late winter and early spring. The state of *basin storage* is also a very important source of nonlinearity; its effect is more pronounced during rainy seasons. Notable attempts in addressing these causes of nonlinearity in streamflows were made by Tong *et al.* (1985) and Kachroo and Natale (1992).

Tong *et al.* (1985) proposed a TARSO (Open-loop Threshold Autoregressive Systems) model for exploring the possibility of constructing simple stochastic models to address the nonlinearity caused by temperature. To examine empirically the possibilities of threshold models in dealing with the nonlinear effects associated with the melting of snow and ice, they used daily observations on flow, precipitation and temperature for two streamflow systems, the Vatnsdalsá and the Jökulsá eystri, in Northern Iceland. They found their model to be more accurate than linear models and the estimated threshold values to be in concordance with the main characteristics of the rivers.

Kachroo and Natale (1992) proposed a *multilinear* model by modifying a simple linear model to address nonlinearity caused by soil moisture. Soil moisture content is an important factor that determines the water storage condition of a basin, which we here on refer as *basin storage*. Their modeling procedure looks for systematic relations between the residuals of a simple linear rainfall-runoff model and the observed discharge as evidence of nonlinearity. Since a simple linear rainfall-runoff model is constrained to have a constant proportionate runoff or gain factor, the presence of systematic relationships (nonlinearity) led them to using a variable gain factor. Their multilinear model (variable gain factor) attributes low gain factor for low soil moisture, medium for medium and high for high soil moisture conditions.

Katchroo and Natale (1992) verified that one may substitute the observed streamflow as an index of the catchment moisture condition. They also proposed a multilinear model that uses weighted least squares to estimate the response functions, and diagrams of the residual error distribution to define low, medium and high flow regions. They found their model to be superior to the simple linear model. The idea of using varying response functions as a function of the prevailing soil moisture conditions was originally suggested by Todini and Wallis (1977).

In this paper, we discuss a NeTAR (Nested Threshold Autoregressive) model for addressing nonlinearities in daily streamflows caused by both temperature and basin storage simultaneously using data for the Oldman River in Alberta, Canada. The NeTAR modeling procedure is discussed in detail in Astatkie *et al.* (1997).

Sources of Nonlinearity in Streamflow Systems

Streamflow generation is a very complex process. For example, consider a dynamic system that has precipitation (rain and snow) input and evapotranspiration and runoff outputs. During periods when there is no snow cover, some portion of the rainfall input infiltrates to subsurface storage. Some of this unsaturated zone subsurface water percolates to the water table and becomes groundwater storage and some evapotranspirates as an output from the system. Both unsaturated zone and groundwater zone water can contribute to streamflow. Some rainfall is captured in surface storage and eventually either infiltrates or evaporates. Some can contribute to streamflow as surface runoff. During periods when there is a snow cover and the surface temperature is below the freezing point, snow or rain will remain on the surface as snow storage. When the air temperature is above the freezing point, the snow will melt; the snowmelt will have the same possible paths as the rain described above except that the ease of infiltration (infiltrability) will depend on the temperature and water content of the surface soil layers. This dynamic system is essentially nonlinear because it depends (nonlinearly) on the state condition, which mainly depends on the temperature and storage conditions of the basin. Moreover, Kavves and Delleur (1984), for example, have shown that the daily streamflow process has seasonal variations in the covariance, has persistence properties that depend on the storage condition of the basin at the specified time origin of the flow process, and is not time reversible. From the statistical point of view, time *irreversibility* is a strong indication of nonlinearity.

A detailed explanation of the major sources of nonlinearity, using real streamflow, temperature and precipitation data, is given below.

Basin Storage

It is well known that catchment response to rainfall is dependent on the state of basin storage at the time of deposit. When the storage is low, the catchment absorbs much of the rainfall. On the other hand, when the storage is high, the catchment absorbs a smaller portion of the rainfall; more of it becomes runoff.

Since measuring the amount of basin storage on a daily basis is almost impossible, we suggest using antecedent flow levels as a proxy. This is mainly following Kachroo and Natale (1992) who used flow values as a proxy for soil moisture content, which is an important factor in determining the state of basin storage. The use of flow values as proxy for initial soil moisture condition can be traced back to the work of Linsley and Ackermann (1942) who found groundwater discharge of streams with continuous flow to be a good index to initial (*i.e.* pre-event) moisture conditions.

To verify this, we investigated the nonparametric regression of current flow, Q_t, on d-days earlier flow, Q_{t-d}, for various rivers and observed piecewise linear functions (with two pieces) similar to that shown in Fig. 1, which shows the super-

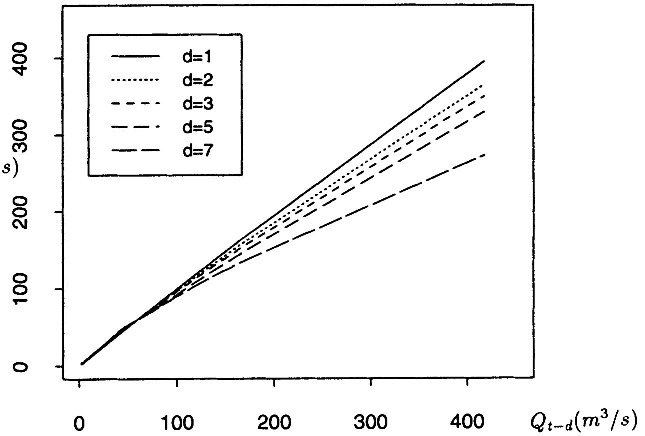


Fig. 1. Plot of the supersmoother regression of Q_t on Q_{t-d} for d = 1,2,3, 5 and 7 for the Oldman River near Brocket.

smoother regression of Q_t on Q_{t-d}, d = 1,2,3,5,7 for the Oldman River based on daily data from 1 January 1988 to 31 December 1990. We give further description of this river p. 331. The supersmoother is a nonparametric regression technique useful for smoothing data. This figure suggests that the slope of the upper component declines as the lag increases, manifesting direct runoff as a short-term phenomenon. It is often assumed that Q_t in the lower component is mainly due to groundwater flow, whereas Q_t in the upper component is mainly due to direct runoff. In our discussion from here on, we consider the lower component as "low basin storage" and the upper component as "high basin storage."

It can be seen in Fig. 1 that the nonlinear relation between Q_t and Q_{t-d} can be approximated by piecewise linear functions. The idea of using different functions, depending upon the prevailing storage conditions of the basin is not new. Todini and Wallis (1977), for example, suggested the same in the Constrained Linear System (CLS) model. For most rivers, either Q_{t-1} or Q_{t-2} is the best indicator of the current state of basin storage. For example, the best indicator for the Oldman River near Brocket is Q_{t-2}.

Direct runoff from rainfall inputs accounts for the high flows of the Oldman River. To illustrate the effect of basin storage Q_{t-2} on flow, we show (in Fig. 2) a three-dimensional perspective plot of $\Delta Q_t = Q_t - Q_{t-1}$ *versus* P_{t-1} and Q_{t-2} when $P_{t-1} > 0$ and $\bar{T} > 9°C$, where $P_{\bar{t}-1}$ is precipitation on the preceding day, and $\bar{T}$ is the average temperature for the preceding three days [$\bar{T}$ = Avg(T_{t-1} ; T_{t-2} ; T_{t-3})]. We used the flow and precipitation values when $\bar{T} > 9°C$ to make sure that the precipitation values are rainfall and flow is not receiving contributions from snowmelt. The condition that $P_{t-1} > 0$ allows us to use the data only when there was a rain on the previous day.

Fig. 2 shows that rainfall below 10 mm/day does not lead to a gain in flow no matter what the basin storage condition. When the storage is low, even a rainfall of 40

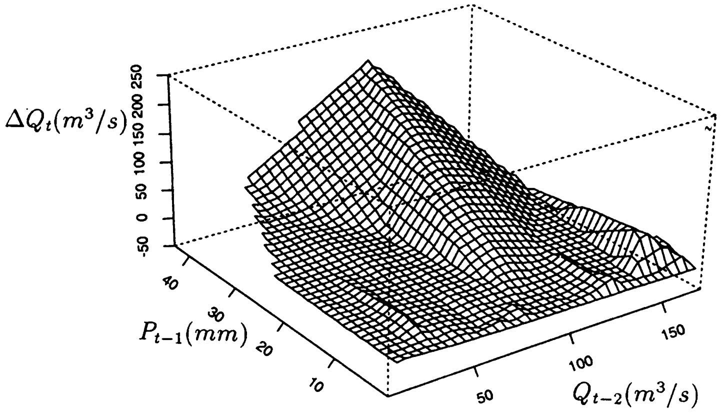


Fig. 2. Three dimensional plot of change in Q_t *versus* P_{t-1} and Q_{t-2}, when $P_{t-1} > 0$ mm and $\bar{T} > 9°C$ for the Oldman River near Brocket. This plot is based on smoothed data.

mm/day has only a small effect on ΔQ_t. The proportion of the rainfall that becomes direct runoff increases as the storage gets higher (as Q_{t-2} increases). Fig. 2 also suggests that a second order polynomial can better approximate the relation between streamflow and rainfall than a linear one as used, for example, by Tong *et al.* (1985) and Kachroo and Natale (1992).

Temperature

Antecedent air temperature values can be used as proxy for the temperature condition of a basin when addressing nonlinearities due to snowmelt and evapotranspiration. Many simple but effective snowmelt models use only air temperature (Watt *et al.* 1989). Although evapotranspiration rates may be calculated from meteorological variables such as air temperature, humidity, wind speed and duration of bright sunshine, temperature can be used as an adequate proxy in many situations.

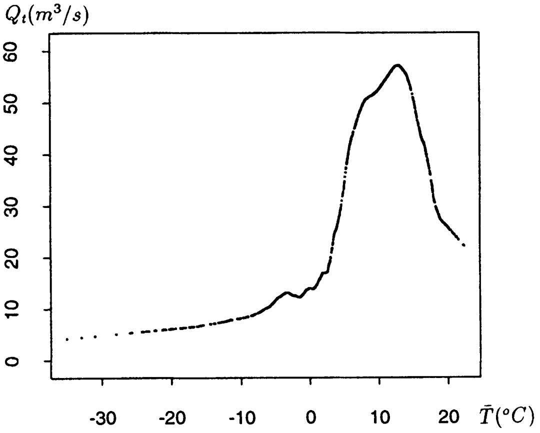


Fig. 3.
Plot of the supersmoother regression of Q_t on $\bar{T}$ for the Oldman River near Brocket.

Fig. 3 depicts a typical relation between flow and temperature for Canadian river basins with seasonal snow accumulation. This figure (the supersmoother of Q_t on $\bar{T}$ for the Oldman River near Brocket) shows that flow increases very slowly for temperatures increasing to 0°C picks up sharply near 0°C because of spring snowmelt and then declines rapidly after snowmelt is over ($\bar{T}$ reaches about 10°C). Fig. 3 suggests the potential for approximating the relation between streamflow and temperature by piecewise linear functions, with possibly three pieces. Other rivers might show only two components, one rising and one declining.

To sum up, the two important sources of nonlinearity in the streamflow process, basin storage and temperature, were considered separately in previous time series work. Kachroo and Natale (1992) considered a multilinear model to deal with the nonlinearity due to basin storage, and Tong *et al.* (1985) considered a TARSO model to deal with the nonlinearity induced by temperature. However, accounting for the nonlinearity due to one source is only half a remedy. We use the NeTAR model that considers both sources simultaneously to model daily flows of the Oldman River. So, our goal is to accurately model the nonlinear response of streamflow to precipitation and temperature inputs under different states of temperature and basin storage by the NeTAR model.

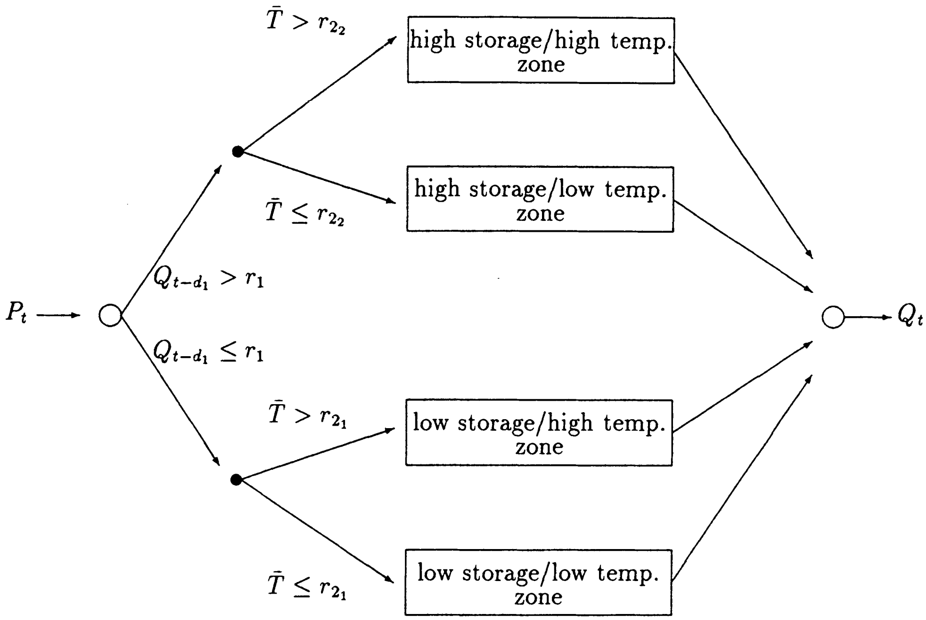


Fig. 4. Structure of a typical NeTAR model for streamflows. Where r_1 is the first stage threshold for distinguishing low and high basin storage and r_{2_1} and r_{2_2} are the second stage thresholds (under low and high basin storage conditions respectively) for distinguishing low and high temperature conditions of a basin.

A typical NeTAR model for streamflows has the structure shown in Fig. 4. The NeTAR model for streamflows has the following form

$$Q_t^{(ij)} = a_0^{(ij)} + \sum_k a_k^{(ij)} Q_{t-k} + \sum_l b_l^{(ij)} T_{t-l} + \sum_m c_m^{(ij)} P_{t-m} + \sum_n d_n^{(ij)} P_{t-n}^2 + \varepsilon_t^{(ij)} \tag{1}$$

where Q_t is flow, P_t is precipitation and T_t is temperature on day t; $i = 1$ for low basin storage and $i \equiv 2$ for high basin storage condition; and $j = 1,2$ (low, high) or $j = 1,2,3$ (low, moderate, high) temperature conditions of the catchment. For a specific i and j, ε_t is a white noise process with zero mean and constant variance, and $Q_0^{(ij)}$ is the average level of $Q_t^{(ij)}$. The inclusion of a second order polynomial of lagged precipitation is to account for such relationships as revealed in Fig. 2.

In general, the NeTAR model given in Eq. (1) consists of four to six zones, purposefully formed to have a very high degree of linearity of the relations and homogeneity of the residuals within a zone. Heterogeneity among zones reflects the merit of NeTAR models over linear ones. We adapt the following summary of NeTAR modeling procedure from Astatkie *et al.* (1997).

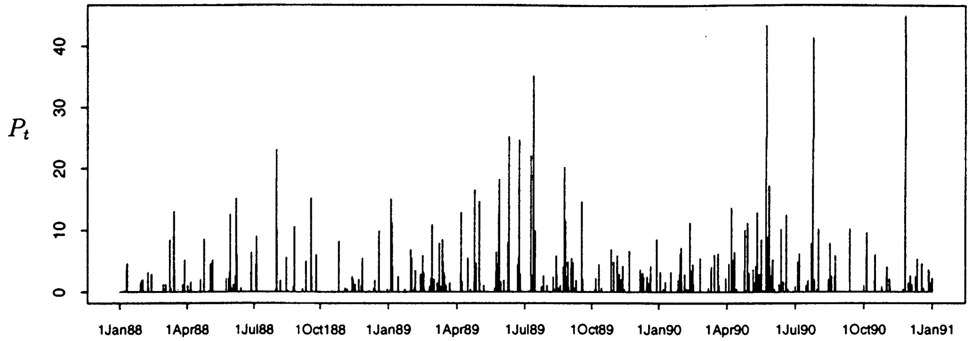
Summary of NeTAR Modeling Procedure

The complete procedure for modeling NeTAR models for streamflows is summarized as follows.

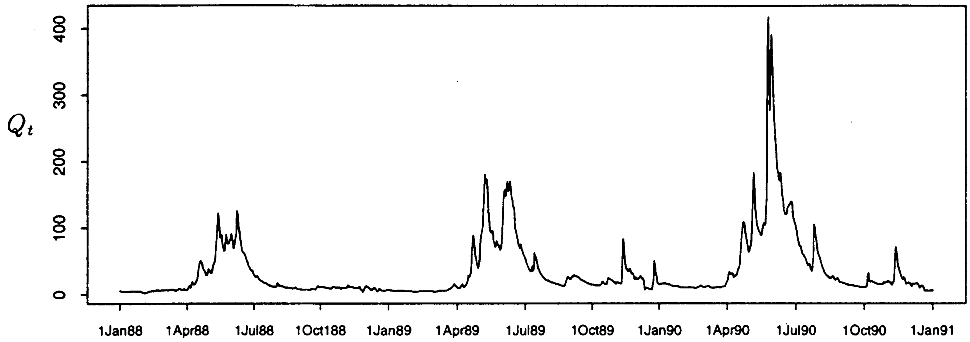
1) Choose a maximum lag k of Q_t based on the patterns in its ACF (autocorrelation function) and PACF (partial autocorrelation function). Use the same lag for P_t and T_t. The value of k is usually chosen to be between seven and ten.
2) Form a data set consisting of ($Q_t, Q_{t-1}, ..., Q_{t-k}, P_t, P_{t-1}, ..., P_{t-k}, P_t^2, P_{t-1}^2, ..., P_{t-k}^2, T_t, T_{t-1}, ..., T_{t-k}$).
3) Estimate the first and second stage threshold values. To do this, first choose either Q_{t-1} or Q_{t-2} as a first stage zone variable and estimate the threshold value that delimits low and high storage conditions as a first stage threshold (r_1). Then estimate the corresponding second stage threshold values (r_{2_1} and r_{2_2}) using $\bar{T}$ as a second stage zone variable. In this step, plots of lagged nonparametric regression estimates are very useful.
4) Form subset data sets, one for each zone, according to the estimated first and second stage threshold values by keeping a tag for the original time order and use a stepwise regression procedure to find a preliminary model. Then do fine tuning using hydrological considerations to identify a final NeTAR model.

Once a final model has been identified, it may be estimated by the least-squares method and then checked for adequacy by examining the reassembled and normalized residuals.

(*a*) Precipitation at Pincher Creek A, the Oldman river basin



(*b*) Flow near Brocket, the Oldman river basin



(*c*) Temperature at Picher Creek A, the Oldman river basin

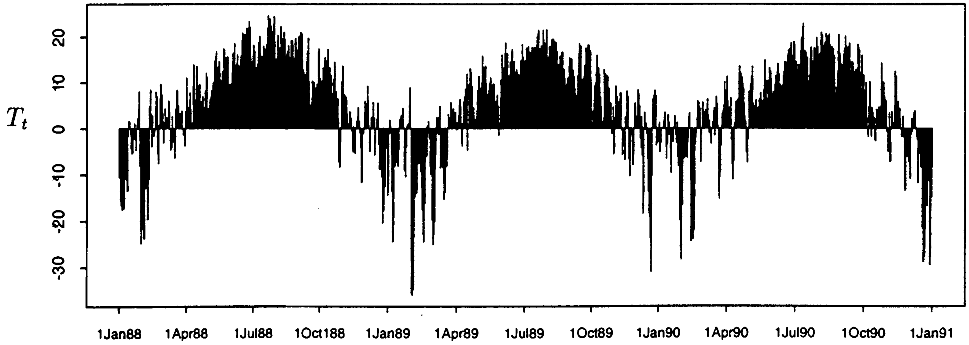


Fig. 5. The Oldman River near Brocket, Alberta. (a) Precipitation (mm/day), (b) Flow (m^3/s), and (c) Temperature (°C), daily data from 1 January 1988 to 31 December 1990.

Table 1 – Profile of the gauging and meteorological stations

Station Name	Station Number	Location Latitude	Location Longitude	Drainage Area	Station For
Oldman River near Brocket	05AA024	49° 33′*N*	113° 49′*W*	4,400 km^2	Flow
Pincher Creek A	3035202	49° 31′*N*	114° 00′*W*	-	Prec. & Temp.
Lethbridge A	3033880	49° 38′*N*	112° 48′*W*	-	Prec. & Temp.

NeTAR Modeling of The Oldman River near Brocket

In this section, we develop and validate a NeTAR model for the Oldman River in Alberta, Canada. Three years of daily data (1 January 1988 to 31 December 1990) on *flow* (Q_t, m^3/s) *precipitation* (P_t, mm), and *temperature* (T_t, °C) were obtained from Environment Canada publications; flow data from *Surface Water Data*, and precipitation and temperature data from *Monthly Record: Meteorological Observations in Canada*. We selected three-year daily data sets (1,096 data points) based on practicability: large enough for a meaningful analysis and small enough considering the availability of published data without interruption.

We selected the Oldman River near Brocket (a natural flow station with a recording gauge) for flow, and the Pincher Creek A station (a meteorological station closest to the gauging station and in the same drainage basin) for precipitation and temperature data. See Table 1 for further information about these stations. The precipitation values are the total rainfall and snowfall (as converted into their rainfall equivalent by Environment Canada) that fell on day t. Average of the daily minimum and maximum temperature readings were used as the temperature for the day.

Precipitation and temperature data were not recorded at the Pincher Creek A station during 1 September 1989 to 31 December 1989. We filled out this gap by data observed at Lethbridge A station, which is the closest alternative meteorological station to Pincher Creek A within the Oldman River basin. The three-year daily precipitation, flow and temperature series are shown in Fig. 5 (a)-(c). We develop a NeTAR model for the Oldman River using two years (1 January 1988-31 December 1989) data, and then validate the model by the third year (1 January 1990-31 December 1990) data in the following sub sections.

NeTAR Modeling using Two Years Data

Based on the pattern observed in the ACF and PACF of Q_t, a maximum lag of $k = 10$ was chosen, and then the best indicator of current state of basin storage was identified to be Q_{t-2}.

The first stage threshold parameter was estimated to be $\hat{r}_1 = 35$ m^3/s, using the supersmoother of Q_t on Q_{t-2} estimates shown in Fig. 6 (a). The supersmoother of Q_t on temperature for low ($Q_{t-2} \leq 35$ m^3/s) and high storage ($Q_{t-2} > 35$ m^3/s) conditions are shown in Fig. 6 (b) and (c) respectively. In Fig. 6 (b) we see an impression of two

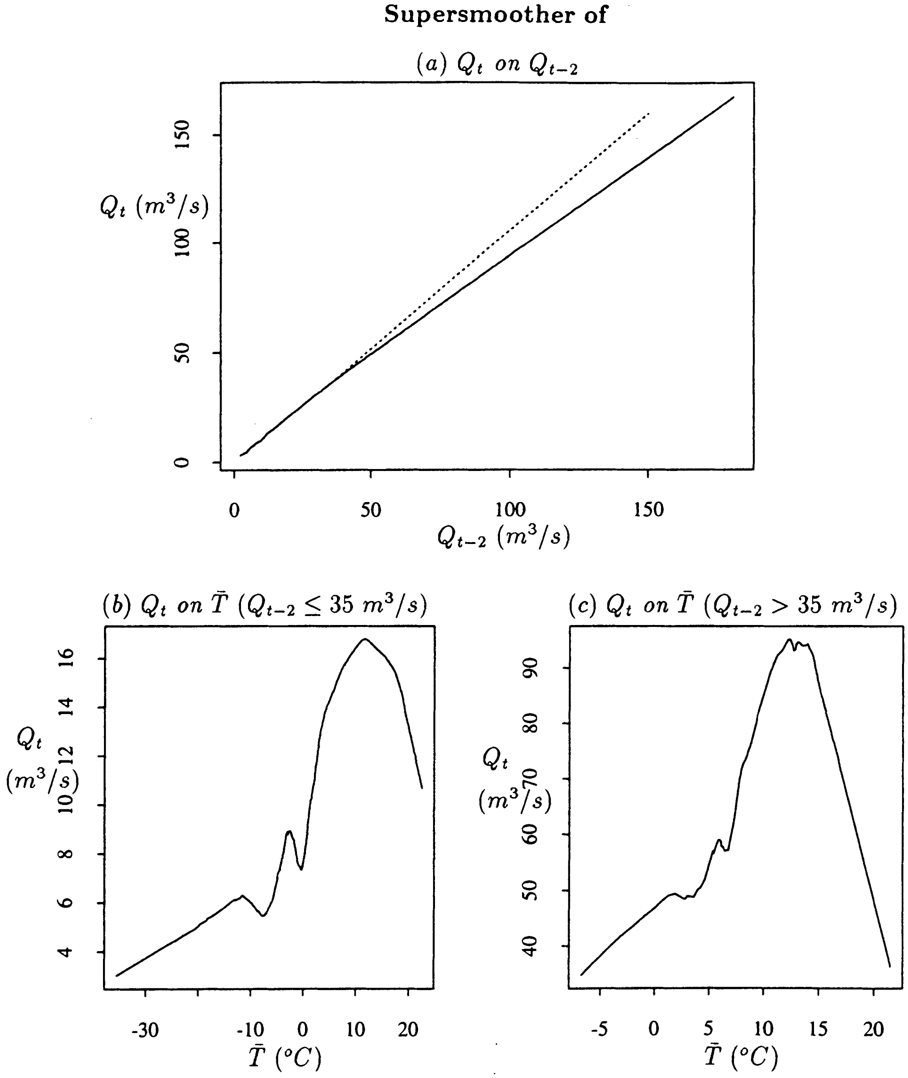


Fig. 6. The Oldman River near Brocket, Alberta. The supersmoother of (a) Q_t on Q_{t-2}, (b) Q_t on $\bar{T}$ when $Q_{t-2} \leq 35\ \text{m}^3/\text{s}$, and (c) Q_t on $\bar{T}$ when $Q_{t-2} > 35\ \text{m}^3/\text{s}$ based on two-year data. The dashed line in (a) is a reference line to reveal the breakpoint.

linear pieces, a rising linear function below ≈11°C and a declining one over ≈11°C. The transition point, as a second stage threshold for the low storage condition, was estimated to be $\hat{r}_{2_1} = 11.1$°C.

Also, Fig. 6 (c) suggests two zones in the high storage region with a threshold value estimated to be $\hat{r}_{2_2} \equiv 12.9$°C. Table 2 describes the NeTAR model zones and their physical interpretations.

Table 2 – Description of NeTAR zones for the Oldman River

Zone No.	Flow and Temp. Range	Portion of year (%)	Time of year and season	Physical insight
1	$Q_{t\text{-}2} \leq 35$ m^3/s and $\bar{T} \leq 11.1°C$	55	Late fall and winter (Oct.-April)	In late fall, low storage implies high infiltration potential & hence no response to rain. In winter, precip. occurs as snow which adds to snowpack.
2	$Q_{t\text{-}2} \leq 35$ m^3/s and $\bar{T} > 11.1°C$	22	Late summer and early fall (July-Oct.)	Same high infiltration potential as Zone 1, but larger rainfalls in Zone 2 cause some runof.
3	$Q_{t\text{-}2} > 35$ m^3/s and $\bar{T} \leq 12.9°C$	13	Late spring & early summer (May-June)	High basin storage and temp. over 0°C results in snowmelt runoff which is generally proportional to temperature.
4	$Q_{t\text{-}2} > 35$ m^3/s and $\bar{T} > 12.9°C$	10	Summer (June-July)	Snow has all melted but flows & hence basin storage remain high. High storage implies low infiltration potential & hence rainfalls yield some runoff.

Base models were identified using the data subsets corresponding to these zones. Then the following final NeTAR model was identified. The estimated standard errors are shown in parentheses.

$$\hat{Q}_t = \begin{cases} \underset{(0.051)}{1.15Q_{t-1}} = \underset{(0.055)}{0.12Q_{t-2}} & \text{if } Q_{t-2} \leq 35\ \mathrm{m}^3/\mathrm{s} \text{ and } \bar{T} \leq 11.1^{0}\mathrm{C} \\ \underset{(0.089)}{1.98Q_{t-1}} - \underset{(0.089)}{0.97Q_{t-2}} + \underset{(0.001)}{0.014P^2_{t-1}} & \text{if } Q_{t-2} \leq 35\ \mathrm{m}^3/\mathrm{s} \text{ and } \bar{T} > 11.1^{0}\mathrm{C} \\ \underset{(0.082)}{1.68Q_{t-1}} - \underset{(0.076)}{0.74Q_{t-2}} + \underset{(0.142)}{0.58T_t} & \text{if } Q_{t-2} > 35\ \mathrm{m}^3/\mathrm{s} \text{ and } \bar{T} \leq 12.9^{0}\mathrm{C} \\ \underset{(0.011)}{0.98Q_{t-1}} + \underset{(0.253)}{0.94P_{t-1}} & \text{if } Q_{t-2} > 35\ \mathrm{m}^3/\mathrm{s} \text{ and } \bar{T} > 12.9^{0}\mathrm{C} \end{cases} \tag{2}$$

The ACF and PACF of the reassembled normalized residuals and squared normalized residuals do not suggest any departure from white noise, confirming the adequacy of the final NeTAR model.

The final NeTAR model Eq. (2) reveals that both basin storage and temperature are important sources of nonlinearity in daily streamflows. This is so because the submodels that were formed according to the magnitude of the state of basin storage and temperature are quite different.

Zone 1, a 'low storage and low temperature' zone, typically represents conditions existing in the late fall and winter. In the fall, basin storage is low; there is a high potential for large abstractions from rainfall inputs and hence very little response to such inputs. In the winter, precipitation occurs in the form of snow which adds to a seasonal snowpack.

Zone 2, a 'low storage and high temperature' zone, typically represents conditions in late summer and early fall. The state of basin storage and the potential for large abstractions is much the same as in Zone 1 but larger rainfall events in this season can result in some runoff.

Zone 3, a 'high storage and low temperature' zone, typically represents late spring and early summer. Basin storage, in the form of seasonal snowpack, is high and air temperature exceeding the melting point result in snowmelt runoff which is approximately proportional to air temperature. Rainfalls during this period are generally small compared to snowmelt.

Zone 4, a 'high storage and high temperature' zone, typically represents conditions in summer. The snowpack has all melted but basin storage and streamflow remain relatively high. The high storage results in a low infiltration potential and hence rainfall events generate some runoff.

Validation of the Oldman River NeTAR Model

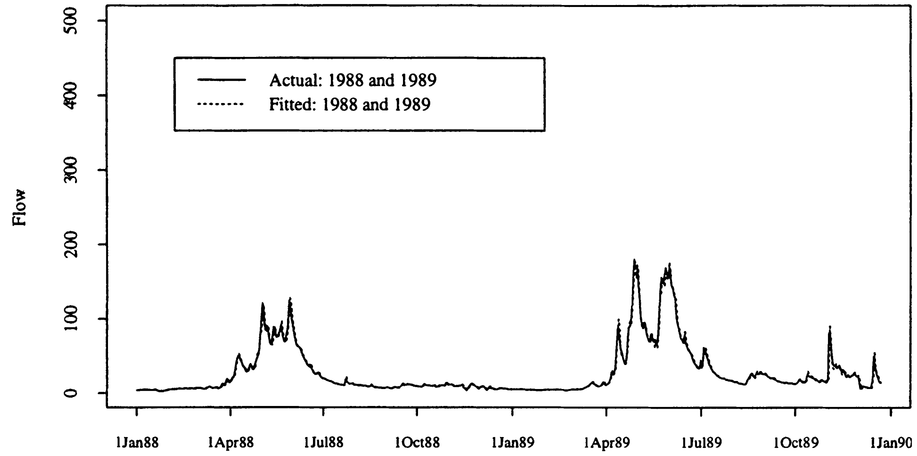
To validate the estimated NeTAR model, we simulated flow values of 1990 (1 January 1990 to 31 December 1990) using model (2). We used the actual values of precipitation (P_{t-1}), temperature (T_t) and antecedent values of flow (Q_{t-1} and Q_{t-2}) to simulate Q_t.

To assess the performance of the final NeTAR model, we plotted the fitted flow values (1 January 1988 to 31 December 1989) and the simulated flow values (1 January 1990 to 31 December 1990) along with the actual values in Fig. 7. This figure suggests that the final NeTAR model fits the calibration period flow values very well and traces the verification period flow values reasonably well.

Conclusions

Temperature and basin storage conditions are shown to be important sources of nonlinearity in daily streamflows of catchments with seasonal snow accumulation. Unlike the TARSO and the Multilinear models that address only one of these sources, the NeTAR model addresses both sources simultaneously. Besides, by looking into the variables and the corresponding coefficients of the submodels one could get useful insights into the dynamics of streamflow systems. To sum up, NeTAR models could be useful for modeling daily streamflows and for understanding the dynamics that generated them.

(a) Actual and fitted values of flow in 1988-89



(b) Actual and simulated, using NeTAR of 1988-89 data, values of flow in 1990

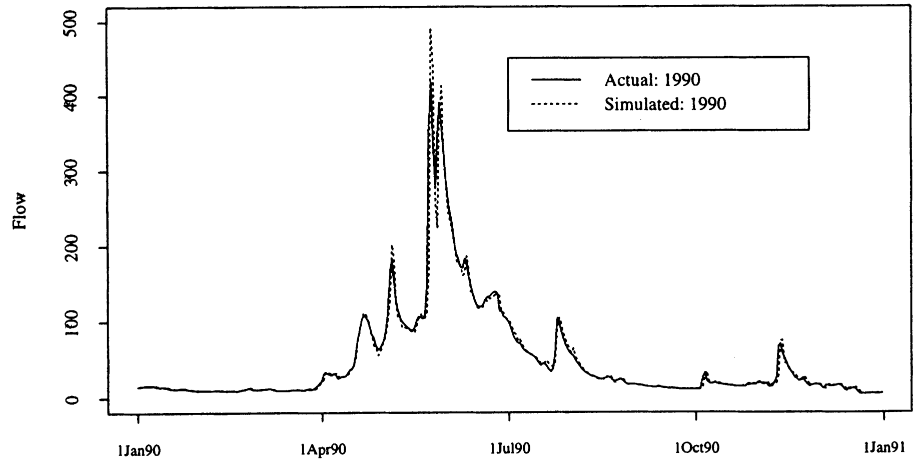


Fig. 7. Flow (m^3/s) of the Oldman River near Brocket, Alberta. (a) Actual (solid) and fitted (broken), 1988-89; (b) actual (solid) and simulated (broken) by the final NeTAR model developed using 1 January 88 to 31 December 89 data.

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