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discussion

Steady-State Crack Propagation in Pressurized Pipelines Without Backfill[1]

L. B. FREUND[2] AND D. M. PARKS.[3] The authors have presented an analysis of an extremely complex physical problem. In order to develop a tractable mathematical model of the pipeline-gas system, a number of wide-ranging assumptions have been made. This approach is justified, of course, by noting that considerable insight into the physical process can be gained from analysis of simple models. With a view toward clarifying the basic assumptions employed in this paper and in the development of other models, we would like to present the following points for discussion.

1 The dynamics of gas flow in the pipeline is described in terms of one-dimensional compressible flow along the axis of the pipe. The one-dimensional equations are modified to account for gas loss through the opening crack, and the rate of mass loss is determined by assuming that conditions are appropriate for application of classical nozzle theory. This theory was developed to describe the rate of discharge of gas from a *large* reservoir through an opening in the reservoir wall with dimensions *small* compared to those of the reservoir. If the reservoir pressure is large with respect to the ambient pressure, then the gas is discharged through the small opening, i.e., the nozzle, with a velocity equal to the local sonic velocity, or with Mach number equal to unity. Thus, in the pipeline analysis it is assumed that the flow along the pipe is one-dimensional, that gas is discharged through the opening crack at the local sonic speed, and that the one-dimensional flow is unperturbed by the outflow through the crack. Using an elastic shell model, which was developed in their reference [1], the authors then compute a pressure decay profile, representing the residual pressure on the pipe walls as a function of distance behind the crack tip. The results in Fig. 1 indicate that the pressure decays to a relatively small value within about one pipe diameter of the tip. Our calculations show that, in order to obtain this rate of pressure decay, crack openings on the order of a pipe diameter are required within this interval. The paper does not discuss the amount of crack opening, and a natural question is the following. If the opening is indeed this large, how can the result be reconciled with the basic assumption of nozzle theory that the opening be small with respect to reservoir dimensions?

2 The fact that the authors presumbly found that the crack flared open very rapidly behind the crack tip, allowing the pressure to decay rapidly, indicates that their shell structure is a very compliant one. In fact, this shell description is based on the plane strain analysis of a very long slit shell under internal pressure which was first discussed in their reference [1]. The primary deformation is thus circumferential bending, a deformation mode to which the shell has comparatively little resistance, which leads to the conclusion of low stiffness or high compliance. This mode of deformation is not consistent with the results of detailed observations of full-scale tests reported in their references [26 and 27]. These authors show that large in-plane tensile strains in the axial direction occur in the pipeline near the propagating crack tip in order to accommodate flaring of the pipe wall behind the tip, and they conclude that this is the primary mode of resistance of the shell to deformation. As is well known, shell structures are far stiffer against in-place deformation than bending deformation. It is thus likely that the authors have considerably underestimated the stiffness of the shell structure.

It should be noted that we have developed a model of the same process, based on a description of deformation corresponding to a far stiffer shell structure (their reference [24]).[4] Using pressures in the same range as those employed by the authors, we found that the rate of shell wall flaring was somewhat lower than expected. A similar conclusion was reached in a more recent study of the coupled shell dynamics and gas dynamics during rapid crack propagation in an initially pressurized elastic membrane shell.[5] It is thus possible that our kinematical assumptions result in a shell structure which is too stiff.

3 In the present analysis, the shell wall is taken to deform elastically in the crack tip region, and plasticity is incorporated in a somewhat *ad hoc* manner at some distance downstream. But for common pipeline steels, which have a high fracture toughness, the results of full-scale tests (the present authors' references [26 and 27]) indicate that extensive plastic deformation is occurring in the entire crack tip region. Indeed, within the region approximately one diameter ahead of the crack tip, axial plastic straining occurs, as noted in the foregoing, in response to the outward radial flaring of the pipe walls, with maximum axial plastic strains of order one percent. Quite close to the crack tip, the circumferential plastic strains grow very large in a Dugdale-like zone. The axial strains remain at a high level even after the crack tip passes a given material point. Thus the crack tip is entirely embedded within a plastically deforming region, so it does not seem clear how the present treatment appropriately models these features.

The present model of the cracked structure is capable of generating only small resistance to deformation. As was noted above, no account is taken of the observed axial plastic straining, through which a large resistance to deformation can be generated.

Even the relatively small resistance of the "plastic hinge" which is included is not fully accounted for, as it is the discontinuity in axial curvature *across* the hinge (rather than its value

[1]By M. F. Kanninen, S. G. Sampath, and C. Popelar, published in the February issue of the JOURNAL OF PRESSURE VESSEL TECHNOLOGY, Vol. 98 Series J, No. 1, 1976, pp. 56-65.

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[4]This work has been published in ASTM STP 590, American Society for Testing and Materials, Philadelphia, 1976.

[5]I. Abou-Sayed and L. B. Freund, *Proceedings of the Eighth SECTAM Conference*, Blacksburg, Va., 1976.

on the "elastic" side) which should be evaluated. But since, from equations (15)-(17), the deformation field is defined only in the elastic range $-l < \xi$, all contributions of the field beyond the hinge are unspecified and are not taken into account. In fact, all effects of the flap region $\xi < -l$ have been lumped into a constant, traveling, axial bending moment of magnitude M_P, applied at $\xi = -l$, and the material points passing through this cross section are tacitly presumed to "disappear," having no further *influence in the problem. Even under the most favorable* conditions, this would seem to be a rather gross simplification of the actual effects of the flap regions, but a particular feature of the present model seems especially suspect. That is, since the location of the hinge is determined from equations (21)-(23) to depend on the ratio of ductile crack propagation velocity to elastic wave speed, the hinge position is extremely close to the crack tip. In fact, from equations (23) and (21) or (22), with $\nu = 0.3$, the *maximum* distance from the crack tip to the hinge is given approximately by $l/R \approx 1.2\ (h/R)^{1/2}$, which is quite small for the thin-shell geometry of typical line pipe. From Fig. 6, we see that for this geometry the maximum distance to the hinge is roughly $l \approx 0.17R = 0.085D$. This is so close to the crack tip that, from Fig. 2, the pressure has decayed only 10 percent from its crack-tip value. Thus, the details of the pressure decay profile which was discussed earlier are not *important in the development of the present model, and it is only* the hoop stress corresponding to the internal pressure at the crack which is important in the final analysis.

In summary, it would seem unlikely that all of the effects of virtually the entire flap region of the shell could be realistically taken into account by lumping them into a prescribed traveling bending moment of constant magnitude applied so close to the crack tip.

4 The paper concludes with a table showing the effects taken into account by the various proposed models for crack propagation in pressurized pipelines. The table indicates that neither dynamic effects nor an energy balance were considered in the model developed by Freund, Parks and Rice.[4] However, the governing equations including inertia effects were derived by means of the principle of virtual work. Thus, energy (or work) is balanced from the outset, and the critical COD fracture criterion, *with a velocity dependent value of COD, can be applied directly.* Inertial resistance to deformation was included in the model, but it was concluded that this resistance was small compared to the very large resistance to in-plane axial straining referred to in the foregoing, and inertial terms were thus neglected in working out details of the model. The validity of this assumption has been born out by more recent, but unpublished, calculations.

Author's Closure

As Professors Freund and Parks have pointed out, the problem of crack propagation and arrest in a pressurized pipeline is extremely complex. At the same time, it is also a highly important practical problem for which quantitative results are urgently needed by the industry. These two considerations necessitate an idealized model. But, in order to have some confidence in its predictive capabilities, the model must have a fundamentally sound basis. We feel that we have adequately demonstrated such a basis in our work. Consequently, to describe our model as evolving from a set of "wide ranging assumptions," we feel, reveals a rather superficial understanding of our work. This feeling is reinforced by their four discussion points which can be countered as follows:

1 The treatment of the gas dynamics in our model is accurately described in the Freund-Parks discussion with the exception of their remarks on the size of the crack opening that accompanies the pressure decay behind the crack tip. While it may be that, in their calculations, the crack opening is of the order of the pipe diameter, in our model it is substantially less. However, this is irrelevant. As Professors Freund and Parks themselves recognize (see their Point 3), the details of the pressure decay are not too important and, in fact, beyond the plastic hinge have no effect whatsoever. Hence, the only relevant question is with regard to the crack opening at the yield hinge position. Typically, this is *substantially less than 1 percent of the pipe diameter* in our calculations. Therefore, it is completely in accord with the assumptions made to compute the pressure decay. Aside from this, it should also be noted that Ives, et al. (see reference [27] of our paper) observed that the pressure decays to zero in one to two diameters behind the crack tip. The predicted decay lengths are also in this range which further establishes credence of the model and suggests perhaps that relatively large crack openings can be accommodated by the model.

2 As was carefully noted in the paper, the "plane strain" shell model referred to here was used *only* to compute the pressure-decay profile behind the crack tip. It has no other bearing on the solution. The equations of classical shell theory form the basis for analyzing the propagating crack in the pipeline. Thus, the conclusions drawn with regard to the full-scale test results here are pointless. This misconception also influences some of the remarks in the third of their points and, therefore, these are also somewhat off the mark.

3 While we recognize that the crack tip is entirely embedded within a large-scale plastically deformed region, in order to make the analysis tractable, we have not tried to include this effect *in our model formulation.* We have instead developed what is in essence a linear elastic fracture mechanics model which ignores crack-tip plasticity. However, the plastic deformation behind the crack tip plays a far more decisive role in the problem and it cannot be ignored. Within the context of our model—which permits spatial variations only in the axial direction (cf, equation (15) of our paper)—the most tenable way in which large-scale plasticity can be admitted is via a plastic yield hinge. So, while the details of our yield-hinge formulation might be argued, it must be recognized that the concept itself is completely consistent with other aspects of the model. One further virtue of this idea is that the crack-driving force expression derived for the model is found not to depend on the deformation in the plastically deformed region. This obviates the difficult analysis job that would otherwise be required for this region. Thus, material does not "disappear" in the model, as Professors Freund and Parks seem to think. Rather, its presence is felt through the yield moment and its location relative to the crack tip. Finally, while Freund and Parks seem to feel intuitively that the plastic yield hinge is unrealistic, we offer the fact that, by including it, the model gives very realistic predictions. Without cognizance of the gross plasticity behind the crack tip, and it might be reemphasized that this could not readily be incorporated into our model in any other way, it would not.

4 The table comparing the various proposed models reflected our somewhat biased view of what the essential features of a pipeline crack propagation model should be. Inertia effects were included primarily because, since we used a steady-state formulation, only a slight increase in complexity was involved. This allowed us to cover a wide range of crack speeds including the brittle crack propagation regime where, as we wanted to point out, inertia effects certainly are important (see reference [1] of our paper). That Freund, et al., have decided to neglect inertia forces merely means that they have chosen to confine themselves to lower crack speeds, albeit the range of interest in ductile crack propagation. Hence, their neglect of inertia was appropriately noted in our table. However, we were probably remiss in not awarding them a "credit star" in the energy balance column.

To summarize, we believe that we have in this paper and its sequel[6] developed a fundamentally sound, logically consistent, albeit simple, mathematical model for crack propagation in a pressurized pipeline. *To the best of our knowledge, our model is* the only theoretical model currently existing— and this includes the model offered by Abou-Sayed and Freund—that enables predictions of crack speed and of the fracture toughness required for crack arrest to be made. Not only can these predictions be made, but, as the comparisons *with* the full-scale tests show, they are reasonably accurate as well. This is not to say that the problem is completely solved. In fact, some areas in which improvement is needed certainly still exist. For this reason, we welcome the comments of Professors Freund and Parks in the spirit that frank exchanges of constructive criticism will benefit all.

[6]C. Popelar, A. R., Rosenfield, and M. F. Kanninen, "Steady-State Crack Propagation in Pressurized Pipelines," to appear in the JOURNAL OF PRESSURE VESSEL TECHNOLOGY.