A micrometeor component of the 1998 Leonid shower

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ABSTRACT

Most astronomers expected a significant meteor shower associated with the Leonid meteoroid stream to appear in 1998 and 1999. An enhanced shower was widely observed in both years, and details can be found in many published articles. In 1998, one remarkable feature was the appearance of a strong component, rich in bright meteors, which appeared about 16 h before the expected maximum of the main shower, but another observed feature was an abnormal peak in the ionosphere characteristic value $f_b E_s$ which was detected about 18 h after the main shower. A very high value of $f_b E_s$ persisted for over an hour. The likely explanation is that the ionosphere was bombarded by an additional swarm of meteoroids, much smaller than those that produce a visible trail or an ionization trail that can be picked up by radio detectors. The different dynamical behaviours between small and large meteoroids are investigated and, in consequence, an explanation for the observed phenomena is offered and 1933 is suggested as being the likely ejection time.

Key words: celestial mechanics – comets: individual: 55P/Tempel–Tuttle – meteors, meteoroids.

1 INTRODUCTION

Of all the meteor showers, the Leonid shower is probably the best known amongst the general public and professional astronomers alike. This fame comes about because of the outburst of very high meteor activity that is associated with it. These outbursts also ensured that the Leonids were both observed and recorded for at least two centuries so that the Leonid shower has played a major part in the historical development of the subject. In particular, the spectacular storms of 1799 and 1833 played a significant part in convincing a sceptical scientific community that streams of solid material could exist in interplanetary space, while the association of the stream with Comet 55P/Tempel–Tuttle, based on orbital considerations (Schiaparelli 1867), was one of the first such pairings to be established. The period of the comet is approximately 33.2 yr, while a typical gap in time between major Leonid storms is around 33 yr (Williams, Johnson & Fox 1986). There is also a coincidence between the perihelion passage time of the comet and the occurrence of a meteor storm. A very large storm was observed in 1966 while good displays were seen in 1998 and 1999, perihelion passage of the comet being in 1965 and 1998. There was little activity observed in the intervening years. This lack of activity in years away from the period around the perihelion passage date of the comet has been discussed by Williams (1997), but is of no particular relevance to the situation that we are about to discuss. The basic science that leads to this coincidence of comet perihelion passage and meteor storm is easy to understand. Meteoroids are ejected from the cometary nucleus with speeds that are much less than the orbital velocity and hence only drift away very slowly from the nucleus, along similar orbits to the comet. Hence there is a high density of meteoroids to be expected close to the cometary nucleus. Detailed modelling of the Leonids, for example by Brown & Jones (1996), Jenniskens (1996), Wu & Williams (1996), Yeomans, Yau & Weissman (1996) and Arlt & Brown (1998), predicted strong showers, and such strong displays were seen and details can be found in many published articles. Earlier, Williams & Wu (1994) had used similar methods to explain the appearances of a new peak in the Perseid shower in terms of the return of its parent comet, 109P/Swift–Tuttle.

Hence the general behaviour of meteor streams has been reasonably well modelled, and again this is not the main topic of concern here. What primarily concerns the general public is the time at which a display is seen, and in particular the ability of the astronomers to predict this beforehand. In principle, this should be straightforward since a meteoroid can only intercept the Earth’s atmosphere and become a meteor when it is passing through one or other of the nodes of its orbit. It can only thus be seen at the precise time when the Earth is at one of these nodes. Hence determining the time of appearance of a shower requires only a determination of the longitude of the ascending node, $\Omega$. Showers have not always appeared when expected. In 1998 a strong
component, rich in bright meteors, appeared about 16 h before the expected maximum of the main Leonid shower. An explanation for this has been given by Asher, Bailey & Emel’yanenko (1999). Other examples of a display taking place at an unexpected time is the new peak in the Perseids that appeared in the early 1990s slightly separated in time from the traditional main peak (e.g. Brown & Rendtel 1996), and already referred to, or the variability on a short time-scale in the Quadrantids (Hughes, Williams & Fox 1981). An other example, which we shall discuss in more detail in a following section, was the unusual level of ionization detected in the ionosphere somewhat after the main Leonid activity in 1998 (Ma, He & Williams 2001), and thought to be due to the impact of a swarm of very small meteoroids.

In order to observe meteor activity at a time that is different from the expected, the longitude of the ascending node \( \Omega \) of relevant meteoroid orbits must be different from the expected value. Now, the longitude of the ascending node is determined only by the intersection of the orbital plane with the ecliptic. Indeed, that is the definition of the node. Hence, in order to change the node, the orbital plane of the meteoroids must be changed. The main purpose of this paper is to discuss mechanisms for doing this. We shall then apply our conclusions to the particular case of the swarm of small meteoroids that was mentioned above.

2 CHANGE OF THE ORBITAL PLANE

To change the orbital plane of a body, it is necessary to change the angular momentum per unit mass \( \mathbf{h} \). Doing this is not difficult; doing it differentially so that meteoroids of different sizes move in different planes is harder. The most obvious effect that discriminates according to size is the Poynting–Robertson effect (Poynting 1903; Robertson 1937). However, this mechanism is in effect a drag and, while it may change both the semi-major axis and the eccentricity of an orbit, it does not change the orbital plane, hence the line of nodes (i.e. \( \Omega \)) remains fixed. On the other hand, gravitational perturbations from the planets can change all the orbital parameters. Indeed, the effect on the Earth is well-known as a cause for changes in the nodes of all meteor streams. However, this effect is mass-independent, and so cannot cause small meteoroids to move in a different plane with a different period from larger ones. There are effects that can produce the desired change: the first is the electromagnetic field, where the charge-to-mass ratio does depend on the mass; and the second is the initial ejection velocity of the meteoroid from the comet, which almost certainly depends on the mass of the meteoroids, and so affects both the orbital plane and the period. We shall discuss these in turn.

In order to evaluate the change of the angular momentum, we have to consider the situation referred to a non-moving reference frame.

In the heliocentric–ecliptic frame, the angular momentum per unit mass, \( \mathbf{h} \), is given by

\[
\mathbf{h} = r \times \mathbf{V} = \left( h_x, h_y, h_z \right) = \left( z \cos \omega - x \sin \omega, x \cos \omega - y \sin \omega, r \cos i \right).
\]

(1)

Also, in terms of the orbital parameters,

\[
\pm h_x = h \sin i \sin \Omega, \\
\mp h_y = h \sin i \cos \Omega,
\]

where \( i \) is the inclination of the orbit. Hence

\[
\tan \Omega = -\frac{h_y}{h_x}
\]

and

\[
(\Delta \Omega) \sec^2 \Omega = -\left( \frac{\Delta h_x}{h_x} - \frac{h_y \Delta h_y}{h_y^2} \right) \tan \Omega \left( \frac{\Delta h_x}{h_x} - \frac{\Delta h_y}{h_y} \right).
\]

Thus

\[
\Delta \Omega = \sin \Omega \cos \Omega \left( \frac{\Delta h_x}{h_x} - \frac{h_y \Delta h_y}{h_y^2} \right) = -\frac{1}{h' \sin^2 i} \left( h \Delta h_x - h_y \Delta h_y \right).
\]

(2)

2.1 The effect of charge

The interplanetary magnetic field is best represented in a simple analytical form (Ma & Xu 1996) by

\[
B = B_0 \frac{r^2}{r^3} r,
\]

where \( r \) is the heliocentric distance and \( B_0 = 4 \times 10^{-9} \text{T} \) with \( r_0 = 1 \text{ au} \). The well-known Lorentz force is given by

\[
F = QV \times B = \frac{QB_0 r_0^4}{r^3} (V \times r) = -\frac{QB_0 r_0^4}{r^3} \mathbf{h},
\]

(4)

where \( Q \) is the charge of a meteoroid and \( V \) is its orbital velocity.

The Lorentz force which acts in a direction perpendicular to the orbital plane will change the angular momentum per unit mass of the meteoroids, that is

\[
\frac{d\mathbf{h}}{dt} = r \times \frac{F}{m} = \frac{QB_0 r_0^4}{m r^3} r \times (V \times r) = \frac{QB_0 r_0^4}{mr^3} \left( \frac{dr}{dt} - \frac{dr}{dt} \right),
\]

(5)

giving

\[
\Delta \mathbf{h} = \frac{QB_0 r_0^4}{mr^3} (r \Delta r - r \Delta r).
\]

(6)

Hence equation (2) becomes, after some algebra,

\[
\Delta \Omega = \frac{1}{h' \sin^2 i \sin \omega} \frac{QB_0 r_0^4}{mr^3} (r \Delta r - r \Delta r) \cdot V.
\]

(7)

We know that \( z = r \sin i \sin (\omega + f) \), so that

\[
\Delta \Omega = \frac{QB_0 r_0^4}{m} \frac{\sin (\omega + f)}{h' \sin \omega} \left( \frac{r}{r} \Delta r - \Delta r \right) \cdot V,
\]

(8)

where \( m \) is the mass of the meteoroid, \( \omega \) is the argument of the perihelion, and \( f \) is the true anomaly.

Let us now estimate the node change arising from the Lorentz force. Since the node changes gradually by this mechanism, we consider the situation after the meteoroid has completed one orbit after the ejection. In this case, \( \Delta r \) is the orbital shift distance in a whole period, \( I_s \), say.

The Lorentz force causes the orbit of the meteoroids to shift in the perpendicular direction. The shift velocity is

\[
v_x(t) = \int_{t_0}^{t} \frac{F}{m} \frac{dr}{dt} = -\frac{QB_0 r_0^4}{m} \int_{t_0}^{t} \frac{1}{r'} dr' = -\frac{QB_0 r_0^4}{mna^3(1-e^2)^{3/2}} \int_{t_0}^{t} (1+e \cos f) df = -\frac{QB_0 r_0^4}{mna^3(1-e^2)^{3/2}} (f + e \sin f - C),
\]

where \( n = 2\pi/P \) is the mean motion, \( P \) is the orbital period, \( e \) is the eccentricity and \( a \) is the semi-major axis of the cometary orbit.
\[ C = f_0 + e \sin f_0, \] where \( f_0 \) is the true anomaly of the ejection position.

The orbital shift distance after a complete revolution owing to the Lorentz force is
\[
I_s = \int_{t_0}^{P_{t+1}} v_i(t) \, dt = -\frac{Q B_0 \sin \hat{\mathbf{r}}_h}{m a^2 (1 - e^2)^{3/2}} \int_{t_0}^{P_{t+1}} (f + e \sin f - C) \, dt
\]
\[
= -\frac{Q B_0 \sin \hat{\mathbf{r}}_h}{m a^2 (1 - e^2)^{3/2}} \int_{t_0}^{P_{t+1}} [n(t - \tau) - C] \, dt, \tag{9}
\]
where \( t_0 \) is the ejection time and \( \tau \) is the time of perihelion passage.

Since \( \tau - t_0 \) is very small compared with \( P \),
\[
I_s \approx -\frac{1}{2 m a^2 (1 - e^2)^{3/2}} \mathbf{p}^2 = -\frac{1}{2 m} \sqrt{\frac{GM_\odot}{a}} \left( \frac{r_0}{a} \right)^2 \left( \frac{P}{a} \right)^2,
\tag{10}
\]
We should note that the analysis above is approximate in that it is assumed that the direction perpendicular to the orbital plane remains constant. This will be so if the calculated distance is small. As we shall see, this is indeed the case.

For the small meteoroids \( b_1 P_1 = 1.13 \times 10^{-3} \text{ g cm}^{-2} \) (cf. Section 4); \( b_1 = 1.13 \times 10^{-3} \text{ cm} \) if \( P_1 = 1.0 \text{ g cm}^{-3} \), then \( m = 6.044 \times 10^{-12} \text{ kg} \). The electric charge that a meteoroid carries cannot be more than a few electrons (Huebner, private communication). We assume that it is equal to that of one electron, \( q = 1.6 \times 10^{-19} \text{ coulomb} \), so \( Q b_1 / m = 1.059 \times 10^{-16} \text{ s}^{-1} \). Substituting orbital parameters of Comet 55P/Tempel–Tuttle, given in Table 1, into equation (10) gives \( I_s \approx -2.796 \times 10^4 \text{ m} \). This is a very small distance, and allowing a slight increase in motion over a few more orbits does not alter this fact.

From equation (7) and the above considerations for \( I_s \), \( \Delta \Omega \) can be numerically calculated at all possible ejection locations. For example, at 1.5 au, where \( V = \left[ (GM_\odot/\alpha)(2a - 1) \right]^{1/2} = 3.312 \times 10^8 \text{ m s}^{-1}, h = [GM_\odot \alpha (1 - e^2)]^{1/2} = 6.078 \times 10^{15} \text{ m}^2 \text{ s}^{-1} \) and \( f = 74^\circ \) (cf. Section 4), we have \( \Delta \Omega \approx 1.82 \times 10^{-10} \text{ rad} \), where \( \phi \) is the angle between \( r \) and \( V \). This is a very small value, and allowing a slight increase in motion over a few more orbits does not alter this. Hence we conclude that the effect of the Lorentz force can be neglected.

### 2.2 The effect of the initial ejection velocity

Here we consider only the effects of the initial ejection velocity,
\[
\Delta h_i = \gamma \Delta \hat{\mathbf{z}} - \Delta \hat{\mathbf{y}},
\]
\[
\Delta h_i = \gamma \Delta \hat{\mathbf{t}} \times \Delta \hat{\mathbf{z}}.
\]
Substituting \( \Delta h_x \) and \( \Delta h_y \) into equation (2), we have
\[
\Delta \Omega = \frac{z}{h \sin i} (h \cdot \Delta V) = \frac{r \sin(\omega + f)}{h \sin i} v \sin \phi,
\tag{11}
\]
where \( v \) is the ejection velocity and \( \phi \) is the angle between the direction of ejection and the orbital plane, so \( v \sin \phi \) is the component of the ejection velocity perpendicular to the orbital plane.

Equation (11) indicates that the ejection speed, the direction of the ejection and the location of the ejection point on the orbit all affect the node of the meteoroid stream formed.

### 3 Change of the Orbital Period

To change the orbital period of a body, it is necessary to change the energy per unit mass \( E \). As already mentioned, gravitational perturbation is mass-independent and so cannot separate small meteoroids from larger ones. The Poynting–Robertson effect can change the orbital period according to size, but can be excluded in our discussion because it is a very long-term effect. Effects that depend on mass and influence the orbital period are the initial ejection velocity and the solar radiation pressure.

Since the force from solar radiation pressure is opposite to the direction of solar gravity, its effect is to ‘weaken’ the gravity so that the quantity \( GM_\odot \) is replaced by \( GM_\odot (1 - \beta) \), where \( \beta = F_r / F_\odot \) (\( F_\odot \) is the force from solar radiation pressure; \( F_r \) is the solar gravity) has the value \( \beta = 5.75 \times 10^{-3} / bp \) (Williams 1997); here, \( b \) is the radius and \( p \) is the bulk density of the ejected meteoroid, both in cgs units. This effect cannot influence the kinetic energy of the ejected meteoroids, so the orbital velocity of the ejected meteoroids at the ejection point should be the same; thus we have
\[
V^2 = GM_\odot \left( \frac{2}{r} - \frac{1}{a} \right) = GM_\odot (1 - \beta) \left( \frac{2}{r} - \frac{1}{a'} \right),
\tag{12}
\]
where \( a' \) is the semi-major axis of the orbit of the meteoroids. Hence
\[
a' = \frac{1 - \beta}{1 - 2a/\beta a}. \tag{13}
\]
For a unit mass of the cometary nucleus, the standard theory of Keplerian motion tells us that
\[
E = -\frac{GM_\odot}{2a}, \tag{14}
\]
and that
\[
a^3 = P^2, \tag{15}
\]
where the semi-major axis \( a \) is in astronomical units and the period \( P \) is in years.

In the weakened gravitational field, the energy per unit mass of the meteoroids is
\[
E' = -\frac{GM_\odot}{2a'}, \tag{16}
\]
and Kepler’s third law gives
\[
a'^3 = (1 - \beta)P'^2, \tag{17}
\]
where \( P' \) is the orbital period of the meteoroids. Hence
\[
\left( \frac{E'}{E} \right)^3 = (1 - \beta) \left( \frac{a'}{a} \right)^3 = (1 - \beta)^2 \left( \frac{P'}{P} \right)^2, \tag{18}
\]
giving
\[
P' = (1 - \beta) P \left( \frac{E}{E'} \right)^{-3/2}. \tag{19}
\]
The radiation pressure only changes the potential energy of the meteoroids relative to the parent comet,
\[
\Delta E_p = -\frac{GM_\odot (1 - \beta)}{r} + \frac{GM_\odot}{r} = \frac{\beta}{\gamma} \frac{GM_\odot}{r}. \tag{20}
\]
The process of ejection also changes the energy of the meteoroids relative to the parent nucleus, but it only changes the kinetic energy.

\[ \Delta E = V \cos \phi \cos \theta + \frac{1}{2} \tau^2, \]

where

\[ V^2 = GM_\odot \left( \frac{2}{r} - \frac{1}{a} \right) \]

is the orbital velocity of the comet at the ejection point, and \( \theta \) is the angle between the directions of \( V \) and the component of the ejection velocity in the orbital plane.

Thus we obtain the result that the total energy of the meteoroids is

\[ E' = E + \Delta E_p + \Delta E_e. \]

By suitable substitution and expansion, an expression for \( P' \) can be found:

\[ P' = P \left[ 1 + \beta \left( \frac{3a}{r} - 1 \right) \right] + \frac{3v}{V_s} \left( \frac{2a}{r} - 1 \right)^{1/2} \cos \phi \cos \theta + \frac{3v^2}{2V_s^2}, \]

where

\[ V_s = \left( \frac{GM_\odot}{a} \right)^{1/2} = 9.266 \times 10^3 \text{ m s}^{-1}. \]

In (23), both \( \beta \) and \( v \) are dependent on the mass of the meteoroids, so the period varies with meteoroid size.

### 4 APPLICATION TO THE LEONIDS

An overview of the 1998 Leonid activity was given by Arlt (1998). Besides the main shower and a component rich in bright meteors which was about 16h before the predicted maximum of the main shower, an unusual level of ionization was detected in the ionosphere about 18h after the main shower.

The \( E_s \) layer is a high-level ionized thin sheet in the ionosphere with an area bigger than 100 km\(^2\) and a thickness of only 1–2 km. The quantity that is measured, called the ionosphere characteristic value, and usually denoted by \( f_s E_s \), is the lowest frequency of the wave that can penetrate the \( E_s \) layer and can be determined by the echo from a higher layer in the ionosphere. The lifetime of the \( E_s \) layer is usually less than 15 min and the value of \( f_s E_s \) is less than 2 MHz. The smooth average value over a month is denoted by \( f_s E' \). If \( f_s E_s \) is bigger than \( f_s E' \) by more than 3 standard deviations and the lifetime is longer than 30 min, the situation is regarded as unusual. If an unusual value of \( f_s E_s \) is detected by two or more neighbouring stations that are separated several hundred kilometres, the \( E_s \) layer is thought to be caused by micrometeors (He & Xu 1997; He 1998).

Observations to survey the ionization effect of the meteor shower were made by several ionosphere observational stations in China during the period 1998 November 14–20. An abnormal peak of the \( f_s E_s \) value was detected by two stations, Guangzhou and Hainan, at the same time (Fig. 1). The very high value of \( f_s E_s \) (sometimes \( \sim 10 \text{ MHz} \)) was maintained for over 1 h. The abnormal phenomenon showed that the ionosphere was bombarded by a swarm of small meteoroids that were not observed in the optical and radio. In other words, the abnormal ionization effect in the ionosphere was caused by a large population of small meteoroids.

The dust swarm peaked about 18 h after the main shower, hence the longitude of the ascending node of the small meteoroids in the swarm of small meteoroids that were not observed in the optical, layer is usually less than 15 min and the value of \( f_s \) and Hainan, at the same time (Fig. 1). The very high value of \( f_s \) was detected by two stations, Guangzhou and Hainan on 1998 November 18.

Leonids was 0.74 different from that of the visible ones. From equation (11), we have

\[ \frac{r_1 \sin(\omega + f_1) \sin \phi_1}{h \sin i} - \frac{r_0 \sin(\omega + f_0) \sin \phi_0}{h \sin i} = 0.013, \]

where the subscripts ‘1’ and ‘0’ denote the respective quantities for small meteoroids and visible ones.

To simplify the problem, the small meteoroids were ejected perpendicular to the orbital plane, while the visible meteoroids were ejected in the orbital plane so that \( \sin \phi_1 = 1 \) and \( \sin \phi_0 = 0 \). This is a special case but gives the maximum separation possible between the two planes. Equation (24) now gives

\[ r_1 \sin(\omega + f_1) \sin \phi_1 = 2.36 \times 10^{13} \text{ (m}^2 \text{ s}^{-1}). \]

According to the model proposed by Whipple (1951), the ejection velocity is given as

\[ v = \frac{K}{(bp)^{0.5}} r^{-1.125} \text{ (m s}^{-1}), \]

where \( K \) is a constant which depends on the cometary radius and the fraction of the incident solar radiation available for sublimation, \( b \) and \( p \) are the same as above, and \( r \) is the heliocentric distance in astronomical units.

In Asher’s (1999) paper, the \( f_0 \) was taken to be \( 10^{-5} \) for visible meteoroids, giving \( b_0 \rho_0 = 5.75 \times 10^{-2} \text{ g cm}^{-2} \), while the ejection velocity at \( r = 1 \text{ au} \) was taken as \( 25 \text{ m s}^{-1} \). For these values, \( K = 6.0 \).

Substituting (26) into (25), we obtain

\[ b_1 \rho_1 = 1.45 \times 10^{-3} \sin^2(\omega + f_1) \frac{1}{r_1^{0.25}}. \]

Hence

\[ \beta_1 = 3.97 \times 10^{-2} \frac{1}{r_1^{0.25}} \sin^2(\omega + f_1). \]

For a meteoroid moving around the Sun, the energy must be negative so that

\[ \frac{1}{2} V^2 - \frac{GM_\odot (1 - \beta)}{r} < 0, \]

giving

\[ \beta < \frac{r}{2a}. \]
Substitution shows that $\beta_1$ does not satisfy equation (30) when $-50.8^\circ \leq f_1 \leq 62^\circ.4$. For ejection within this range of true anomaly the meteoroids would have to be so small that they would be lost from the Solar system. The detected small meteoroids must thus be ejected farther from perihelion.

From equations (27) and (28), we can calculate the values of $b_1P_1$ and $\beta_1$ for various heliocentric distances, and these are shown in Table 2. The three values of $r_1$ given in the table correspond to values of the true anomaly of $\pm 74^\circ 6$, $\pm 94^\circ 4$ and $\pm 106^\circ 4$, respectively.

From Table 2, we see that all the values of $\beta_1$ should be of the order of $5 \times 10^{-2}$. We will take the average value from the table, $\beta_1 = 5.07 \times 10^{-2}$, for small meteoroids in our discussion. This corresponds to $b_1P_1 = 1.13 \times 10^{-3} \text{g cm}^{-2}$.

The small meteoroids were 18 h behind the main Leonid meteor shower in 1998, and this resulted from orbital period changes between the small and visible meteorites. Hence

$$n_1P_1' - n_0P_0' = 18 \text{h}, \quad (31)$$

where $n_1$ and $n_0$ are the number of orbits completed by the small and the visible meteoroids between ejection and observation.

As mentioned above, the small meteoroids were assumed to be ejected perpendicular to the orbital plane, so that $\cos \theta_0 = 0$, while the visible ones were in the orbital plane, so that $\cos \theta_0 = 1$.

If we assume that the ejection of both small and visible meteoroids occurred at the same position $r$, from (23) and (31) we obtain

$$n_1 \left[ 1 + \beta_1 \left( \frac{3a}{r} - 1 \right) \right] + \frac{5.566 \times 10^{-4}}{r^{1/25}}$$

$$-n_0 \left[ 1 + \beta_0 \left( \frac{3a}{r} - 1 \right) \right] + \frac{8.101 \times 10^{-3}}{r^{1/25}} \left( \frac{2a}{r} - 1 \right)^{1/2} \cos \theta_0$$

$$+ \frac{1.094 \times 10^{-5}}{r^{1/25}} = 6.189 \times 10^{-5}. \quad (32)$$

Because $\beta_1$ is much bigger than $\beta_0$, the period of the small meteoroids is longer than that of the visible ones. So $n_1 < n_0$ is not possible, and $n_1$ is smaller than $n_0$. Indeed, by keeping the largest terms only, equation (32) gives

$$\frac{n_0}{n_1} = 1 + \beta_1 \left( \frac{3a}{r} - 1 \right). \quad (33)$$

Ejection, which is gas-driven, cannot generally occur at heliocentric distances greater than 3 au, so the range of $n_0/n_1$ can be determined from equation (33) as

$$1.47 \leq \frac{n_0}{n_1} \leq 2.56. \quad (34)$$

Simple integer pairs that satisfy inequality (34) are 2:1, 3:2, 4:2, 5:2, 5:3, 6:3, 6:4, 7:3, 7:4 and so on.

Table 3 shows the possible ejection positions corresponding to some of the above ratios for five possible directions of ejection velocity of the visible meteoroids. The values with asterisks denote heliocentric distances that are impossible because the small meteoroids will be lost from the system, and thus they do not satisfy inequality (30). As can be seen, the results are the same when $n_0/n_1 = 2:1, 4:2$ and 6:3, for example.

The dependence on $\cos \theta_0$ (i.e. the direction of ejection) is also slight: for example, for $n_0/n_1 = 2:1$, $r$ varies from 1.385 au, when ejection is in the same direction as the cometary motion, to 1.493 au when ejection is in the opposite direction to the motion. The most probable range of $\cos \theta_0$ is 0.5 to $-0.5$, roughly $30^\circ$ to the solar direction, so the most probable ejection position is between 1.41 and 1.47 au.

McNaught & Asher (1999) suggested that three Leonid dust trails encountered the Earth in 1998 and that they had completed four, five and six orbits. Therefore, if McNaught & Asher are correct, the possible ejection positions for Leonids seen in 1998 are around 1.44, 2.12 and 2.78 au (see Table 3).

As trails get older, they spread both along the trail and perpendicular to it, mainly because of the perturbation from Jupiter. The sharp outburst discussed in this paper cannot thus be very old. However, from Table 3 we conclude that it is not possible that the visible meteors seen in the Leonids 1998 completed only one revolution after ejection, since $n_0$ is at least 2. Hence the meteoroids observed in 1998 must have been ejected during the returns of Comet 55P/Tempel–Tuttle in 1933 or earlier. In this case, the ejection would have been around 1.44 au. We believe that this is more likely than 2.12 and 2.78 au and that, for the sharp peak, 1933 is more likely to be the ejection time than earlier years. However, these data given above are insufficient for us to distinguish between this solution and that of McNaught & Asher.

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**Table 2.** Values of $b_1P_1$ and $\beta_1$ obtained from equations (27) and (28) when ejection occurred at different positions.

<table>
<thead>
<tr>
<th>$r_1$ (au)</th>
<th>$f_1$</th>
<th>$b_1P_1$ (g cm$^{-2}$)</th>
<th>$\beta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>285.4</td>
<td>1.29 $\times$ 10$^{-3}$</td>
<td>4.48 $\times$ 10$^{-2}$</td>
</tr>
<tr>
<td>1.5</td>
<td>74.6</td>
<td>1.11 $\times$ 10$^{-3}$</td>
<td>5.18 $\times$ 10$^{-2}$</td>
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<td>1.17 $\times$ 10$^{-3}$</td>
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<td>1.22 $\times$ 10$^{-3}$</td>
<td>4.73 $\times$ 10$^{-2}$</td>
</tr>
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</tr>
<tr>
<td>2.5</td>
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<td>1.13 $\times$ 10$^{-3}$</td>
<td>5.11 $\times$ 10$^{-2}$</td>
</tr>
</tbody>
</table>

**Table 3.** The ejection positions and the relative angles ($\cos \theta_0$) of the ejection of the visible meteoroids for different revolution numbers.

<table>
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<tr>
<th>$n_0$</th>
<th>$n_0 = 3$, $n_0 = 4$, $n_0 = 5$, $n_0 = 6$, $n_0 = 7$</th>
<th>$n_1$</th>
<th>$n_1 = 3$, $n_1 = 4$</th>
<th>$\cos \theta_0$</th>
<th>$r$ (au)</th>
<th>$r$ (au)</th>
<th>$r$ (au)</th>
<th>$r$ (au)</th>
<th>$r$ (au)</th>
</tr>
</thead>
<tbody>
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<td>$n_1 = 3$, $n_1 = 4$</td>
<td>$\cos \theta_0$</td>
<td>$r$ (au)</td>
<td>$r$ (au)</td>
<td>$r$ (au)</td>
<td>$r$ (au)</td>
<td>$r$ (au)</td>
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<td>2.7283</td>
<td>1.3851</td>
<td>0.9039*</td>
<td>2.0734</td>
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<tr>
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<td>1.4132</td>
<td>0.9356*</td>
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<td>1.0567</td>
<td>1.8744</td>
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<td>1.4404</td>
<td>0.9685*</td>
<td>2.1249</td>
<td>1.4404</td>
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<td>1.4670</td>
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<td>1.4670</td>
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acknowledges the support of the Royal Society KC Wong Fellowship.

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