Timing the millisecond pulsars in 47 Tucanae

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ABSTRACT

In the last ten years, 20 millisecond pulsars have been discovered in the globular cluster 47 Tucanae. Hitherto, only three of these pulsars had published timing solutions. Here we improve upon these three and present 12 new solutions. These measurements can be used to determine a variety of physical properties of the pulsars and of the cluster. The positions of the 15 pulsars have been determined with typical errors of only a few mas and they are all located within 1.2 arcmin of the cluster centre. Their spatial density within that region is consistent with a distribution of the type \( n(r) \propto r^{-2} \), with a sudden cut-off outside four core radii. Two pulsars have a projected separation of only 0.12 arcsec, and could be part of a triple system containing two observable pulsars. We have measured the proper motions of five of the pulsars: the weighted mean of these, \( \mu_\alpha = (6.6 \pm 1.9) \text{ mas yr}^{-1} \) and \( \mu_\delta = (-3.4 \pm 0.6) \text{ mas yr}^{-1} \), is in agreement with the proper motion of 47 Tucanae based on Hipparcos satellite data. The period derivatives measured for many of the pulsars are dominated by the dynamical effects of the cluster gravitational field, and are used to constrain the surface mass density of the cluster. The pulsar accelerations inferred from the observed period derivatives are consistent with those predicted by a King model using accepted cluster parameters. We derive limits on intrinsic pulsar parameters: all the pulsars have characteristic ages greater than 170 Myr and have magnetic fields smaller than \( 2 \times 10^9 \text{ Gauss} \); their average characteristic age is greater than \( \sim 1 \text{ Gyr} \). We have also measured the rate of advance of periastron for the binary pulsar J0024−7204H, \( \dot{\omega} = (0.059 \pm 0.012) \text{ yr}^{-1} \), implying a total system mass of \( 1.4^{+0.9}_{-0.8} M_\odot \) with 95 per cent confidence.

Key words: binaries: general – pulsars: general – globular clusters: individual: 47 Tucanae.

1 INTRODUCTION

Among the globular clusters in the Galactic system, 47 Tucanae (hereafter 47 Tuc) holds the record for the number of known pulsars: 20 to date (Manchester et al. 1991; Robinson et al. 1995; Camilo et al. 2000). Camilo et al. estimated the total number of potentially detectable pulsars in this cluster to be at least 200. The known pulsar population of 47 Tuc is very different from the population in the Galactic disc: all of the pulsars have periods less than 8 ms, and 13 are members of binary systems. Radio images of the cluster (Fruchter & Goss 2000; McConnell & Ables 2000) show three scintillating point sources, the positions of which coincide with the brightest known pulsars in the cluster: 47 Tuc C, D and J. It is therefore improbable that any bright pulsars remain to be discovered, even with the severe selection effects against the detection of binaries with very short orbital periods and pulsars with very short rotational periods noted by Camilo et al. (2000).

In addition to this set of pulsars, there is a collection of other exotic objects near the core of 47 Tuc: at least nine X-ray sources (Hasinger, Johnston & Verbunt 1994; Verbunt & Hasinger 1998) and more than 20 blue stragglers (Guhathakurta et al. 1992). Evidence has also been found for large numbers of cataclysmic variables in the core (Edmonds et al., in preparation). The high stellar density necessary for the formation of these objects may increase the rate of exchange of stars in binaries (Hut, Murphy & Verbunt 1991), leading to the formation of low-mass X-ray binaries and millisecond pulsars, a process addressed by Rasio, Pfahl & Rappaport (2000) for the specific case of 47 Tuc.
Table 1. Parameters of the globular cluster 47 Tuc.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre RA, α17 Tuc (J2000)</td>
<td>00h24m05s29.0 ± 0.28</td>
<td>(De Marchi et al. 1996)</td>
</tr>
<tr>
<td>Centre Dec, δ17 Tuc (J2000)</td>
<td>-72°04'52&quot;3 ± 1&quot;3</td>
<td>(De Marchi et al. 1996)</td>
</tr>
<tr>
<td>Distance, D</td>
<td>5.0 ± 0.4 kpc</td>
<td>(Reid 1998)</td>
</tr>
<tr>
<td>Age</td>
<td>10.0 ± 0.4 Gyr</td>
<td>(Gribin et al. 1997)</td>
</tr>
<tr>
<td>Total mass</td>
<td>(1.67 ± 0.04) × 10^6 M⊙</td>
<td>(De Marchi et al. 1996)</td>
</tr>
<tr>
<td>Tidal radius</td>
<td>40' (58 pc)</td>
<td>(Da Costa 1979)</td>
</tr>
<tr>
<td>Core radius, r_c (r_c)</td>
<td>23.1 ± 17.0 (0.6 pc)</td>
<td>(Howell, Guhathakurta &amp; Gilliland 2000)</td>
</tr>
<tr>
<td>Escape velocity</td>
<td>58 km s(^{-1})</td>
<td>(Webbink 1985)</td>
</tr>
<tr>
<td>Central density</td>
<td>-1 × 10^5 M⊙ pc(^{-1})</td>
<td>(Pryor &amp; Meylan 1993)</td>
</tr>
<tr>
<td>Central line-of-sight velocity dispersion, v_z(0)</td>
<td>11.6 ± 1.4 km s(^{-1})</td>
<td>(Meylan &amp; Mayor 1986)</td>
</tr>
<tr>
<td>Proper motion in RA, μ_α</td>
<td>7.0 ± 1.0 mas yr(^{-1})</td>
<td>(Odenkirchen et al. 1997)</td>
</tr>
<tr>
<td>Proper motion in Dec, μ_δ</td>
<td>-5.3 ± 1.0 mas yr(^{-1})</td>
<td>(Odenkirchen et al. 1997)</td>
</tr>
</tbody>
</table>

The new observations presented here bring the number of pulsars in 47 Tuc to 15. In Section 2 we describe the observations, data reduction and analysis used to obtain the new timing solutions. We present these solutions in Section 3. The new solutions lead to a wealth of astrophysical results which we discuss in the rest of the paper. In Section 4 we use high-precision astrometry to investigate the pulsar distribution in 47 Tuc and the proper motion of the cluster. In Section 5 we derive some constraints for the surface mass density of the cluster, and limits on the characteristic ages and magnetic fields of the pulsars. In Section 6 we discuss some characteristics of the binary systems, and present the rate of advance of periastron measured for 47 Tuc H. Finally, in Section 7, we summarize our results and briefly discuss the prospects for future timing observations of the pulsars in 47 Tuc.

### 2 OBSERVATIONS AND DATA PROCESSING

Most of the early observations of 47 Tuc were made at 660 MHz using the 64-m radio telescope at Parkes, Australia, between 1989 and 1991 (Manchester et al. 1990; 1991; for a summary of all observations see Table 2). These observations led to the discovery of ten millisecond pulsars. Further observations of 47 Tuc at 430 MHz between 1991 and 1993 (Robinson et al. 1995) resulted in the discovery of one more pulsar, B0021–72N, and phase-coherent timing solutions for 47 Tuc C and D, the two brightest isolated pulsars in the cluster. For the remaining nine pulsars then known, the paucity of detections prevented the determination of any further coherent timing solutions.

1 Because all pulsars are detected in the same telescope beam pattern, and precise positions were not available originally, the pulsars were named B0021–72A, B, etc. Objects A, B and K were later found to be spurious, so the list of pulsars in 47 Tuc starts with B0021–72C, now referred to as J0023–7204C, or 47 Tuc C.
In August 1997, observations of 47 Tuc were resumed at Parkes after a four-year gap. The cluster was observed on 126 d during the following 24 months. The majority of these observations were made using the central beam of a multi-beam system (Lyne et al. 2000) at a central frequency of 1374 MHz (using the central beam of a multi-beam system (Lyne et al. 2000) at a central frequency of 1374 MHz (λ20 cm) with a bandwidth of 288 MHz. Incoming signals from two orthogonal polarizations were down-converted and filtered in a 2 × 96 × 3-MHz filter bank. After summing the polarization pairs, the resulting voltages were one-bit sampled every 125 μs and, together with accurate time referencing, stored on magnetic tapes for later analysis.

Recording the raw data in this way has two benefits. First, it allows off-line searches for new pulsars to be carried out. This strategy has been remarkably successful, essentially doubling the number of pulsars known, many of which are undetectable most of the time because of interstellar scintillation (Camilo et al. 2000). Secondly, for timing purposes, it allows us to refine the timing models iteratively, resulting in the ephemerides reported here.

In the timing analysis, the raw data are de-dispersed and folded according to an initial ephemeris for each pulsar. In the early stages, this ephemeris is determined from variations in the observed rotational period of the pulsar in the discovery and confirmation observations. It is important that the pulsars be detected frequently, otherwise it is impossible to count unambiguously the number of pulsar rotations between two different epochs. The 20-cm observations made after 1997 August made use of the excellent sensitivity of the Parkes multi-beam system and resulted in a large increase in the detection rate for all the previously known pulsars in 47 Tuc when compared to the earlier observations at 660 MHz. This was despite the lower flux densities at 1400 MHz.

The integrated pulse profiles obtained by folding the data at the predicted pulse period are then cross-correlated with a low-noise ‘standard’ pulse profile (see Camilo et al. 2000 for the 20-cm profiles). This allows the determination of topocentric pulse times-of-arrival (TOAs), referred to the observatory time standard. This was related to UTC National Institute of Standards and Technology (NIST) by a radio link to the Tidbinbilla Deep Space Station and from there to NIST by a GPS common-view system. We then use the TEMPO software package\(^2\) to calculate the corresponding barycentric TOAs using the assumed pulsar position and the JPL DE200 solar system ephemeris (http://ssd.jpl.nasa.gov). The differences between the measured and predicted TOAs are used to improve the parameters of the ephemeris (for further details of this process see, e.g., Taylor 1992). In the early stages, the improved ephemeris is used to reprocess the raw data. This iterative process increases the number and quality of the TOAs, which in turn are used to improve the ephemeris.

After determining the timing solutions for the 1997–1999 period, we re-analysed the raw data from the earlier observations (see Table 2). This resulted in a substantial increase in the number and quality of TOAs, especially for pulsars in binary systems, because of the much improved orbital ephemerides.

3 COHERENT TIMING SOLUTIONS FOR 15 PULSARS

The coherent timing solutions obtained from the analysis described in the previous section result in a wealth of high-precision astrometric, spin and (for the binary pulsars) orbital information. Table 3 summarizes the current timing status for the pulsars known in 47 Tuc. The orbital parameters for 47 Tuc S and T were first determined using a new technique described by Freire, Kramer & Lyne (2001), while the orbital parameters of the remaining binaries had been determined earlier (Robinson et al. 1995; Camilo et al. 2000). The timing solutions were obtained as follows.

Because the dispersion in proper motions among pulsars is expected to be very small (see Section 4.3), we assume that all pulsars have the average proper motion of the cluster, the value of which we take to be that determined from Hipparcos data (Table 1). Using TEMPO, the available 1991–1999 TOAs were fitted to a model for each pulsar containing celestial coordinates, spin parameters, and binary elements where relevant. Dispersion measures (DM) were obtained separately by measuring frequency-dependent delays across the 288-MHz bandwidth available at 1400 MHz, and the resulting values coincide with those given by Camilo et al. (2000). We also fitted astronomically meaningless time offsets between groups of TOAs obtained at different frequencies and with different time resolution. We do this because it is difficult to make a proper absolute alignment of pulse profiles obtained at different frequencies owing to variations in pulse shapes.

In Table 4 we present the positions, rotational parameters, and DMs obtained in this way for 15 pulsars. The corresponding timing residuals for the seven isolated pulsars are displayed in Fig. 1 as a function of time.

The orbital elements for the eight binary pulsars with timing solutions are presented in Table 5, while the corresponding timing residuals are shown in Fig. 2 as a function of time, and in Fig. 3 as a function of orbital phase.

For each pulsar we list the five Keplerian parameters: binary period \(P_b\), projected semimajor axis light travel time \(x\), time of passage through the ascending node \(T_{\text{asc}}\) and, where measurable, longitude of periastron \(\omega\) and eccentricity \(e\). For 47 Tuc H we indicate the time of passage through periastron, rather than through ascending node, and we also list the measured rate of advance of

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\(^2\)http://pulsar.princeton.edu/tempo

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periastron $\omega$. For low-eccentricity binaries (all but 47 Tuc H), in which some orbital parameters are highly covariant in standard binary fits, we used the binary model ELL1 as implemented in \textsc{tempo} to determine the solutions presented (see Lange et al. 2001).

Finally, we attempted to measure proper motions for those pulsars for which we had a combination of a long time-baseline and generally high-quality data. We did this with a straightforward \textsc{tempo} fit, and list the results in Table 6 for five pulsars. We also determined proper motions by comparing the positions of the pulsars in 1992–1993 and 1998–1999: for each of the two independent position measurements, no proper motion was assumed, and all TOAs used at each epoch had the same frequency (430 MHz in early 1990s, 1400 MHz in late 1990s), thus avoiding any problems with the alignment of standard profiles at different frequencies. In these fits we used the accurate rotational and binary parameters obtained previously, so only the celestial coordinates were determined proper motions by comparing the positions of the pulsars and the low detection rate, particularly in the

All uncertainties presented in Tables 4–6 are our best estimates of realistic 1$\sigma$ confidence levels, and in most cases are twice the formal fit uncertainties from \textsc{tempo}.

The main factors limiting the precision of the timing solutions presented in this paper are the relatively low signal-to-noise ratio of the pulsars and the low detection rate, particularly in the observations of the early 1990s. For some of the pulsars, the 20-cm flux densities are so low that they are detectable only in about 10 per cent of all observations (see Camilo et al. 2000). Additionally, the data obtained between 1989 July and 1991 May were not used in any of the fits because the difference between the Parkes clock and UTC is not known to better than 50 s. This introduces significant errors that would compromise several of the measurements.

### 4 PULSAR POSITIONS

The timing solutions presented in the previous section bring the number of pulsars in 47 Tuc with accurately known positions to 15, and with measured proper motions to five. For easy reference, we derive from the coordinates in Table 4 the east-west ($x$) and north-south ($y$) offsets of the pulsars relative to the cluster centre. These are presented in Table 7 and depicted in Fig. 4. In the remainder of this section, we discuss the implications of these positional measurements.

#### 4.1 Radial distribution

The radial distribution for the pulsars in 47 Tuc was first addressed by Rasio (2000), based on an earlier version of some of the results we present here (Freire et al. 2000). We now outline the main characteristics of this distribution and draw some conclusions from it.

The most striking characteristic of the pulsar distribution is that all the pulsars in 47 Tuc with measured positions lie within 1.2 arcmin (3 core radii) of the centre of the cluster, despite the fact that the tidal radius of 47 Tuc is about 40 arcmin (Table 1). This distribution, as we shall now see, is not an artefact introduced by the size and shape of the Parkes telescope radio beam pattern, which has a half-power radius of $\approx 7$ arcmin at 1400 MHz.

The flux densities for most of the pulsars have been calculated by Camilo et al. (2000). Among the pulsars with a known solution, 47 Tuc N has the lowest flux density ($0.03 \pm 0.01 \text{ mJy}$). Supposing that this value is the lower limit for the flux density of a pulsar for which we can obtain a timing solution, weak pulsars like 47 Tuc U, with a flux density of 0.06 $\pm 0.01 \text{ mJy}$, are detectable in a circle with a radius of at least 7 arcmin. There are 10 pulsars with at least this flux density, and the fact that none of these is seen outside a radius of 1.2 arcmin is therefore a true feature of the pulsar distribution, and not an artefact arising from the shape of the beam. It should also be noted that the original surveys at 660 MHz (Manchester et al. 1991), with a larger telescope beam, covered an area approximately 100 times the size of the roughly arcmin-scale central region in which we now localize the 15 pulsars.

The present pulsar distribution is essentially an equilibrium distribution. The characteristic ages of these pulsars are of the

### Table 4

Celestial coordinates and rotational parameters for 15 pulsars in 47 Tuc at the reference epoch of MJD 51 000. In this and in the following tables, the numbers in parentheses are 1$\sigma$ confidence-level uncertainties in the last digits quoted. The positions are referred to the J2000 equinox. For all pulsars, these parameters were fitted assuming that the pulsar has the proper motion of the cluster given in Table 1 (see Section 4.3). For the DMs, the error is one or less in the last digit quoted.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>Right Ascension (h m)</th>
<th>Declination (° ″)</th>
<th>Period (ms)</th>
<th>$P$ $(10^{-20})$</th>
<th>DM (cm$^{-3}$ pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>00 23 50.3511(2)</td>
<td>−72 04 31.486(1)</td>
<td>5.756779980968(5)</td>
<td>−4.985(2)</td>
<td>24.6</td>
</tr>
<tr>
<td>D</td>
<td>00 24 13.8776(2)</td>
<td>−72 04 43.8323(9)</td>
<td>5.3575732850382(5)</td>
<td>−0.333(2)</td>
<td>24.7</td>
</tr>
<tr>
<td>E</td>
<td>00 24 11.1013(4)</td>
<td>−72 05 20.131(3)</td>
<td>3.536329147529(7)</td>
<td>9.852(2)</td>
<td>24.2</td>
</tr>
<tr>
<td>F</td>
<td>00 24 03.8519(2)</td>
<td>−72 04 42.799(1)</td>
<td>2.6235793491667(3)</td>
<td>6.451(1)</td>
<td>24.4</td>
</tr>
<tr>
<td>G</td>
<td>00 24 07.956(1)</td>
<td>−72 04 39.683(7)</td>
<td>4.0403791457482(8)</td>
<td>−4.215(2)</td>
<td>24.4</td>
</tr>
<tr>
<td>H</td>
<td>00 24 06.6898(7)</td>
<td>−72 04 06.789(3)</td>
<td>3.210340709441(1)</td>
<td>−0.162(5)</td>
<td>24.4</td>
</tr>
<tr>
<td>I</td>
<td>00 24 07.932(1)</td>
<td>−72 04 39.664(5)</td>
<td>3.484992064038(1)</td>
<td>−4.59(1)</td>
<td>24.4</td>
</tr>
<tr>
<td>J</td>
<td>00 24 59.4040(1)</td>
<td>−72 03 58.7720(9)</td>
<td>2.100635458586(2)</td>
<td>−0.978(4)</td>
<td>24.6</td>
</tr>
<tr>
<td>L</td>
<td>00 24 03.770(2)</td>
<td>−72 05 56.90(1)</td>
<td>4.3461680057845(2)</td>
<td>−12.19(2)</td>
<td>24.4</td>
</tr>
<tr>
<td>M</td>
<td>00 23 54.485(3)</td>
<td>−72 05 30.72(1)</td>
<td>3.676643219590(3)</td>
<td>−3.832(6)</td>
<td>24.4</td>
</tr>
<tr>
<td>N</td>
<td>00 24 09.1835(9)</td>
<td>−72 04 28.875(7)</td>
<td>3.0539543473732(1)</td>
<td>−2.186(2)</td>
<td>24.6</td>
</tr>
<tr>
<td>O</td>
<td>00 24 04.6492(5)</td>
<td>−72 05 53.751(3)</td>
<td>2.6433432956679(7)</td>
<td>3.032(6)</td>
<td>24.4</td>
</tr>
<tr>
<td>Q</td>
<td>00 24 16.488(1)</td>
<td>−72 04 25.149(7)</td>
<td>4.033181182805(2)</td>
<td>3.412(2)</td>
<td>24.3</td>
</tr>
<tr>
<td>T</td>
<td>00 24 08.541(2)</td>
<td>−72 04 38.91(2)</td>
<td>7.584479792133(9)</td>
<td>29.47(6)</td>
<td>24.4</td>
</tr>
<tr>
<td>U</td>
<td>00 24 09.8325(5)</td>
<td>−72 03 59.667(3)</td>
<td>4.342826691451(2)</td>
<td>9.524(3)</td>
<td>24.3</td>
</tr>
</tbody>
</table>
order of $10^9$ yr, while the relaxation time (the time it takes for a pulsar to change kinetic energy significantly through interactions with other stars) in the core of 47 Tuc is of the order of $10^8$ yr (Djorgovski 1993). Therefore, the pulsars are expected to be in thermal equilibrium with the remaining stars, and their distribution should be merely a function of their mass and of the potential of the cluster. Indeed, the de-projected pulsar distribution is consistent with $n(r) \sim r^{-2}$ until about 4 core radii, decreasing sharply outside this radius (see Fig. 5).

This concentration of pulsars near the centre of the cluster is a consequence of mass segregation: if all objects in a cluster have reached thermal equilibrium, i.e., all stellar populations have a similar average kinetic energy, then the most massive populations, among which are the pulsars, will have smaller velocities, and therefore will dwell deeper in the potential well of the cluster. To quantify this, we note that the kinetic energy of the lighter stars in the cluster core, with mass $m_{\text{ms}}$, is limited by the escape velocity $v_e = \sqrt{-2W(0)}$. Equating the resulting limit of kinetic energy to that of the pulsars results in a limit to the velocity of pulsars bound in the cluster core:

$$v_{\text{p, max}}^2 = \frac{(m_{\text{ms}}/m_p)v_e^2}{2},$$

where $m_p$ is the pulsar mass. As pulsars move away from the centre of the cluster under the influence of the gravitational potential of the cluster, their velocities decrease, eventually reaching zero at a maximum possible distance $r_{\text{lim}}$ from the centre of the cluster.

The potential energy $U(r)$ at $r_{\text{lim}}$ is calculated from conservation of energy:

$$U(r_{\text{lim}}) = m_p W(r_{\text{lim}}) = m_p W(0) + m_p v_{\text{p, max}}^2/2,$$

where $W(r)$ is the cluster potential. Using equation (1) we obtain

$$W(r_{\text{lim}}) = W(0) \left( 1 - \frac{m_{\text{ms}}}{m_p} \right).$$

This limit, with $m_{\text{ms}} = 0.8 M_\odot$ and $m_p = 1.45 M_\odot$ (the average

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Post-fit timing residuals as a function of date for seven isolated pulsars. In this and in Figs 2 and 3 the crosses, circles and filled diamonds represent residuals at 1400, 660 and 430 MHz respectively. At right we display corresponding histograms of timing residual/uncertainty for individual TOAs.}
\end{figure}
Table 5. Orbital parameters for the eight binary pulsars in 47 Tuc with known coherent timing solutions. For 47 Tuc H the value listed under $T_{\text{asc}}$ is the time of passage through periastron.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>$P_b$ (days)</th>
<th>$x$ (sec)</th>
<th>$T_{\text{asc}}$ (MJD)</th>
<th>$\omega$ (°)</th>
<th>$e$</th>
<th>$\dot{\omega}$ ($°$ yr$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>2.256844818(6)</td>
<td>1.981839(4)</td>
<td>51000.4194595(8)</td>
<td>218.5(7)</td>
<td>0.000318(4)</td>
<td>...</td>
</tr>
<tr>
<td>H</td>
<td>2.3576965(5)</td>
<td>2.152812(7)</td>
<td>51000.97359(6)</td>
<td>110.494(9)</td>
<td>0.070557(5)</td>
<td>0.059(12)</td>
</tr>
<tr>
<td>I</td>
<td>0.2206649386(2)</td>
<td>0.0404087(6)</td>
<td>51000.0416882(3)</td>
<td>...</td>
<td>&lt;0.001</td>
<td>...</td>
</tr>
<tr>
<td>J</td>
<td>0.135943404(2)</td>
<td>0.045157(5)</td>
<td>51000.021155(2)</td>
<td>...</td>
<td>&lt;0.0004</td>
<td>...</td>
</tr>
<tr>
<td>Q</td>
<td>1.18008405(1)</td>
<td>1.462200(7)</td>
<td>51000.985847(2)</td>
<td>132(9)</td>
<td>0.00008(1)</td>
<td>...</td>
</tr>
<tr>
<td>T</td>
<td>1.12617678(2)</td>
<td>1.33847(3)</td>
<td>51000.317048(3)</td>
<td>55(6)</td>
<td>0.00037(4)</td>
<td>...</td>
</tr>
<tr>
<td>U</td>
<td>0.4291056829(6)</td>
<td>0.526953(5)</td>
<td>51000.0705011(6)</td>
<td>341(7)</td>
<td>0.00015(2)</td>
<td>...</td>
</tr>
</tbody>
</table>

Figure 2. Post-fit timing residuals as a function of date for eight binary pulsars and corresponding histograms of timing residual/uncertainty for individual TOAs.
mass for isolated and binary pulsars), is represented by a dashed line in Fig. 6, where we use a King model for the gravitational potential of 47 Tuc (King 1966; Phinney, private communication). From this we derive $r_{\text{lim}} \sim 5r_c$, which is very close to the inferred $4r_c$. Therefore, the confinement of the pulsars in the inner region can be explained, to first order, as resulting from the shape of the potential and the upper limit for the kinetic energy of the pulsars. The fact that all pulsars are located close to the core is a constraint on any future more accurate model of the gravitational potential of the cluster.

Spitzer (1987) demonstrates that the distributions of two stellar species of mass $m_i$ and $m_j$ in thermal equilibrium with all stars in a cluster are related by

$$n_i(r) \propto n_j(r)^{m_i/m_j}.$$  \hspace{1cm} (4)

Therefore, two stellar populations with similar masses in thermal equilibrium should have similar radial distributions. Some of the most massive blue stragglers known in 47 Tuc have masses of 1.3–1.6 $M_\odot$ (Gilliland et al. 1998). Because this is the mass range expected for pulsars (Thorsett & Chakrabarty 1999), including binaries such as those in 47 Tuc (Camilo et al. 2000), both populations should have similar radial distributions, which is apparently the case (Rasio 2000). However, we caution that the
the pulsars in M15 is
the dominant stellar species should be of the type only the most massive blue stragglers with that of pulsars, and in homogeneous, having a relatively wide range of masses. population considered by Rasio (2000) is not altogether near the centre of M15 is 0.9-M
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### Table 6. Proper motions of five pulsars in 47 Tuc. The average is obtained by weighting the proper motions by the inverse of each uncertainty.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>( \mu_x ) (mas yr(^{-1}))</th>
<th>( \mu_y ) (mas yr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>7.8(1.6)</td>
<td>-3.2(1.0)</td>
</tr>
<tr>
<td>D</td>
<td>7.7(1.8)</td>
<td>-2.6(1.6)</td>
</tr>
<tr>
<td>E</td>
<td>9.1(2.3)</td>
<td>-3.9(2.1)</td>
</tr>
<tr>
<td>F</td>
<td>6.6(2.6)</td>
<td>-3.7(2.0)</td>
</tr>
<tr>
<td>J</td>
<td>4.6(0.8)</td>
<td>-3.7(0.8)</td>
</tr>
<tr>
<td>Average:</td>
<td>6.6(1.9)</td>
<td>-3.4(0.6)</td>
</tr>
</tbody>
</table>

### Table 7. East–west (x) and north–south (y) offsets of 15 pulsars from the centre of the cluster, assumed here to be exactly at \( \alpha = 00^\circ 24^\prime 05^\prime 29^\prime \) and \( \delta = -72^\circ 04^\prime 52^\prime 3^\prime \). The errors in the offsets are 2 or less in the last digits quoted. The angular distance of each pulsar from the centre of the cluster, \( \theta_\perp \), has an accuracy limited by the uncertain absolute position of the cluster (Table 1).

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>x (arcmin)</th>
<th>y (arcmin)</th>
<th>( \theta_\perp ) (arcmin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-1.1541</td>
<td>0.3469</td>
<td>1.21</td>
</tr>
<tr>
<td>D</td>
<td>0.6634</td>
<td>0.1411</td>
<td>0.68</td>
</tr>
<tr>
<td>E</td>
<td>0.4489</td>
<td>-0.4639</td>
<td>0.65</td>
</tr>
<tr>
<td>F</td>
<td>-0.1111</td>
<td>0.1583</td>
<td>0.19</td>
</tr>
<tr>
<td>G</td>
<td>0.2060</td>
<td>0.2103</td>
<td>0.29</td>
</tr>
<tr>
<td>H</td>
<td>0.1088</td>
<td>0.7585</td>
<td>0.77</td>
</tr>
<tr>
<td>I</td>
<td>0.2041</td>
<td>0.2106</td>
<td>0.29</td>
</tr>
<tr>
<td>J</td>
<td>-0.4547</td>
<td>0.8921</td>
<td>1.00</td>
</tr>
<tr>
<td>L</td>
<td>-0.1174</td>
<td>-0.0767</td>
<td>0.14</td>
</tr>
<tr>
<td>M</td>
<td>-0.8347</td>
<td>-0.6404</td>
<td>1.05</td>
</tr>
<tr>
<td>N</td>
<td>0.3008</td>
<td>0.3904</td>
<td>0.49</td>
</tr>
<tr>
<td>O</td>
<td>-0.0495</td>
<td>-0.0242</td>
<td>0.06</td>
</tr>
<tr>
<td>Q</td>
<td>0.8651</td>
<td>0.4525</td>
<td>0.98</td>
</tr>
<tr>
<td>T</td>
<td>0.2512</td>
<td>0.2232</td>
<td>0.34</td>
</tr>
<tr>
<td>U</td>
<td>0.3509</td>
<td>0.8772</td>
<td>0.94</td>
</tr>
</tbody>
</table>

4.2 A triple system with two pulsars?

There is a remarkable coincidence between the celestial coordinates of 47 Tuc G, an isolated pulsar, and 47 Tuc I, a binary pulsar (see Table 7). The projected angular distance between these two pulsars is only 120 mas.

To estimate the chance probability of finding two pulsars in such close proximity, suppose that pulsars are randomly distributed in a disc of radius 18 arcsec (the projected angular distance of these two pulsars from the centre of the cluster). For a given pulsar, the probability of finding a second one at a projected distance of 0.12 arcsec or less is proportional to the area of a disc of radius 0.12 arcsec divided by the area of a disc of radius 18 arcsec or \( \sim 5 \times 10^{-5} \). The probability of being at a distance larger than 0.12 arcsec is 0.99995. For the five pulsars known within 18 arcsec, we have 10 different possible pairs. The probability of not finding any pulsar within 0.12 arcsec of any other pulsar is 0.99995\(^{10} = 0.99995 \), i.e., the overall probability of finding one or more pulsars projected within 0.12 arcsec of any other is \( 5 \times 10^{-4} \), about one in 2000.

If the pulsars are near to each other but not interacting significantly, then the acceleration caused by the cluster should be similar for both of them. In Section 5 we see that their values of \( P/P \) are similar, with a difference of \( 3 \times 10^{-18} \) s\(^{-1} \), in a possible range \( \sim 20 \) times larger. Therefore, the combined probability of finding the pulsars with such close projected separation and with such similar accelerations is about one in 40,000.

Given this low formal probability of chance coincidence, it is worth considering that these two pulsars may be in a hierarchical triple system with a major axis of at least 600 au. A system with two components 600 au apart is not likely to remain bound for long in the dense environment of a globular cluster. By considering the stellar flux with one-dimensional speeds of \( v_i(r) \), an estimate...
for the mean time a binary system can survive in a dense environment (i.e., before it gets hit by another cluster member) is

\[ t \approx n(r) \sigma v_z(r) / 2 \]

In this expression \( n(r) \) is the density of stars, \( v_z(r) \) is the one-dimensional stellar velocity dispersion near the binary and \( \sigma \) is the cross-sectional area for the interaction of the binary with a star from the cluster. The King model for 47 Tuc predicts \( n(r) \approx 4 \times 10^4 \text{ pc}^{-3} \) for this location. With \( v_z(r) \approx 10 \text{ km s}^{-1} \) we estimate \( t \approx 10^5 \text{ yr} \). This time is comparable to the orbital period of the putative binary system, and \( 10^{12} \text{ yr} \) the age of the cluster. Thus, despite the low formal probability of a chance coincidence, this simple calculation argues against the reality of such an association. Ultimately, the reality or otherwise of this system can be tested by future measurements of higher period-derivatives for the two pulsars.

### 4.3 Proper motions

The proper motions listed in Table 6 are displayed in Fig. 7, together with the value measured for the proper motion of the cluster by Odenkirchen et al. (1997). At the 2\( \sigma \) level all these proper motions are consistent with each other.

If a pulsar is moving with a transverse velocity \( v_p \) (km s\(^{-1}\)) relative to the centre of the cluster, the difference in proper motions is

\[ \Delta \mu = 0.21 v_p / D \text{ mas yr}^{-1} \]

where \( D \) is the distance to the cluster in kpc. The escape velocity of 47 Tuc is about 58 km s\(^{-1}\) (Table 1). Therefore, in order to detect a pulsar in an escape trajectory at the 3\( \sigma \) level, we would have to measure relative proper motions of about 2.4 mas yr\(^{-1}\) with a precision better than 0.8 mas yr\(^{-1}\), and we are approaching this level for 47 Tuc J (Table 6). If the pulsars are in thermal equilibrium with the surrounding stars, their relative motions are of the order of the central velocity dispersion or less, i.e., \( \approx 12 \text{ km s}^{-1} \). Thus, detecting their relative motions implies measuring relative proper motions of about 0.5 mas yr\(^{-1}\) with a precision of 0.15 mas yr\(^{-1}\). The typical precision for the positions of the weakest pulsars is around 5 mas. In order to obtain a precision of 0.15 mas yr\(^{-1}\) for these weak pulsars, we would have to measure the positions of the pulsars again in \( \approx 50 \) years’ time.
However, for the brightest pulsars, with positional accuracies of about 1 mas, it will suffice to make another measurement in about 10 yr, using current data-acquisition systems.

The assumption of thermal equilibrium for the pulsars implies that, with current timing precision, a measurement of the proper motions of these pulsars is in effect a measurement of the proper motion of the cluster. We can therefore make a weighted average of the five proper motions and determine the motion of the cluster, knowing that the peculiar motions of the pulsars should be smaller than the errors in the individual measurements. The weights chosen are simply the inverse of the uncertainties in each coordinate. For the currently observed proper motions, we find the weighted average value to be $\mu_a = (6.6 \pm 1.9)$ mas yr$^{-1}$ and $\mu_b = (-3.4 \pm 0.6)$ mas yr$^{-1}$. The uncertainties are taken to be the sum in quadrature of the weighted dispersion of the values of proper motion for the pulsars about the average, and 0.5 mas yr$^{-1}$, which accounts for the expected actual dispersion of proper motions.

5 PULSAR ACCELERATIONS IN THE CLUSTER POTENTIAL

Nine of the 15 pulsars in Table 4 have negative period derivatives ($P$). This has been observed before for some pulsars located in globular clusters (e.g. Wolszczan et al. 1989). Rather than being a result of intrinsic spin-up, negative period derivatives are thought to be caused by the acceleration of the pulsar towards the Earth in the cluster potential (see Fig. 8).

In Section 5.1 we use the observed period derivatives of the pulsars to estimate lower limits for the surface mass density of the cluster. We then consider the aptness of a King model together with accepted parameters for 47 Tuc in describing the accelerations experienced by the pulsars as inferred from the period derivatives (Section 5.2). We derive limits for the ages and magnetic fields of the pulsars in Section 5.3.

The observed period derivative, $P_{\text{obs}}$, is the sum of the intrinsic spin-down of the pulsar, $P_{\text{int}}$, and the effect of the acceleration along the line of sight, $a_l$. This sum may be negative if a negative $a_l$ contribution is not exceeded by a positive $P_{\text{int}}$; i.e., a negative $P_{\text{obs}}$ implies $|a_l| > P/P_{\text{int}}$. It also implies $|a_l| > |P/P_{\text{obs}}|$.

The acceleration along the line of sight has several components, denoted by $a_S, a_G,$ and $a_C$ in

$$\left(\frac{P}{P_{\text{obs}}}\right) = \frac{a_S}{c} + \frac{a_G}{c} + \frac{a_C}{c} + \left(\frac{P}{P_{\text{int}}}\right)$$

where the terms represent the following:

1. The centrifugal acceleration (Shklovskii 1970), given by

$$\frac{a_S}{c} = \frac{\mu^2 D}{c}$$

This term, using the values in Table 1, amounts to $a_S/c = (+9 \pm 3) \times 10^{-19}$ s$^{-1}$.

2. The difference in Galactic acceleration along the line of sight between a given object and the barycentre of the solar system ($a_G$). This is a function of the object’s Galactic latitude $b$, longitude $l$, and distance from the solar system $D$. For 47 Tuc we have $b = -44.9^\circ$, $l = 305.9^\circ$ and $D = 5.0 \pm 0.4$ kpc. Using Paczynski’s (1990) model of the gravitational potential of the Galaxy, we obtain $a_G/c = (-4.5 \pm 0.2) \times 10^{-19}$ s$^{-1}$.

3. Accelerations arising from the gravitational field of the globular cluster and its individual stars ($a_C$). This is the most interesting contribution from an astrophysical point of view.

Contributions 1 and 2 total $(+5 \pm 3) \times 10^{-19}$ s$^{-1}$. Henceforth, $(P/P_{\text{obs}})$ indicates the measured value of $P/P$ minus these contributions, and $a_l$ refers solely to $a_C$. All conclusions in this section apply only to the pulsars with known timing solutions.

5.1 Projected surface mass density of 47 Tuc

In Fig. 8 are shown some useful geometrical parameters we use in the remainder of this section. The plane $\Pi$ passes through the centre of the cluster and is perpendicular to the line of sight, and the core radius is given by $r_c = D\theta_c$. A particular observed pulsar has unknown distance to the centre of the cluster, $r$, and to $\Pi$, $l$ – we know only the projected distance of the pulsar to the centre of the cluster, $R_\perp = D\theta_\perp$. The acceleration along the line of sight, $a_l$, is the only component potentially detectable from the Earth.

We now derive constraints for the surface mass density of the cluster in its inner regions. According to Phinney (1993), the following expression is valid to within $\sim 10$ per cent for all plausible cluster models, and for all $\theta_\perp$:

$$\frac{a_l}{c}(\theta_\perp) = 0.1 \frac{GM_{\text{cyl}}(< \theta_\perp)}{c^2 D^2 \theta_\perp^2}$$

$$= 5.1 \times 10^{-19} \left(\frac{\Sigma(< \theta_\perp)}{10^3 M_\odot \text{pc}^{-2}}\right) \text{s}^{-1},$$

where $M_{\text{cyl}}(< \theta_\perp)$ is the mass of all the matter with a projected distance smaller than $\theta_\perp$, and $\Sigma(< \theta_\perp)$ is the corresponding projected surface mass density. The limit on $\Sigma$ does not depend on estimates of the distance to the cluster or core radius. The pulsars with negative observed period derivatives provide a lower bound on $a_l$ and therefore, with equation 8, on $\Sigma$ and $M_{\text{cyl}}$. These limits are presented in Table 8.
5.2 Accounting for the pulsar accelerations

We have used a King model of 47 Tuc to calculate the gravitational potential as a function of distance to the centre of the cluster, \( W(r/r_c) \), divided by \( v_l(0)^2 \) (Fig. 6). The input for such a model is a single parameter, the logarithm of the core radius divided by the tidal radius (King 1966).

The radial derivative of \( W(r/r_c)/v_l(0)^2 \) yields a ‘normalised acceleration’, \( a_o(r/r_c) \). In order to calculate accelerations \( A \) in \( \text{m s}^{-2} \), we need two more cluster parameters: the central dispersion \( o \), can be obtained for each pulsar by subtracting the intrinsic \( P/P \) from \( (P/P)_{\text{obs}} \). Unfortunately, the intrinsic period derivative is not known. We derive limits on this quantity in Section 5.3, where we also find that the average characteristic age of the pulsars is greater than \( \sim 1 \) Gyr. Here we assume that each pulsar has a characteristic age of 3 Gyr and subtract the corresponding \( P/P \) from each observed value (listed in Table 10, below) to obtain an estimate for \( A_o \). These, along with values of \( A_o \) calculated from equation (10), are presented in Table 9.

We are now in a position to compare the ‘observed’ and average predicted accelerations. We do this for each pulsar in the last column of Table 9. This value is significantly biased for some individual pulsars by the subtraction of a fixed \( P/P = 0.5 \times 10^{-17} \text{ s}^{-1} \), our first-order attempt at accounting for the intrinsic period derivatives. However, the average of all values in the last column of the table should be approximately unity, if the modelling described above is to be consistent with the actual accelerations experienced by the pulsars – which indeed it is, despite considerable uncertainty: \( \langle A_o/A_o(R_o) \rangle = 1.2 \pm 0.7 \).

As an additional consistency check on the King model of the cluster potential, we performed a Monte Carlo simulation of the pulsar population of 47 Tuc. The aim of this simulation is to see, given reasonable input assumptions, whether we can reproduce the distribution of \( (P/P)_{\text{obs}} \).

In the simulation, we generate a spherically symmetric population of \( 10^6 \) pulsars with a radial distribution of the type \( n(r) \propto r^{-2} \) and a cut-off of 4 core radii. Each of the pulsars was randomly assigned an age from a flat distribution ranging between 500 Myr and 10 Gyr – the mean \( P/P = 0.5 \times 10^{-17} \text{ s}^{-1} \) corresponding to what was assumed before. These assigned ages correspond to the intrinsic \( P/P \) of each of the model pulsars. Finally, given the position of each of the model pulsars relative to the cluster centre, we calculate the contribution to \( P/P \) from the acceleration in the cluster determined from the King model with \( D = 5.0 \text{ kpc}, v_l(0) = 11.6 \pm 1.4 \text{ km s}^{-1}, \theta_c = 23.1 \text{ arcsec} \). By adding this contribution to the intrinsic \( P/P \) we have, for each pulsar, a model observed \( P/P \) which can then be directly compared to the sample of 15 pulsars for which we have timing solutions.

The results of this simulation are shown in Fig. 9, where we present a cumulative plot of \( (P/P)_{\text{obs}}/l_{\text{max}}(R_c)/\text{c} \) for the real and simulated samples. Given the relatively small size of the observed distribution, and the straightforward simulation that we have performed, the agreement between the model and the observed samples is good.

Our overall conclusion is therefore that a King model for 47 Tuc, with \( D = 5.0 \pm 0.4 \text{ kpc}, v_l(0) = 11.6 \pm 1.4 \text{ km s}^{-1}, \theta_c = 23.1 \text{ arcsec} \) and a small contribution from the intrinsic \( P/P \) of the pulsars, provides a good description for the observed values of \( P/P \).

\[ A_o = \frac{\sqrt{r_c^2 - r^2} - \lambda^{-1} \int_0^l A(r/r_c) \frac{1}{r} \frac{dl}{r}}{\theta_c}, \]

where \( \lambda \) is a normalising constant such that \( \lambda^{-1}/r^2 \) is the local linear density of simulated pulsars, and \( N \) is their total number in a column along the integration path.

How do these ‘average’ theoretical accelerations along each line of sight compare with the observed accelerations? The observed acceleration, \( A_o \), can be obtained for each pulsar by subtracting the intrinsic \( P/P \) from \( (P/P)_{\text{obs}} \). Unfortunately, the intrinsic period derivative is not known. We derive limits on this quantity in Section 5.3, where we also find that the average characteristic age of the pulsars is greater than \( \sim 1 \) Gyr. Here we assume that each pulsar has a characteristic age of 3 Gyr and subtract the corresponding \( P/P \) from each observed value (listed in Table 10, below) to obtain an estimate for \( A_o \). These, along with values of \( A_o \) calculated from equation (10), are presented in Table 9.

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Our overall conclusion is therefore that a King model for 47 Tuc, with \( D = 5.0 \pm 0.4 \text{ kpc}, v_l(0) = 11.6 \pm 1.4 \text{ km s}^{-1}, \theta_c = 23.1 \text{ arcsec} \) and a small contribution from the intrinsic \( P/P \) of the pulsars, provides a good description for the observed values of \( P/P \).
5.3 Limits on pulsar ages and magnetic fields

We now obtain one-sided limits for the characteristic age and the inferred surface dipole magnetic field strength of the pulsars, by calculating the maximum intrinsic period derivative for each pulsar, $P_{\text{int max}}$. First, suppose we have a reliable estimate for the maximum possible acceleration for the line of sight of each pulsar, $a_{\text{int max}}(R_\perp)$. Next, assume that each pulsar actually experiences such a maximum (negative) acceleration. Under such conditions, an observed $|P/P|$ that is different from $|a_{\text{int max}}(R_\perp)/c|$ is caused by a finite (positive) intrinsic $P/P$. Hence, $(P/P)_{\text{int}} < |a_{\text{int max}}(R_\perp)/c| + (P/P)_{\text{obs}}$.

After determining $P_{\text{int max}}$ we derive a minimum characteristic age of the pulsar, $\tau_c > P/(2P_{\text{int max}})$, and an upper limit for the magnetic field, $B < 3.2 \times 10^{19}(PP_{\text{int max}})^{1/2}$ Gauss.

Only a reliable upper limit for the acceleration along each line of sight remains to be calculated. Phinney (1993) calculated the maximum acceleration expected near the centre of a globular cluster,

$$a_{\text{int max}}(R_\perp) = \frac{3}{2} \frac{v_z^2(R_\perp)}{\sqrt{v_z^2 + R_\perp^2}},$$

(11)

This expression is valid to within $\sim 10$ per cent for $R_\perp \leq 2\tau_c$, provided that the function $v_z(R_\perp)$ is accurately known.

The values of $P/P$ observed for the pulsars (reflecting in part the gravitational potential of the cluster) do not appear to decrease greatly with $R_\perp$ (Fig. 10), so we first assume conservatively that $a_{\text{int max}}(R_\perp)$ does not decrease with $R_\perp$. Therefore, the absolute upper limit for $a_{\text{int max}}(R_\perp)$ is $a_{\text{int max}}(0)$. Within the constraints imposed by the parameters and uncertainties in Table 1, the maximum possible value for $a_{\text{int max}}$ at the centre of the cluster is obtained with $D = 4.6$ kpc, $v_z(0) = 13.0$ km s$^{-1}$ and $\theta_c = 21.4$ arcsec: according to equation (11), $a_{\text{int max}} = 5.74 \times 10^{-17}$ s$^{-1}$.

The limits for the pulsar parameters obtained with this constant maximum acceleration are presented in Table 10. All characteristic ages are larger than 170 Myr and all pulsars have magnetic fields lower than $2.4 \times 10^9$ Gauss. These values are consistent with those measured for Galactic millisecond pulsars (e.g. Camilo, Thorsett & Kulkarni 1994).

Note that these limits are extremely conservative, and are independent of any detailed modelling of the cluster. It is nevertheless clear that even with such a crudely over-estimated $a_{\text{int max}}(R_\perp)$ we can derive useful limits for the parameters of the pulsars.

By modelling the cluster and obtaining better constraints for $a_{\text{int max}}(R_\perp)$, more stringent limits can be derived. For this reason, we now compare the observed pulsar $P/P$ with the maximum accelerations along each line of sight calculated from the King model (Fig. 10). The limits for the pulsar parameters derived from this model are also presented in Table 10. As expected, these are more constraining than those derived before. In particular, the characteristic ages for 47 Tuc C and M exceed 3 Gyr, and a simple average of the lower limits on age for all pulsars yields $\tau_c > 0.9$ Gyr, while the magnetic field strengths of 47 Tuc I and M are less than $3 \times 10^8$ Gauss, very close to the lowest values observed among Galactic disc pulsars. These values depend, of course, on the particular cluster model used in calculating the accelerations.

Limits for the characteristic ages and magnetic fields of the pulsars in M15 have been derived by Anderson (1992), using a mass model for the cluster based on the surface luminosity density and assuming a constant mass-to-light ratio. The majority of pulsars in that cluster have characteristic ages $\sim 10^{10}$ yr (Anderson 1992), but there are signs of a relatively recent burst in pulsar formation. At least two pulsars have maximum characteristic ages $\sim 10^9$ yr and magnetic fields of about $(10-20) \times 10^9$ Gauss. This is

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Figure 9. Cumulative distributions of $(P/P)_{\text{obs}}/|a_{\text{int max}}/c|$ for 15 observed pulsars, and for a simulated population (smooth curve), normalized by the observed distribution (see Section 5.2).

Figure 10. The values for $(P/P)_{\text{obs}}$ (Table 10) are plotted versus $\theta_\perp$ for the 15 pulsars with coherent timing solutions (stars). We also plot the maximum acceleration along each line of sight calculated from a King model with the nominal distance and central velocity dispersion (solid line), and accounting for the uncertainties in those quantities (dashed lines). For all the curves and pulsar positions, we assumed $\theta_\perp$ to have its nominal value. See Table 1 for cluster parameters and their uncertainties.
attributed to the ongoing core collapse in M15, leading to much increased recycling of pulsars. In 47 Tuc, on the other hand, there are no visible signs of such a recent burst in pulsar formation: all the known pulsars for which a timing solution has been obtained have characteristic ages (probably much) greater than 170 Myr, weak magnetic fields, and very short rotational periods.

6 Binary pulsar systems in 47 Tuc

The binary systems containing pulsars in 47 Tuc are divided into two main groups, segregated by companion mass (Camilo et al. 2000). The first is composed of binaries with very short orbital periods (1.5–5.5 hr) and companion masses ~0.03 M\_\odot. We have timing solutions for three of these (47 Tuc I, J, and O). The second group has orbital periods in the range 0.4–2.3 d and companion masses ~0.2 M\_\odot; we refer to this group as ‘normal’ binaries. Of these, 47 Tuc E, H, Q, T and U have a timing solution at the moment.

Coherent timing solutions give extremely detailed orbital information, and often allow the measurement of very small eccentricities. Most millisecond pulsar–white dwarf systems in the Galactic disc have very low, but measurably significant, eccentricities (e.g. Camilo 1999). These are thought to result from tidal interactions between the neutron star and convective cells in the envelope of its companion during the giant phase of its evolution, and a relationship between orbital period and eccentricity can be derived for such systems (Phinney 1992). The millisecond pulsar–white dwarf systems in globular clusters often have larger eccentricities than those in the disc of the Galaxy (Phinney 1992). These are thought to result from interactions with other stars in the cluster: the denser the environment, the more likely one is to find a relatively large degree of eccentricity in the system.

The five eccentricities measured for binary pulsars in 47 Tuc (for the normal binaries 47 Tuc E, H, Q, T and U; see Table 5) are all much larger than one would expect for similar systems in the Galactic disc, and are therefore, presumably, fossil remnants of gravitational interactions between the binary systems and other cluster stars. Rasio & Heggie (1995) calculate the expected value of this eccentricity for a system with a particular binary period that has interacted with other stars in a region of known density for a certain length of time. Unfortunately we do not know the relevant densities or interaction-time scales for the pulsars observed in 47 Tuc with any degree of certainty, possibly not even to an order of magnitude. Nevertheless, taking plausible estimates for density and time-scales, we derived some ‘predicted’ eccentricities. Those for 47 Tuc E, Q, and T are within an order of magnitude of the observed values, which is as close to agreement as we can expect to attain given the uncertainties in input parameters. The computed eccentricity for 47 Tuc U is two orders of magnitude below the observed value, while that for 47 Tuc H is under-predicted by a factor of 1000.

One possible explanation for the unexpectedly large eccentricity of 47 Tuc U, and possibly that of 47 Tuc H, is that they have in the past spent a considerable amount of time in a region with a much higher stellar density than they are found in at present. For instance, if 47 Tuc L (an isolated pulsar) had spent 10\(^{10}\) yr in its present location while part of a 2.35-d binary, its computed eccentricity would be 0.15. Therefore, some of the pulsars we observe relatively far from the centre of the cluster may have non-circular orbits in the cluster potential which take them through higher-density regions periodically, or may have been ejected from higher-density regions to their present locations, leading to some of the large eccentricities measured.

Alternatively, the high eccentricity of the 47 Tuc H binary system, by far the largest in the cluster, could indicate that it obtained its present companion as an already-formed white dwarf through an exchange interaction. However, such exchanges tend to produce even higher eccentricities (Phinney 1992).

The large eccentricity of 47 Tuc H has permitted a measurement of its rate of advance of periastron: \(\omega = (0.059 \pm 0.024)\times 10^{-6}\) yr\(^{-1}\) (Table 5). Assuming that this advance is entirely a result of general relativity, and not of any tidal effects (which is likely, since both stars are presumably degenerate and have negligible dimensions...
Hipparcos is consistent with the proper motion of the cluster measured with value calculated from averaging the proper motions of the pulsars further timing measurements are required to investigate this.

Two of the pulsars (47 Tuc G and I) have a projected separation of only 600 au, and similar $P/P_0$ (Thorsett & Chakrabarty 1999). Although it is unlikely, they may belong to the first system known with two detectable pulsars, but their confinement near the centre of the cluster was once, or has been for a long time, in a high-density state. In that case the number of millisecond pulsars in the cluster depends heavily on its previous dynamical history. This could explain why clusters with similar properties (core density, total mass, stellar mass distribution, metallicity) have large differences in their pulsar populations, in both number and kind (Kulkarni & Anderson 1996). More complete searches for pulsars in globular clusters, a better understanding of selection effects and a good determination of the evolutionary states of globular clusters will be needed to address this question.

A continuing timing programme will lead to additional and improved measurements of the proper motions of the pulsars. A

Figure 11. Mass–mass diagram for the 47 Tuc H system. The allowed range of total mass (sloping straight line surrounded by dashed lines) is derived from the measured rate of advance of periastron and its 2-$\sigma$ uncertainty. The area below the thick curving line is excluded by the measured mass function and the requirement that $\cos i \geq 0$. There is a 95 per cent a priori probability that the system lies at $\cos i = 0.95$. The vertical lines indicate the average measured mass for neutron stars in the radio pulsar population, $1.35 \pm 0.04 M_\odot$ (Thorsett & Chakrabarty 1999).
measurement of the proper motions of the pulsars relative to the cluster would add constraints to the cluster mass model. Continued timing will also allow a better determination of the orbital parameters of 47 Tuc H, and will test whether 47 Tuc G and I are physically associated. Additional solutions for known pulsars may also be determined, and new pulsars will almost certainly be discovered. Continued monitoring of this cluster is therefore likely to remain a worthy enterprise.

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