2D modelling of the light distribution of early-type galaxies in a volume-limited sample – I. Simulations with artificial data

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ABSTRACT

This paper is the first of two dedicated to the application of a two-dimensional (2D) fit to the light distribution of 73 early-type galaxies belonging to the Virgo and Fornax clusters, a sample of which is 80 per cent complete, volume and magnitude limited down to $M_B = -17.3$, and highly homogeneous. Two empirical models of the surface-brightness distribution of the early-type galaxies have been tested: the first uses a single spheroid represented by the $r^{1/n}$ Sersic law, while the second is characterized by two components, ‘bulge’ and ‘disc’, by means of the $(r^{1/n} + \text{exp})$ laws. The $\chi^2$ fitting routine (MINUIT), used to fit the 2D light distribution of real galaxies, is tested here on artificial galaxy images that were built with structural parameters chosen randomly from a large fixed interval. In this paper we present the properties of the two models, describe the minimization technique, and discuss the results of the tests of the 2D fit obtained from simulated artificial galaxies. By exploiting these tests we were able better to constrain the errors affecting the structural parameters, and the strategies that should be adopted to get the best fit.

Key words: galaxies: elliptical and lenticular, cD – galaxies: fundamental parameters – galaxies: photometry – galaxies: structure.

1 INTRODUCTION

In recent years several works have pointed out the advantages of the two-dimensional (2D) fitting technique for the study of the light distribution of spiral and lenticular galaxies (Byun & Freeman 1995; Scorza & Bender 1995; de Jong 1996a,b; Michard 1998; Moriondo, Giovanardi & Hunt 1998; Scorza et al. 1998), in particular with respect to the classical 1D analysis based on decomposition into two components of the light profiles.

We want to take advantage of this fact for the study of the light distribution of 73 early-type galaxies (Es and lenticulars belonging to the Virgo and Fornax clusters already studied extensively by Caon, Capaccioli & Rampazzo (1990) and Caon, Capaccioli & D’Onofrio (1994) (hereinafter referred to as C2Da,b, respectively, or together as C2D), testing two different models of their surface-brightness distribution: a one-component model, given by a single spheroid that follows the $r^{1/n}$ Sersic (1968) law, and a two-component model, in which the 2D light distribution is represented by a $r^{1/n}$ spheroid and an additional exponential component that may represent a normal disc, a small inner disc, an extended outer halo, or the central core of the galaxies.

The $r^{1/n}$ law has already been successfully adopted to fit the light profiles of early-type galaxies, dwarfs and bulges of spirals (Caon, Capaccioli & D’Onofrio 1993; Andredakis & Sanders 1994; D’Onofrio, Capaccioli & Caon 1994; Young & Currie 1994; Graham & Prieto 1999). In the 2D model the $r^{1/n}$ law represents a single spheroid with a constant surface density distribution along ellipses of similar flattening.

The second model is similar to the classical parametric bulge to disc (BD) decomposition, generally used to separate the light distribution of spiral and S0 galaxies. Such decomposition has been used efficiently to investigate the structure of these galaxies (see e.g. the review of Simien & de Vaucouleurs 1986; Capaccioli & Caon 1990a,b,c), in particular when the dust does not affect the light distribution too much. The problem is whether the BD decomposition can be adopted to fit the whole Hubble sequence, from pure Es to normal spirals. Our model is in fact quite unusual either for early-type or spiral galaxies. The use of the $r^{1/n}$ law, for example, already implies that the bulge is no more an homologous component represented by the $r^{1/q}$ de Vaucouleurs law. In the same way, the use of an exponential component does not necessarily imply the existence of a thin disc. It is well known in fact that the exponential law is widely used to represent dwarf galaxies. Furthermore the $r^{1/n}$ law with $n = 1$ is similar to an exponential law.

It follows that the use of the $(r^{1/n} + \text{exp})$ model has to be considered an experiment. It may result in a simple mathematical exercise, but this can be judged only a posteriori looking at the properties of the fits, at the statistical correlations among the structural parameters, and at the physical processes that originate the galactic components.
Let us consider, for example, the class of E galaxies. In these objects it is very difficult to detect the presence of a faint new component in the dominating light of the bulge. However, we already know that E discs are a well proven category of object (Capaccioli 1987; Carter 1987; Bender 1988; Michard 1994; Scorza & Bender 1995). In order to detect such small inner discs, new approaches alternative to the BD decomposition have been also followed by Simien & Michard (1990), Scorza & Bender (1995) and Michard (1998), adopting a technique that is able to reproduce the isophotal contours of E galaxies with no constraints upon the surface-brightness distribution, but assuming only the existence of a thin disc.

Small discs are not the only components that can be resolved in E galaxies. Sometimes large cores and extended outer halos may be considered as photometric components of E galaxies, and all these structures can in principle be detected by means of a 2D fit with a two-component model.

Despite the fact that the classical BD decomposition method is probably less accurate than the previously cited methods explicitly devised for the purpose of detecting small faint components, it has the great advantage that it always gives a result that can be used in a statistical way.

Note that the use of the terms ‘bulge’ and ‘disc’ (within quotation marks) are used here only to specify the component that follows the $r^{1/4}$ law and the exponential law respectively, i.e. without their classical meaning. We do not speak of ‘physical’ components at this time, but only of ‘mathematical’ components. Later we will see that to these components may also be attributed a physical significance.

In the present paper we describe the characteristics of our selected models (Section 2), the minimization technique (MINUIT, CERN program library) (Section 3), and we check, by means of simulations of artificial galaxy images, the results of the 2D fit (Section 4). These tests are necessary to verify the accuracy of the fit, the cross-correlations among the structural parameters that may be induced by the 2D fitting algorithm, and the behaviour of the errors that affect the extracted parameters.

In Paper II we address the problem of the fit of real galaxies, discuss the errors that should be attributed to the galactic parameters, present the results of the individual fits, and show the correlations occurring among the parameters. Finally, we briefly comment on our results in the context of current theories of galaxy formation and evolution.

2 2D MODELS OF THE SURFACE BRIGHTNESS DISTRIBUTION

Model 1 is the one-component model, which uses the simple $r^{1/4}$ law and has already been studied by Caon et al. (1993) for the sample of galaxies using the 1D light profiles. It is a six-parameter model given by the equation

$$I(R) = I_0 \text{dex}[{-k[(R/R_k)^{1/4} - 1]}],$$

where $R = [([x_1])^2 + [y/(b/a)]^2]^{1/2}$ is the distance of the pixel $(i,j)$ from the galaxy centre, $k = 0.868 \times n - 0.142$ is a variable that depends on $n$ and is constant for each model ($k = 3.33$ for $n = 4$ as in the de Vaucouleurs $r^{1/4}$ law), $R_k$ is the effective radius, $I_0$ the corresponding effective surface brightness, and $(b/a)$ is the flattening of the isophotes. An exponent $c$ greater or less than two realizes a moderate degree of boxiness or discyness of the isophotes, respectively. A further free parameter that does not enter in the above formula is the position angle PA of the major axis. The model also includes a number of fixed parameters that also enter in the best-fit procedure. They are the coordinates of the centre of the galaxy $(x_c, y_c)$, the sky background $I_{sky}$, and the sky surface brightness $\mu_{sky}$. We took all these parameters from the high quality photometric data of CDF. The six free parameters are therefore $\mu_c = -2.5 \log(I_c), R_c, (b/a)_c, n, c$ and $PA$.

Model 2 is a modification of the model analysed by Byun & Freeman (1995). It is a two-component model in which the light distribution of the ‘bulge’ is chosen to follow a $r^{1/4}$ law, while the ‘disc’ has an exponential distribution. The model has therefore nine parameters. The equations are

$$I_b(R_b) = I_b \text{dex}[{-k[(R/R_b)^{1/4} - 1]}] \quad \text{(bulge)} \quad (2)$$

$$I_d(R_d) = I_d \exp[-R_d/R_b] \quad \text{(disc)} \quad (3)$$

where $k$ is the same as before, $R_b = [([x_i])^2 + [y/(b/a)]^2]^{1/2}$ and $R_d = [([x_i])^2 + [y/(b/a)]^2]^{1/2}$ give respectively the distance of the pixel $(i,j)$ from the galaxy centre for the two components, and the indices $b$ and $d$ refer to the ‘bulge’ and ‘disc’ parameters. $R_c$ and $I_c$ are the effective radius and the effective surface brightness of the bulge component, $R_b$ and $I_b$ the scalelength and the central surface brightness of the exponential component. The remaining parameters are the same as before. The nine free parameters are: $\mu_c = -2.5 \log(I_c), R_c, \mu_b = -2.5 \log(I_b), R_b, (b/a)_b, (b/a)_d, n, c$, and $PA$.

The last parameter, PA, is assumed to be the same for both the ‘disc’ and ‘bulge’ components. Obviously an intrinsic twist of the ‘bulge’ component cannot be realized by the one-component model, but we will see in Paper II that the combination of the two components provides a twist of the major axis position angle and a varying ellipticity because of the different contribution of each component (in terms of luminosity) in each pixel of the CCD frame.

Both models have been applied to the whole CDF sample without distinguishing a priori E and SO galaxies. Of course, galaxies disturbed by additional components, such as bars, lenses, rings and dust lanes cannot be satisfactory dealt with by the present 2D models.

The preference for the $r^{1/4}$ law with respect to the classical $r^{1/4}$ law is due to the increasing evidence that not only the E galaxies seem to follow such light distribution, but also the bulges of spirals (Frogel 1988; Andredakis & Sanders 1994; Andredakis, Peletier & Balcells 1995; Courteau, de Jong & Broeils 1996; de Jong 1996a,b). However, in order to be conservative, we performed all the fits also by keeping the exponent $n$ fix to 4.

The index $c$, added to both models, realizes a moderate degree of boxiness ($c > 2$) or discyness ($c < 2$) for the ‘bulge’ component. The use of this parameter needs further explanation. The boxiness and discyness of the isophotes are generally believed to be related to the process of galaxy formation (Bender, Döbereiner & Möllenhoff 1988a,b; Kormendy & Bender 1996). Low-luminosity Es are isotropic and rotationally supported and show discy isophotes, while high-luminosity Es are anisotropic, slow rotators and have preferentially boxy isophotes. The two groups also have different radio and X-ray properties. In general, discy Es have power-law cores, while boxy Es have flat cores and central density cusps (Lauer et al. 1995; Faber et al. 1997). However, recent numerical simulations of merger remnants indicate that the dichotomy between boxy and discy E galaxies may originate from variations in the mass ratio of the merger components (Barnes 1998; Naab, Burkert & Hernquist 1999), and the observed scatter...
in the kinematic and isophotal properties of both classes can be explained in terms of projection effects. If this explanation can be accepted, the disciness of the isophotes is not necessarily linked to the presence of a disc component, but may be connected to the structure of the bulge of the merger remnant or to a combination of properties of the bulge and disc components. In this case our hypothesis of \( c < 2 \) (a moderate degree of disciness for the ‘bulge’ component) can be accepted as a working hypothesis. Anyway, in order to be conservative, we also perform the fits by keeping \( c = 2 \) as a fixed parameter, but we will see that the solutions for the structural parameters are not strongly dependent on this.

The values of \((b/a)_b\) and \((b/a)_d\) are kept free in the present analysis. Actually, if the new component is a thin disc we have problems to justify values of \((b/a)_b\) larger than \((b/a)_h\). This occurs sometimes in the fit of large Es, where small and round inner discs are found. The round shape of such discs is probably the result of the seeing effect. However, since the new component may be also a core extension (or an extended halo) we could accept a \((b/a)_d\) flattening larger than \((b/a)_h\). In any case the best fits give only few values of \((b/a)_d\) marginally larger than \((b/a)_h\) (often within the errors). It follows that solutions of this kind are probably not present in our analysis. To avoid confusion, we have fitted the galaxies once more by imposing \((b/a)_h\) greater than \((b/a)_d\). The results for the structural parameters do not change significantly.

The 2D BD decomposition is in fact not strongly affected by the flattening of a small inner component, because it is a small structure extended on few pixels.

### 3 THE MINIMIZATION ALGORITHM

The best fitting solution is found by minimizing \( \chi^2 \) given by the formula

\[
\chi^2 = \sum_i \frac{(I_{\text{gal}} - I_{\text{mod}})^2}{\sigma_i^2}
\]

where \( I_{\text{gal}} \) is the intensity of each pixel \( i \) of the galaxy frame made of \( n \) pixels, \( I_{\text{mod}} \) is the intensity of the model calculated through Equations (2) and (3), and \( \sigma_i^2 \) is the total variance of each pixel (see below).

The MINUIT algorithm provides the best fitting parameters simultaneously for each of the adopted models and their errors calculated in proximity of the minimum \( \chi^2 \). Unfortunately, this kind of nonlinear problem is affected by the presence of many local secondary minima (Schombert & Bothun 1987; Byun & Freeman 1995), in particular when the data are not complete (as in this case, since many galaxies of our sample are larger than the CCD frames). It was therefore necessary to check \textit{a posteriori} the reliability of the solutions, by looking at the 2D residuals, and at the \( a_b, \epsilon \) and PA profiles (the classical parameters that characterize the isophotes of the galaxies) plotted against the corresponding CCD + Schmidt data derived by C2D.

Finally, an \( F \)-test has been performed using the \( \chi^2 \) distributions in order to check the hypothesis that solutions with similar reduced \( \chi^2 \) (\( \chi^2_r \)) have the same variance. It could happen in fact that the difference between the two solutions (obtained by Models 1 and 2) is not statistically significant. We have therefore compared the \( \chi^2 \) distribution obtained with the \( r^{1/n} \) model with that obtained by the \((r^{1/n} + \exp)\) model. The mean and the variances of the two distributions are used by the \( F \)-test to check how much they are ‘significantly’ different. This test, preliminarily verified on artificial galaxy images, has been performed for all the galaxies of our sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 3</th>
<th>Run 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>45</td>
<td>37</td>
<td>44</td>
<td>38</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>5.0</td>
<td>37.1</td>
<td>1.7</td>
<td>8.5</td>
</tr>
<tr>
<td>( R_h )</td>
<td>3.9</td>
<td>19.4</td>
<td>1.3</td>
<td>3.8</td>
</tr>
<tr>
<td>( I_c )</td>
<td>5.2</td>
<td>1.8</td>
<td>1.7</td>
<td>8.1</td>
</tr>
<tr>
<td>( R_c )</td>
<td>3.1</td>
<td>9.5</td>
<td>2.5</td>
<td>4.9</td>
</tr>
<tr>
<td>( n )</td>
<td>1.6</td>
<td>12.1</td>
<td>2.5</td>
<td>1.9</td>
</tr>
<tr>
<td>( c )</td>
<td>0.4</td>
<td>0.8</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>( (b/a)_h )</td>
<td>0.2</td>
<td>0.6</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>( (b/a)_d )</td>
<td>1.4</td>
<td>14.8</td>
<td>2.0</td>
<td>2.8</td>
</tr>
<tr>
<td>PA</td>
<td>0.01</td>
<td>0.07</td>
<td>0.02</td>
<td>0.03</td>
</tr>
</tbody>
</table>

The goodness of fit is indeed a complicated problem in this context. In order to use the \( \chi^2 \) as an estimate of the goodness of the 2D fit, it is important to have an accurate measurement of the errors that are associated with each pixel of the image. Both for real and artificial galaxies, we considered only the read-out noise and the photon noise. The total variance in each pixel was calculated as

\[
\sigma_i^2 = \sigma_{\text{read}}^2 + \sigma_{\text{phn}}^2 + \sigma_{\text{sky}}^2.
\]

The first term \( \sigma_{\text{read}}^2 \) is known from the characteristics of each CCD chip, the second \( \sigma_{\text{phn}}^2 \) and third \( \sigma_{\text{sky}}^2 \) terms are respectively proportional to the number of detected photocharges from the galaxies and the sky. Other sources of error, such as flattening, intrinsic variation in galaxy brightness, and the presence of background objects that affect real images, give only a small contribution. Anyway, stars and background galaxies have been accurately masked in each frame of our galaxy sample.

An important source of uncertainty is the sky background, in particular for faint galaxies and for the largest galaxies that fill the CCD frame. Previous tests made by de Jong 1996a,b and Saglia et al. (1997) showed that an error of \( \sim 1 \) per cent on \( I_{\text{sky}} \) might be quite dangerous for the results of the fit. Our simulations with artificial galaxies indicate, on the other hand, that this small error has no dramatic consequences for the final result of the fit (see Table 1). Furthermore, in our data \( I_{\text{sky}} \) and \( \mu_{\text{sky}} \) are known with great accuracy (on average \( \sim 0.53 \) per cent) from the ‘global mapping’ technique adopted by C2D. The global mapping technique uses (at the same time) the CCD and the Schmidt light profiles; it is from the match of the two profiles that \( I_{\text{sky}} \) can be derived with very small uncertainty. With such a degree of accuracy, the errors on the structural parameters due to the uncertainty in the sky background can be considered small.

The same error matrix (the frame with the errors attributed to each pixel) has been used for calculating the total \( \chi^2 \) of the \( r^{1/n} \) and \((r^{1/n} + \exp)\) models. This gives a first indication of the best-fitting model. The reduced \( \chi^2 \) is generally found for real galaxies in the interval \([1, 2]\) for fits of good quality. For artificial galaxies the \( \chi^2_t \) is much closer to 1, except for particular cases (see below).

An \( F \)-test is necessary when the \( \chi^2_t \) of the two models are similar, if we want to distinguish the best-fitting model. We do this test in Paper II, listing the results for each galaxy of our sample.

### 4 SIMULATIONS OF ARTIFICIAL GALAXIES

Before applying the adopted models to real galaxies we tested on artificial galaxy images the reliability of the fitting routine and the systematic effects that may influence the final results. Fifty-one artificial galaxies have been created, following the procedure

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developed by Byun & Freeman (1995), but using the $r^{1/n}$ law for the 'bulge' and the classical exponential formula for the 'disc'.

The structural parameters were randomly chosen from fixed intervals spanning a wide spectrum of BD ratios, scalelengths, and inclinations (appropriate approximately to mimic the conditions found in our sample of Virgo and Fornax galaxies assumed at a distance of 18.3 Mpc). The BD ratios vary from $10^{-3}$ to $10^3$ (quite a large interval, but we will see that goodness of fit can be obtained

Figure 1. Input structural parameters (abscissa) versus output parameters for artificial galaxies. Units are mag arcsec$^{-2}$ for $\mu_0$ and $\mu_e$, and kpc for $R_e$ and $R_h$. In the left panels we plot input versus output. In the right panels we plot the absolute value of the percentage errors for the parameters extracted in different runs (from left to right: run 1 to 4; see text). The dashed line marks the median error. The errors on the surface-brightness parameters were calculated as $\Delta I/I$. 

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only in a smaller range), the effective surface brightness $\mu_e$ and the central disc brightness $\mu_0$ vary within the interval $[17, 27] \text{mag arcsec}^{-2}$, the effective radius between 0.05 and 10 kpc, the scalelength of the discs between 0.05 and 25 kpc, and the $(b/a)_b$ and $(b/a)_d$ flattening within $[0.4, 1.0]$ and $[0.1, 1]$, respectively. The free Sersic exponent $n$ varies between 1 and 15, while the boxy/discy parameter $c$ varies between 1.6 and 3. The position angle PA was fixed to 0 for all the artificial galaxies.

The galaxies were built over a grid of $400 \times 400$ pixels. Adopting the spatial scale of most of our CCD images (0.463 arcsec pixel$^{-1}$), we have a frame of $\sim 3.1 \times 3.1\text{arcmin}$, a value approximately equal to that of many frames of real galaxies ($\sim 2.5 \times 3.8 \text{arcmin}$). The artificial galaxies were created in surface-brightness units, and subsequently transformed in Intensity using a constant sky level of 50 ADU (a value very close to that of most real galaxies) and a constant sky surface brightness $\mu_{\text{sky}} = 22.5 \text{mag arcsec}^{-2}$. A Poissonian noise was added, simulating the photon and read-out noise statistics of the CCD frames. These values have been chosen in order to get artificial galaxies with S/N ratios as similar as possible to those of real galaxies (the S/N ratio on real galaxy images varies from 0.5 to $> 50$ on a single pixel depending on galaxy sizes and luminosities).

The artificial galaxy images have been convolved with a point spread function (PSF), given by a single Gaussian with a FWHM of 1.5 pixels (the FWHM of real galaxies ranges from 0.9 to 1.5 arcsec). This simulates the effect of seeing with sufficient accuracy, even if the existence of extended wings of the PSF may still affect the fit. In order to avoid as much as possible the influence of seeing in the fit we masked out the inner galaxy region. Two masks have been used: the first excludes from the fit a circular area with a diameter of 10 pixels ($\sim 4.6\text{arcsec}$), while the second is bigger (16 pixels in diameter corresponding to $\sim 7.4\text{arcsec}$). The two masks were used to test the seeing effects on the parameters extracted from the minimization algorithm.

In total, 4 runs of MINUIT have been applied to each artificial galaxy. In the first, the fit was applied on the galaxy masked by the big mask and the initial conditions were chosen from a ‘by eye’ decomposition of the light profiles; in the second, the fit was performed on the same image but starting from random initial conditions; in the third, the fit was still performed on this frame but changing the sky background of 1 per cent; finally, in the fourth run the fit was applied on the galaxy masked by the big mask and the initial conditions were chosen from ‘by eye’ initial conditions.

MINUIT performs a nonlinear fit of the 2D surface-brightness distribution and provides a simultaneous solution for all the structural parameters, together with their ‘mathematical’ uncertainty. With this term we mean that mathematically the solution has a very small error due to the calculation of the error in the proximity of a minimum, but the ‘true’ solution may be far from this (possibly local) minimum. The ability of MINUIT in discovering the true minimum is found to be strongly dependent on the starting conditions. In particular, for real galaxies this is due not only to the nonlinearity of the problem but also to the cross-talk of some of the parameters (such as $\mu_e$ and $R_e$, or $\mu_0$ and $R_0$).

In general, the agreement between input and output parameters is satisfactory (see Fig. 1). The value of the $\chi^2$ is generally less than 2 (as in Schombert & Bothun 1987), and it can be used as an index for the goodness of fit, since the error distribution in pixels with the same intensity level is approximately Gaussian. For artificial galaxies, we considered only those with $\chi^2 < 1.05$ to be good fits (only these models are plotted in Fig. 1), given the almost perfect gaussian errors distribution. Notably, the galaxies rejected with this strong constraint ($\sim 20$ per cent) are generally those with the larger values of the Sersic index $n$. This can easily be understood since the big values of $n$ produce highly peaked central fluxes that are strongly affected by convolution with the PSF. Galaxies with a Sersic index $n$ larger than 10 are very badly reproduced by the adopted models.

The best way to derive a more robust estimate of the errors on the structural parameters would be to use a bootstrapping technique, but this is too time consuming in this context (too many galaxies), so we preferred to compare the input/output differences for all the structural parameters used in the models.

Fig. 1 shows a comparison between the input and the output structural parameters derived by MINUIT for the whole sample of artificial data. The input (abscissa) versus the output parameters are plotted in the left panels of the figure. In the right panels we plot the absolute value of the percentage input/output difference for each of the four mentioned runs of MINUIT (from left to right: run 1, big mask + ‘by eye’ initial conditions; run 2, big mask + random initial conditions; run 3, big mask + ‘by eye’ initial condition + 1 per cent difference in the sky background; run 4, small mask + ‘by eye’ initial conditions).

In Table 1 we list the absolute values of the percentage (median) errors of the structural parameters derived for the artificial galaxies. The columns give the median errors for the parameters extracted in the different runs. It is apparent that the input parameters are recovered by the fitting algorithm with a low percentage error, except when random initial conditions are chosen. The largest uncertainties are found for the central and the effective surface brightness ($\mu_0$ and $\mu_e$), for the disc scalelength and the effective radius ($R_0$ and $R_e$), for the free Sersic exponent $n$, and for the ‘disc’ flattening $(b/a)_d$ in decreasing order. The first row gives the total number of objects retained after the application of our constraint $\chi^2 < 1.05$.

Since the masked frames reproduce quite well the parameters of the input galaxies, we choose to adopt the median statistical uncertainty of these models as the most probable error that affects the structural parameters of real galaxies. Of course, the large spread around the median apparent in some cases implies that larger errors may be present in the final data. Note that large errors affect almost all the parameters when the initial conditions are completely random.

It is also interesting to note in Fig. 1 that $\mu_0$ and $\mu_e$ are reproduced nearly as well as $R_0$ and $R_e$, despite the fact that the central surface-brightness parameters are strongly dependent on the values of the central pixels, where the higher S/N ratio is observed.

The seeing forces a smaller value of the Sersic exponent $n$ (in particular when $n$ is large) and consequently a redistribution of the ‘bulge’ parameters. It also affects the $(b/a)_b$ and $(b/a)_d$ flattening, in particular for the smaller ‘discs’, which appear artificially round, but we do not find any correlation between $n$ and the $(b/a)_b$ and $(b/a)_d$ parameters (see below).

In Fig. 2 we plotted the residuals $\Delta m = m(\text{input}) - m(\text{output})$ of the total magnitudes of the artificial galaxies calculated using the original input data and the structural parameters derived by MINUIT. Nearly 45 galaxies (approximately 88 per cent of the total sample) have a total luminosity within 0.1 mag of the input value. However, occasionally large errors may occur, in particular for the faintest galaxies. It is difficult to establish a certain origin for these large errors: they can originate from the effects of seeing that affect the region of the fit with the higher S/N ratio, or from the wrong initial
conditions of the parameters, or from a cross-talk among some of the parameters.

In Fig. 3 we present the luminosity function of the artificial galaxies moved to the distance of the Virgo cluster. The dashed line represents the luminosity function derived from the output of the fit, and it is apparent that it well reproduces the input histogram. Note that the two distributions are almost identical for the most luminous galaxies. The same happens for the LF of the single 'bulge' and 'disc' components (Fig. 4).

The absence of mutual correlations among the structural parameters of artificial galaxies and among their errors is apparent from Figs 5 and 6. Both figures have been plotted for the run number 4, but the same results are obtained for the other runs. The figures demonstrate that no spurious correlations are induced by the fitting algorithm, i.e. the random choice of the parameters is well conserved by the fitting procedure. Mutual correlations among the structural parameters are absent in Fig. 5 but this is not the case for the real galaxies (see Paper II). The residuals $D$ (true value – value found by MINUIT) are also not mutually correlated (see Fig. 6). A smooth correlation is apparent for $Dm$ and $D\log(R_e)$ (but with a correlation coefficient equal to 0.5). These figures confirm that the 2D fit is working well.

We already stressed that the level of the sky background is a dangerous parameter for the fit. Using artificial galaxies we look at the errors on the structural parameters introduced by an error of 1 per cent in the level of the sky background $I_{\text{sky}}$. Note that, owing to the cross-talk of the parameters, the error on the sky background propagates on the whole fit. However, this small error does not seem to produce a strong variation in the median error of the structural parameters with respect to that obtained in Run 1 and 4. Since for the real galaxies we know that the sky background error is lower than 1 per cent (see Paper II), we are confident that our final results are only marginally affected by this problem.

A further source of error that should be considered is that linked to the coordinates $(x_c, y_c)$ of the centre of the galaxy. In our simulations, and for the real galaxies where the centre is known with the accuracy of a fraction of a pixel, we considered the peak of the gaussian (fitted to the centre of the galaxy) as the true centre of the fit. In this way we keep fixed two potentially free parameters and consequently we cannot accurately test the effects of this kind of error. However, we have verified that a change of 1 pixel in the coordinate of the centre always produces a big increase in the value of the $\chi^2$ (this test was done for the whole sample of objects). Unfortunately, an error in the centre has the effect of producing

\[\Delta m\]

Figure 3. Histogram of the luminosity function of artificial galaxies moved to the distance of the Virgo cluster. The solid line enclosing the shaded area marks the original input data; the dashed line gives the results of the fit obtained from the seeing convolved and masked frames.

Figure 4. (Upper panel) Histogram of the luminosity function of the 'disc' component of the artificial galaxies moved to the distance of the Virgo cluster. As in the previous figure, the solid line enclosing the shaded area marks the original input data while the dashed line gives the results of the fit obtained from the seeing convolved and masked frames. (Lower panel) The same as the upper panel but for the luminosity function of the 'bulge' component.
systematic residuals after the subtraction of the 2D model to the original galaxy. These are indeed observed in many of our galaxies (see Paper II), but it should be remembered that the origin of the systematic residuals is not only due to the errors affecting the structural parameters, but also to the fact that real galaxies are never perfectly reproduced by the models (small subcomponents or substructures may be present). In any case, it will be seen that the observed flux deviations are often of the order of a few per cent.

It is clear from Table 1 that the nonlinear fitting algorithm is particularly sensitive to the initial conditions. If they are too far from the true solution the fit can converge to the wrong local secondary minimum. We have also seen that a good way for choosing the initial condition is that of fitting the light profiles of the galaxies. Since the initial guess depends on knowledge of the morphology of the object, this procedure can always introduce a subjective bias in the solutions. We therefore repeated the fit of real and artificial galaxies starting each time from different initial conditions, in order to see if the true minimum had really been found.

It is also important to check the ability of MINUIT in recovering the presence of a small inner exponential component. We have therefore built an artificial galaxy with a small inner 'disc' embedded in a large 'bulge' (the BD ratio of this galaxy is $\sim 1000$). The input structural parameters of this toy galaxy are listed in column 2 of Table 2. The other columns give respectively: the results of the fit obtained with the $r^{1/n}$ model using a central mask of 18 pixels (in diameter) and no convolution with the PSF (col. M1); the results of the fit obtained by the $r^{1/n}$ model on the same masked image (col. M2); the results of the $(r^{1/n} + \exp)$ model using a smaller mask of 8 pixels (in diameter) and no seeing convolution (col. M3); the same as col. M3 for the $r^{1/n}$ model (col. M4); the results of the $(r^{1/n} + \exp)$ model for the galaxy convolved with a PSF of 2 pixels (FWHW) and masking the central part with a mask of 8 pixels in diameter (col. M5); the results of the $r^{1/n}$ model on the masked image of col. M5 (col. M6).

The analysis of these data shows that the $r^{1/n}$ and $r^{1/n} + \exp$ models are indistinguishable when the mask is large (18 pixels). The inner disc is very small and not detected. The two $\chi^2$ distributions (of Models 1 and 2) are almost identical and the $F$-test gives $F = 1.002$ (which means that the two distributions are indistinguishable). On the other hand the fit made with the smaller mask (8 pixels) gives $F = 1.170$ and the two $\chi^2$ distributions are now different. The $(r^{1/n} + \exp)$ model provides a better fit than the $r^{1/n}$ model.

When the fit is applied to the image convolved with the gaussian PSF and masked as before, we still find that the $(r^{1/n} + \exp)$ model is preferable. In this case in fact the $F$-test gives a value of 2.13 and there is no doubt that the two distributions are different. In other words the fit is better with the $(r^{1/n} + \exp)$ model, but one can suspect that the seeing simulates the presence of an inner component and increases the value of $F$. Unfortunately, the only way to determine whether the inner components are a product of seeing effects is to fit the galaxy and the PSF simultaneously. However, we shall see in Paper II that the values of the structural parameters found in our ground-based images are mostly

Figure 5. Mutual correlations among the structural parameters of the 2D fit. $R_e$ and $R_0$ are expressed in kpc; $\mu_0$ and $\mu_e$ are in mag arcsec$^{-2}$. 
Figure 6. Mutual correlations among the residuals of the 2D fit. $\Delta R_e$, $\Delta R_h$ and $\Delta n$ are in logarithmic units (e.g. $\Delta R_e = \log(R_e(\text{fit})) - \log(R_e(\text{true}))$).

Table 2. The structural parameters of our toy galaxy. Column 1 lists the parameters, column 2 lists the input structural parameters and columns 3 (M1) to 8 (M6) list the output structural parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Orig.</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
<th>M6</th>
</tr>
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<tbody>
<tr>
<td>$\mu_0$</td>
<td>18.5</td>
<td>18.4</td>
<td>18.0</td>
<td>18.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_h$</td>
<td>2.0</td>
<td>2.6</td>
<td>2.6</td>
<td>2.7</td>
<td>2.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>20.3</td>
<td>20.3</td>
<td>20.3</td>
<td>20.3</td>
<td>20.3</td>
<td>20.3</td>
<td></td>
</tr>
<tr>
<td>$R_e$</td>
<td>65.7</td>
<td>65.7</td>
<td>66.4</td>
<td>66.1</td>
<td>66.5</td>
<td>65.9</td>
<td>66.9</td>
</tr>
<tr>
<td>$n$</td>
<td>4.0</td>
<td>3.9</td>
<td>4.0</td>
<td>4.1</td>
<td>4.2</td>
<td>4.2</td>
<td>4.5</td>
</tr>
<tr>
<td>$c$</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$b/a_0$</td>
<td>0.88</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$b/a_2$</td>
<td>0.77</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>1.002</td>
<td>1.007</td>
<td>1.010</td>
<td>1.015</td>
<td>1.057</td>
<td>1.081</td>
<td></td>
</tr>
</tbody>
</table>
conserved when the fits are performed with the \textit{HST} images. We can therefore be confident that, once the seeing region is masked-out, \textsc{minuit} is able to separate bulge and disc components with BD ratios as big as 1000.

In conclusion we have shown that, using a mask that eliminates from the fit the central region of the galaxies, it is still possible to extract the original input parameters. The dimension of the mask should be chosen as large as possible in order to avoid the influence of seeing but also as small as possible to avoid the loss of small inner components. As a result of our experience with artificial galaxies, we use here a mask of 8 pixels in diameter. This mask was also applied to the fitting of real galaxies. Galaxies with BD ratios as large as 1000 can be correctly found using this mask by the 2D fit, if accurate initial conditions are used.

5 CONCLUSIONS

We have tested, by means of simulations of artificial galaxies, two possible models of the 2D surface-brightness distribution of early-type galaxies. The first model, represented by the $r^{1/n}$ law, uses a single spheroidal component; the second, represented by the $(r^{1/n} + \exp)$ laws, uses instead two components, which we call ‘bulge’ and ‘disc’. In Paper II we exploit this experience for the fit of 73 objects belonging to the Virgo and Fornax clusters.

We have discussed the adopted 2D fitting algorithm (MINUIT), based on a $\chi^2$ minimization, and analysed the various sources of error affecting the results of the fit. We have shown that the effects of seeing must necessarily be removed or taken into account in the fitting procedure, and that, using a small mask that removes from the fit the inner region of the galaxies, we can avoid the large influence of seeing.

We have also discussed the other sources of error that affect the fit and provided a table with the median percentage errors on each structural parameter. This error can be attributed also to the parameters of the real galaxies. They should of course be interpreted as the most probable errors affecting the galactic parameters, and we must keep in mind that larger errors may be present in the final result.

In Paper II we shall present the results found for the real galaxies, the relationships occurring among the structural parameters, and a discussion of the data in the context of the current theories of galaxy formation.

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