On the structure of the inner Crab Nebula

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ABSTRACT
Origin of the jet-like feature in the inner Crab Nebula is discussed. Because self-collimation processes in ultrarelativistic pulsar winds are extremely ineffective, it is suggested that the collimation occurs beyond the termination shock where the flow is already mildly (or non-) relativistic. It is argued that the shock shape is highly non-spherical because the energy flux in the pulsar wind decreases towards the axis. The shock near the axis should be much closer to the pulsar than at the equator and therefore the jet looks as if it originates directly from the pulsar.


1 INTRODUCTION
Pulsars lose their rotational energy predominantly by generation of an ultrarelativistic, magnetized winds that power surrounding synchrotron nebulae. A remarkable jet-torus structure found in the inner Crab Nebula (Brinkmann, Aschenbach & Langmeier 1985; Hester et al. 1995; Weisskopf et al. 2000) provides new clue to understanding of energy conversion mechanisms in plerions. It is generally believed that at the inner boundary of the X-ray torus (about 0.1 pc from the pulsar) a shock front is located where the wind energy releases. Optical wisps and knots were also found in this region (Scargle 1969; Hester et al. 1995). The jet–torus structure seems to be a generic property of pulsar wind nebulae as it was recently confirmed by Chandra observations of the Vela pulsar (Pavlov et al. 2000, 2001; Helfand, Gotthelf & Halpern 2001) and PSR B1509–58 (Kaspi et al. 2000).

The most puzzling feature of the inner nebula is the X-ray jet. It is frequently suggested that hoop stresses in the pulsar wind may collimate the flow. However careful considerations show that such a collimation is extremely ineffective in ultrarelativistic flows because electric force nearly compensates hoop stresses (Tomimatsu 1994; Bogovalov 1997; Beskin, Kuznetsova & Rafikov 1998; Chiueh, Li & Begelman 1998; Bogovalov & Tsinganos 1999; Lyubarsky & Eichler 2001). Namely the curvature radius of the poloidal field line (or of the flow line, which is the same) may be estimated as

$$R_c \sim \gamma^2 \left( 1 + \frac{1}{\sigma_0} \right),$$

where $r$ is the cylindrical radius, $\gamma$ the local flow Lorentz factor, $\sigma_0$ the local ratio of the Poynting flux to the matter energy flux. One can see that in an ultrarelativistic wind the flow lines are nearly straight and collimation is possible, if possible at all, only near the axis.

2 COLLIMATION IN THE PULSAR WIND
Bogovalov (2001) investigated collimation in relativistic winds numerically and found that a cylindrical jet actually forms near the axis of the flow. However the fraction of the collimated poloidal flux is rather small,

$$\frac{\psi}{\psi_0} < \frac{1}{2\sigma_0},$$

where the poloidal flux function is defined from the poloidal field strength by

$$B_p = \frac{1}{r} \nabla \psi \times \mathbf{e}_\phi,$$

and the subscript 0 is referred to the values at the equator of the flow. It is important to note that the fraction of the total energy carried by such a jet is much less than $\psi/\psi_0$ because Poynting flux decreases with latitude. For a monopole-like configuration, which was considered by Bogovalov, the electromagnetic fields may be described, not too far from the centre, by Michel's (1973) solution for the force-free case:

$$\psi = \psi_0 (1 - \cos \theta), \quad B_\phi = \frac{\psi_0 \Omega \sin \theta}{cR}, \quad E = \frac{\Omega}{c} \nabla \psi,$$

where $R$ and $\theta$ are the spherical radius and the polar angle, correspondingly, $\Omega$ the angular velocity of the central body. Estimating the Poynting flux and adding a spherically symmetric matter energy flux, one can write the total energy flux as

$$F = \frac{\Omega^2 \psi_0}{4\pi c^2 R^2} \left( \sin^2 \theta + \frac{1}{\sigma_0} \right).$$

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Here the first term in the brackets refers to the Poynting flux and the second one to the matter energy flux. One can see that in the collimated part of the flow (2), the Poynting flux is comparable to the matter energy flux and the fraction of the total energy carried by the jet is
\[ \frac{E_{\text{jet}}}{E_{\text{tot}}} \approx \frac{1}{\sigma_0}. \]  
(6)

For \( \sigma_0 \sim 10^4 \), which is generally accepted for the Crab, this fraction is extremely small. A fraction of energy carried by the jet may be larger if one takes into account possible logarithmic collimation at very large distances from the pulsar, however, estimates show (Lyubarsky & Eichler 2001) that for generally accepted pulsar parameters, this fraction remains too small to explain the observed jet-like feature.

Decreasing of the Poynting flux with latitude seems not to be a specific feature of monopole-like winds but quite a general feature of axisymmetrical flows. The Poynting flux \( P \propto EB_0 \propto r \Omega_B B_0 \). Taking into account that \( B_0 \propto r^{-1} \int j \, dr \), where \( j \) is the poloidal current density, one can see that \( P \propto r^2 \rightarrow 0 \) as \( r \rightarrow 0 \) unless \( B_0 \) or \( j \) are singular at the axis of the flow. The situation with oblique rotators is less clear; however in the split monopole configuration the flow near the rotation axis is the same both for the oblique case and for the axisymmetrical one (Bogovalov 1999). So one can conclude that the jet in the Crab is unlikely to be explained by collimation in the pulsar wind. Let us consider another possibility.

3 COLLIMATION BEYOND THE TERMINATION SHOCK

Let the pulsar wind flow radially until the termination shock. In the shock the flow decelerates, moreover beyond the shock the flow is subsonic (more exactly subfastmagnetosonic) and therefore it may decelerate smoothly further on. According to equation (1), hoop stresses rise with decreasing velocity and therefore they eventually squeeze the flow towards the axis, forming a jet-like feature. Taking into account that the energy flux in the pulsar wind decreases with latitude (see equation 4), one can anticipate that the termination shock is highly non-spherical; at the axis the shock should be much closer to the pulsar than at the equator. Therefore the ‘jet’ looks as if it originates from the pulsar. Note that the observed jet is definitely non-relativistic because it is seen at large angle. This supports the idea that the ‘jet’ is formed not in the relativistic pulsar wind but in the mildly (or non-) relativistic flow beyond the shock.

The position of the shock is determined from the conditions of energy and momentum balance. In principle, numerical simulations are necessary to find the downstream parameters and the shock shape self-consistently. However, one can roughly sketch the shape of the shock simply by equating the ram pressure of the wind to the downstream pressure (see Eichler 1982; Levinson & Eichler 2000) and assuming that the last is roughly constant, as in any subsonic flow without mass forces. In ultrarelativistic flows the ram pressure is nearly equal to the energy flux. According to equation (4), the energy flux along the axis is \( \sigma \) times less than in the equatorial plane at the same radius. If the pressure beyond the shock is roughly the same at the axis and at the equator, the ratio of the distance to the shock at the axis, \( z_0 \), to the equatorial radius of the shock, \( R_0 \), should be very small,
\[ \frac{z_0}{R_0} \sim \frac{1}{\sqrt{\sigma_0}}, \]  
(7)

so a cusp should form.

The shape of the cusp, \( R(\theta) \), may be found taking into account that only the normal to the shock component of the momentum flux contributes to the pressure balance; then the approximate balance condition should be written in the form
\[ F \sin^2 \eta = \text{const}, \]  
(8)

where \( \eta \) is the impact angle of the flow. Taking into account that the upstream flow is radial, one can easily express \( \eta \) via \( \theta \) and \( dR/d\theta \). Then one can write equation (7), with the aid of equation (4), as
\[ \frac{dR}{d\theta} = R_0 \left( \sin^2 \theta + \frac{1}{\sigma_0} \right). \]  
(9)

Expanding around \( \theta = 0 \), one can get the solution at \( \theta \ll 1 \):
\[ R/R_0 = \frac{1}{\sqrt{\sigma_0}} + \frac{1}{2} \theta^2. \]  
(10)

In cylindrical coordinates it reads as
\[ r^2 = 2z^2 \left( \frac{z}{R_0} - \frac{1}{\sqrt{\sigma_0}} \right), \]  
(11)

so there is a cusp, \( r \propto z^{3/2} \), truncated at \( z \sim R_0/\sqrt{\sigma} \).

Of course equation (8) is very rough because the assumed pressure balance is fulfilled only to within an order of magnitude and the downstream pressure is actually not constant, especially if one takes into account hoop stresses (Begelman & Li 1992). Nevertheless such a simple approach already demonstrates the basic qualitative feature of the shock shape, namely the cusp at the axis. The distance to the cusp may be something larger if one takes into account the jet from the pulsar itself, however this distance should be anyway significantly less than \( R_0 \) because the power of the jet is very small. In this case a truncated cusp is formed by a bow shock produced by the jet. One can assume that this shock is observed as the optical knot 1 located at the distance \( \sim 1500 \, \text{au} \) from the pulsar and having the shape of an arc (Hester et al. 1995).

4 DISCUSSION

The observed structure of the inner Crab nebula may be qualitatively described as follows (Fig. 1). The wind propagates nearly radially from the pulsar to the termination shock transferring the spin-down energy. It is important that a significant fraction of this energy is transferred by MHD waves excited in the wind by the obliquely rotating pulsar magnetosphere. In the equatorial belt a specific wave is built from the oscillating equatorial current sheet (Michel 1982; Coroniti 1990; Bogovalov 1999). Such a structure is called the striped wind. At higher altitudes, where magnetic field does not change sign, fast magnetosonic waves may transfer a significant amount of energy. The waves generally easily decay because at the wavelength scale non-ideal processes are operating (Usov 1975; Michel 1982; Coroniti 1990; Melatos & Melrose 1996). Recent analysis shows (Lyubarsky & Kirk 2001) that the dissipation may not be so rapid as it was thought before because the flow significantly accelerates during the wave dissipation and this dilates the dissipation time-scale. However, in the shock the flow decelerates and the waves dissipate immediately. Therefore beyond
the shock only the averaged magnetic field remains, the energy of oscillating magnetic field being already converted into the plasma energy. So the condition \( \sigma \sim 1 \) obtains just beyond the shock. 

More specifically, the average magnetic field is zero at the equator, reaches the maximum value at an intermediate latitude (depending on the angle between the magnetic and rotation axes) and falls to zero at the axis. In the equatorial belt, where the Poynting flux is maximal according to equation (4), most of the energy is carried by the striped wind and therefore beyond the shock where the wave energy has already released. According to equation (1), hoop stresses should effectively squeeze the flow toward the axis. Moreover because the post-shock energy density increases toward the equator, gradients of plasma and magnetic pressure will also squeeze the flow. When the toroidal field lines shrink, the magnetic energy is converted into heat (\( \sigma \) further decreases in this process) and as a result a hot region arises near the axis. An additional energy release is provided near the axis by the kink instability (Lyubarskii 1992; Begelman 1998), which destroys the regular toroidal magnetic field and transforms the energy of the toroidal field partly into the thermal energy and partly into the energy of a chaotic magnetic field. Therefore the synchrotron emissivity should have a maximum near the axis making impression of a jet. So the observed jet-like feature may be attributed to collimation and energy release beyond the termination shock.

**ACKNOWLEDGMENTS**

I am grateful to David Eichler for stimulating discussions and encouraging me to write this Letter.

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