Research Note

Damping of $P$ and $S$ at short distances

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The study of damping in the Earth's shell by Jeffreys & Crampin (1970) concerns average properties of the shell based on the damping of the 14-monthly nutation and the appearance of $S$ at larger distances. Application to $S$ at short distances indicated (Jeffreys 1965) a damping of 5-15 per cent at a distance of 7°, which is nowhere near enough to account for the observed decrease in amplitude. There is much information from free vibrations and surface waves; this indicates much more severe damping than average at depths up to 400 km or so, but the estimates are also averages over a considerable range of depth. To get estimates of the distribution with depth we need, as a starting-point, values for small depths. Small depths and short periods are accessible in the body waves $P$ and $S$. Information is given in a paper by L. Ruprechtova (1959). Reduced amplitudes $A^*$ are given on a logarithmic scale (to base 10), magnitudes and periods of maximum movement being allowed for. They show a considerable scatter; the periods for $P$ are mostly about 4 s, those of $S$ about 8 s. $A^*$ decreases with $\Delta$ from about 2° to 11° for $P$, 4° to 13° for $S$, and then rises continuously for both to a maximum at 20°.

For $P$ the change in $A^*$ over the range 2° to 11° is about $-1.9$. No allowance has been made for ordinary spreading. The theory of this is complicated; provisionally I take it as contributing a factor $1/\Delta$ to the amplitude, giving a decrease of about 0.7 in $\log_{10}(1/\Delta)$, the remainder is $-1.2$, giving $-0.13$ per degree. For $SH$ the change in 4° $< \Delta < 13°$ is $-1.5$, giving, with similar treatment, $-0.11$ per degree. (Data for $SV$ are not given; it is known to be more difficult to identify.) Since, however, the theory of damping is given in terms of exponential functions these changes in logarithms must be multiplied by 2.3, giving $-0.30$ and $-0.25$ per degree.

For $S$ the damping factor, according to the modified Lomnitz law, is (Jeffreys & Crampin 1970; Jeffreys 1970, p. 47)

$$\exp \left\{ -\frac{q x}{2} \hat{a}^2(\alpha - 1)! \gamma^{x - \alpha} \sin \frac{1}{2} \pi \alpha \right\}.$$ 

The exponent for an increase of distance by $\beta/\gamma$ radians increases by $-1/2Q$ in the usual notation, so that the increase in a given interval of $x$ is $-\gamma[x]/2Q\beta$. Then 

$$q \hat{a}^2(\alpha - 1)! \gamma^{x - \alpha} \sin \frac{1}{2} \pi \alpha = 1/Q.$$ 

With the data for $S$

$$\gamma = 2\pi/8^s = 0.8/1^s, \quad \beta = 4.4 \text{ km/sec}, \quad [x] = 1^\circ = 111 \text{ km}$$

we have

$$Q^{-1} = 0.025.$$
For $P$, with about the usual ratio of velocities, the exponent takes an extra factor $4/9$, and $\gamma = 1\cdot6$/sec. Then

$$Q^{-1} = 0\cdot032.$$  

$Q$ depends little upon period, since $\alpha$ is about 0.2, and the results are consistent since some of the data are rough. This damping is much more severe than any reported for greater depths.

The law gives also a shift of phase of $\cot \pi \alpha \times$ times the exponent. With $\alpha = 0\cdot2$ this factor is about 3. For $S$ the result amounts to about 0.9 radians per degree, say 1.5 periods at $10^\circ$. Since the data are stated to be usually for the second swing this is of the right order of magnitude.

The depth (below the Moho) reached by a ray emerging at $10^\circ$ is about 80 km; it varies approximately as $\Delta^{3/2}$. Beyond $11^\circ$ for $P$ and $13^\circ$ for $S$ the amplitudes rise again. This could mean that the rays penetrate a layer where the damping is much less severe, and the distance travelled in the layer of strong damping diminishes. However the amplitudes at the maximum about $20^\circ$ are about the same as about $6^\circ$ in spite of the $1/\Delta$ factor. Some focusing effect is probably still called for.

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Since the above was written I have had a paper by S. J. Gibowicz (1972). He gives analogous data for the North and South Islands of New Zealand. The drop in his log $(A/T)$ from $\Delta = 2^\circ$ to $13^\circ$ is 2.00 for the North Island; from $2\cdot1^\circ$ to $13\cdot7^\circ$ it is 1.7 for the South Island, with considerable irregularity in both cases. He infers values of $Q$ from 70 to 160 for the North Island and 130 to 180 for the South Island. He emphasizes the existence of considerable local differences.

References