Pion Rescattering Correction to the 
Decay Rate of Hyper-Triton

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The pionic decay rate of the hyper-triton is calculated, including pion rescattering effect. It is shown, that this effect is as important as the effect of the Pauli principle previously discussed by several authors. The agreement with the observed decay rate is satisfactory.

§ 1. Introduction

Recently, the accuracy of observed decay rates of light hyper-nuclei have been greatly improved, and detailed discussion of them has become possible. Among various decay modes, the pionic decay is the most important one for light elements. The observed decay rates are usually much different from that of the free $\Lambda$-hyperon, and this suggests the importance of the effects of neighbouring nucleons. The most important of them will be due to the Pauli principle, and this has already been discussed by several authors.

The purpose of this paper is to investigate another interesting effect, namely, that of the rescattering of the emitted pion by neighbouring nucleons, in the case of the hyper-triton. The reason why we choose the hyper-triton is as follows.

1) Its non-pionic decay is very rare (the branching ratio being less than a few %), and therefore the pionic decay rate is unambiguously known from the total decay rate.
2) The observed decay rate is very large, i.e.

$$\Gamma(H')/\Gamma(\Lambda) = 1.9 \pm 0.5.$$  

It is known that the effect due to the Pauli principle enhances the decay rate by the factor of about 1.3 in the case of hyper-triton with spin $J = 1/2$. Though the experimental error must be taken into consideration, the Pauli principle does not seem to explain the fast decay completely, and thus it is possible that the pion rescattering effect plays an important role.

3) The wave function of the hyper-triton is fairly well known, and this reduces the ambiguity of our calculation.

In the case of nuclear matter, the pion rescattering effects are the subject
of so called "dimesonics". The effect on the pionic decay rate is due to the renormalization of pion wave function, and the result of Miyazawa gives $Z^2 = 1.3$. This encourages our expectation that the rescattering effect enhances the decay rate to the desired order of magnitude. Of course, for the hyper-triton, the concept of nuclear matter cannot be applied, and if we can assume that the emitted pion is scattered at most once in the nucleus, the correction term is expressed as a two body operator, (see Eq. (2·3) and Fig. 1b). Note that this is a kind of induced pion emission term briefly discussed in reference 4).

In § 2, we derive this two body operator using the static theory and a kind of impulse approximation for the pion-nucleon interaction.

In § 3, we calculate the pionic decay rate of the hyper-triton. A closure approximation is used to obtain the closed expression for the decay rate.

In § 4, we give some qualitative arguments on the pionic decay of the other light hyper-nuclei.

§ 2. The effective pionic decay operator

Let us derive the effective pionic decay operator including the rescattering correction. As was mentioned previously, we assume that the emitted pion is scattered by nucleons at most once before it gets out of the nucleus. For the pion-nucleon scattering, we use the static theory and a kind of impulse approximation which assumes that the nucleon-nucleon interaction can be neglected during the propagation of the pion and its interaction with a nucleon. Then the effective pionic decay operator can be readily written down according to the Feynman diagrams illustrated in Fig. 1a and 1b. It is given in the second quantized form as follows:

$$F(k, t) = \int d(1) d(1') \langle 1' | F^{(0)}(k, t ; r_i) | 1 \rangle a^*_n (1') a (1)$$

$$+ \int d(1) d(1') d(2) d(2') \langle 1', 2' | F^{(0)}(k, t ; r_i, r_j) | 1, 2 \rangle$$

$$\times a^*_n (1') a^*_n (2') a_n (2) a (1) ,$$

Fig. 1a. Feynman diagram for $F^{(0)}$

Fig. 1b. Feynman diagram for $F^{(1)}$
where \( a_4(1) \) is the annihilation operator of the \( A \)-hyperon at 1 (representing spin and isotopic spin as well as coordinate), and the integral \( \{d(1) \) is understood to involve the summation with respect to the discrete variables.

\[ F^{(0)}(k, t ; r_1) \text{ and } F^{(1)}(k, t ; r_1, r_2) \text{ are given by} \]

\[
F^{(0)}(k, t ; r_1) = (s + p \left( \frac{\alpha_1 \cdot k}{k_0} \right) ) \tau_{1t} \exp(-ikr_1),
\]

\[
F^{(1)}(k, t ; r_1, r_2) = -\sum_{\ell} \frac{d^p k'}{(2\pi)^2} M_\ell(k, t ; k', t') \left( \frac{1}{k'^2 - k_0^2 - i\varepsilon} \right) \left( s + p \left( \frac{\alpha_1 \cdot k'}{k_0} \right) \right) \times \tau_{1t} \exp(-ikr_2 + ik'(r_2 - r_1)) \]

\[
\times \exp\left(-ik_0 |r_1 - r_3| \right) \exp(-ikr_2).
\]

Here, \( s \) and \( p \) are the s-wave and p-wave decay amplitudes of the \( A \)-hyperon, respectively, and \( k_0 \) is the magnitude of the pion momentum for the free \( A \)-decay. \( k \) is the momentum of the pion emitted from hyper-triton, whose magnitude in the static limit is equal to \( k_0 \), though we write in the following by the different notation \( k \), and \( t \) is its isotopic spin. The matrix \( \tau \) is defined by regarding the \( A \)-hyperon as the \(-\frac{1}{2}\) component of an isotopic doublet, according to the \( \frac{1}{2} \) rule. \( M_\ell(k, t ; k', t') \) is the amplitude of the scattering of the pion by the nucleon \( 2 \), the initial and the final states being \( (k', t') \) and \( (k, t) \), respectively. In the energy region of our interest, only s-wave and p-wave scatterings are important, and \( M \) is given by

\[
M(k, t ; k', t') = -4\pi \sum_\alpha P_\alpha(k, t ; k', t') h_\alpha(\omega_0),
\]

where \( P_\alpha \) is the well-known operator selecting the eigenchannel \( \alpha \), and \( h_\alpha(\omega_0) \) is the T-matrix in the channel \( \alpha \) at the energy \( \omega_0 = \sqrt{k_0^2 + m_\pi^2} \), chosen so as to be finite at threshold. These notations are the same as those of reference 9).

To obtain the numerical results in the next section, we approximate \( h_\alpha \) by the scattering length, and the following values are used for them:

\[
\begin{align*}
    h_3 &= -0.10 \, m_\pi^{-1}, & h_1 &= 0.17 m_\pi^{-1}, & \text{for } s\text{-wave}, \\
    h_{33} &= 0.23 m_\pi^{-2}, & h_{31} &= -0.05 m_\pi^{-3}, \\
    h_{13} &= -0.06 m_\pi^{-3}, & h_{11} &= -0.02 m_\pi^{-3}, & \text{for } p\text{-wave}.
\end{align*}
\]

§ 3. The pionic decay rate of hyper-triton

Having obtained the effective decay operator, we now calculate the pionic decay rate of the hyper-triton. The total pionic decay rate is given by
\[ \Gamma_{\pi} (\Delta H^3) = 2\pi \sum_{l} \left( \frac{d^3k}{(2\pi)^3} \right) \sum_{n} \left| \langle n | F(k, t) | \Delta H^3 \rangle \right|^2 \frac{1}{2\omega_k} \delta (\omega_k + \varepsilon_n - \Delta m) \]

\[ = \frac{1}{8\pi^2} \sum_{l} d\Omega_k \sum_{n} \left[ \left| \langle n | F(k, t) | \Delta H^3 \rangle \right|^2 \right] \frac{1}{k_n} k_n \delta (\Delta m - m_\pi - \varepsilon_n), \quad (3.1) \]

where \(|n\rangle\) is a three nucleon state with energy \(\varepsilon_n\) (measured from the ground state of triton), \(\Delta m\) is the mass difference \(\Delta m = m(\Delta H^3) - m(H^3)\), and \(k_n\) is defined by \(V_{k_n^2 + m_\pi^2} = \Delta m - \varepsilon_n\).

In order to simplify the expression (3.1) and also to avoid the ambiguities due to our ignorance of the correct wave function of \(|n\rangle\), we use here a closure approximation which consists in neglecting the \(n\)-dependence of \(\psi\) and \(\theta\)-function. The justification of this approximation is discussed in reference 3), and we only note here that in the static limit it becomes exact since in this case \(\varepsilon_n\) is to be neglected.

The completeness relation gives

\[ \sum_{n} \left| \langle n | F(k, t) | \Delta H^3 \rangle \right|^2 = \langle \Delta H^3 | F^*(k, t) F(k, t) | \Delta H^3 \rangle. \quad (3.2) \]

We express \(|\Delta H^3\rangle\) as follows:

\[ \langle \Delta H^3 \rangle = \frac{1}{\sqrt{2}} \int d(1) d(2) d(3) \psi^* (1, 2, 3) a_s^*(1) a_n^*(2) a_n^*(3) |0\rangle, \quad (3.3) \]

where \(\psi^* (1, 2, 3)\) is the wave function of hyper-triton normalized to unity, i.e.

\[ \int d(1) d(2) d(3) | \psi (1, 2, 3) \rangle^2 = 1. \quad (3.4) \]

Contracting the operators, and retaining only the first order terms with respect to \(F^{(1)}(k, t; r_1, r_2)\), we obtain

\[ \langle \Delta H^3 | F^*(k, t) F(k, t) | \Delta H^3 \rangle \]

\[ = \int d(1) d(1') d(2) d(2') d(3) \psi^* (1', 2, 3) \langle 1'| F^{(0)*} | 1 \rangle \langle 1' | F^{(0)} | 1 \rangle \psi (1, 2, 3) \]

\[ - 2 \int d(1) d(1') d(2) d(2') d(3) \psi^* (2', 1', 3) \langle 1'| F^{(0)*} | 1 \rangle \langle 2' | F^{(0)} | 2 \rangle \psi (1, 2, 3) \]

\[ - 4 \text{Re} \int d(1) d(1') d(2) d(2') d(3) \psi^* (2', 1', 3) \langle 1'| F^{(0)*} | 1 \rangle \langle 2' | F^{(0)*} | 2' \rangle \times \langle 1' | F^{(0)} | 1, 2 \rangle \psi (1, 2, 3) \]

\[ + 4 \text{Re} \int d(1) d(1') d(2) d(2') d(3) \psi^* (1'', 2', 3) \langle 1'' | F^{(0)*} | 1 \rangle \times \langle 1' | F^{(0)} | 1, 2 \rangle \psi (1, 2, 3) \]

\[ + \text{3-particle correlation terms.} \quad (3.5) \]

In the following, we neglect the last terms, since it is well known that in the nucleus, the three particle correlation is much smaller than the two particle correlation.
Corresponding to the above expression (3.5), we divide \( \Gamma_\pi (\mathcal{H}^3) \) as follows:

\[
\Gamma_\pi (\mathcal{H}^3) = \Gamma_\pi^{(0)} + \Gamma_\pi^{(1)} + \Gamma_\pi^{(2)} + \Gamma_\pi^{(3)}.
\]

(3.6)

The first term \( \Gamma_\pi^{(0)} \) is the simplest and is given by

\[
\Gamma_\pi^{(0)} = \frac{3k}{2\pi} \left( |s|^2 + |p|^2 \frac{k^2}{k_0^2} \right).
\]

(3.7)

For \( k = k_0 \), this is just the decay rate of the free \( \Lambda \)-hyperon. The second term \( \Gamma_\pi^{(1)} \) represents the effects of the Pauli principle and it is given by

\[
\Gamma_\pi^{(1)} = \frac{k}{\pi} \left\{ \frac{3}{2} - \frac{1}{2} \langle \mathbf{r}_1 \cdot \mathbf{r}_3 \rangle \right\} \left\{ \frac{1}{2} \left( |s|^2 + |p|^2 \frac{k^2}{k_0^2} \right) + \frac{1}{3} \langle |s|^2 - \frac{1}{2} |p|^2 \frac{k^2}{k_0^2} \rangle \langle \mathbf{r}_1 \cdot \mathbf{r}_3 \rangle \right\} \int \exp(ikr) g(r) d^3r,
\]

where \( \langle \cdot \rangle \) denotes the expectation value, and \( g(r) \) is given by

\[
g(|\mathbf{r}_1 - \mathbf{r}_3|) = \int \phi^*(\mathbf{r}_2, \mathbf{r}_1, \mathbf{r}_3) \phi(\mathbf{r}_1, \mathbf{r}_3, \mathbf{r}_4) d^3r \left( \frac{r_5 - r_1 + r_3}{2} \right).
\]

(3.8)

(3.9)

As is mentioned in § 1, this term has been investigated in references 2), 3) and 4). We have given the above expression only for completeness.

The third and the fourth terms are the corrections due to the pion rescattering which are the subject of this paper. \( \Gamma_\pi^{(3)} \) has a similar structure to \( \Gamma_\pi^{(1)} \), as is understood from the similarity of their Feynman diagrams:

\[
\Gamma_\pi^{(3)} = \frac{k}{\pi} \left\{ \frac{3}{2} - \frac{1}{2} \langle \mathbf{r}_1 \cdot \mathbf{r}_3 \rangle \right\} \cdot 4\pi \Re \left\{ \frac{1}{2} \left( |s|^2 h_1 + 3 |p|^2 h_3 k^2 \right) \right\}
\]

\[
+ \frac{1}{2} \left( |s|^2 h_1 + |p|^2 h_3 k^2 \right) \langle \mathbf{r}_1 \cdot \mathbf{r}_3 \rangle \int \exp(ikr) g(r) r d^3r \}
\]

(3.10)
The appearance of only $h_1$ and $h_{11}$ is apparent from the consideration of the spin and the isotopic spin conservations. The structure of $\Gamma^{(3)}_s$ is a little complicated. It is given by

$$
\Gamma^{(3)}_s = \frac{2k}{\pi} \cdot 4\pi \text{Re}[A] \int f(r) j_0(\kappa r) e^{ik\kappa r} r dr \\
+ \frac{B}{k} \int f(r) j_1(\kappa r) (1 - ik\kappa r) e^{ik\kappa r} dr \\
+ \frac{C}{k^3} \int f(r) j_3(\kappa r) (3 - 3i\kappa r - k^2 r^2 r_0) e^{ik\kappa r} dr,
$$

where

$$
A = |p|^2 k^3 \left\{ \frac{1}{3} (4h_{33} + 2h_{31} + 2h_{13} + h_{11}) \\
+ \frac{2}{9} (2h_{33} - 2h_{31} + h_{13} - h_{11}) \langle \sigma_1 \cdot \sigma \rangle \\
+ \frac{2}{9} (2h_{33} + h_{31} - 2h_{13} - h_{11}) \langle \tau_1 \cdot \tau \rangle \\
+ \frac{4}{27} (h_{33} - h_{31} + h_{13} + h_{11}) \langle \sigma_1 \cdot \sigma \rangle \langle \tau_1 \cdot \tau \rangle \\
+ |s|^2 \{2 (h_3 + h_1) + \frac{2}{3} (h_3 - h_1) \langle \tau \cdot \tau \rangle \},
$$

$$
B = k^3 \left\{ |s|^2 (4h_{33} + 2h_{31} + 2h_{13} + h_{11}) + \frac{|p|^2}{k_0^2} (2h_3 + h_1) \\
+ \frac{2}{3} |s|^2 (2h_{33} + h_{31} - 2h_{13} - h_{11}) + \frac{|p|^2}{k_0^2} (h_3 - h_1) \langle \tau_1 \cdot \tau \rangle \},
$$

$$
C = |p|^2 k^4 k_0^3 \left\{ \frac{2}{3} (4h_{33} + 2h_{31} + 2h_{13} + h_{11}) \\
- \frac{2}{9} (2h_{33} - 2h_{31} + h_{13} - h_{11}) \langle \sigma_1 \cdot \sigma \rangle \\
+ \frac{4}{9} (2h_{33} + h_{31} - 2h_{13} - h_{11}) \langle \tau_1 \cdot \tau \rangle \\
- \frac{4}{27} (h_{33} - h_{31} + h_{13} - h_{11}) \langle \sigma_1 \cdot \sigma \rangle \langle \tau_1 \cdot \tau \rangle \},
$$

and $f(r)$ is defined by

$$
f(|r_1 - r_2|) = \int \phi^*(r_1, r_3, r_4) \phi(r_1, r_2, r_0) d^3(\frac{r_0 - r_1 + r_3}{2}).
$$
Now we introduce the following quantities:

\[ 1 + d_0 = \frac{\Gamma_x^{(0)}}{\Gamma(A)}, \]
\[ d_1 = \frac{\Gamma_x^{(1)}}{\Gamma(A)}, \]
\[ d_2 = \frac{\Gamma_x^{(2)}}{\Gamma(A)}, \]
\[ d_3 = \frac{\Gamma_x^{(3)}}{\Gamma(A)}, \]

where \( \Gamma(A) \) is the decay rate of the free \( A \)-hyperon which, as was mentioned previously, is equal to \( \Gamma_x^{(k)} \) for \( k = k_0 \). In order to determine \( d \)'s, we must know the ratio \( \frac{|\rho|^2}{|s|^2 + |\rho|^2} \), expectation values \( \langle \tau_1 \cdot \tau_2 \rangle \), and \( \langle \sigma_1 \cdot \sigma_2 \rangle \), and the radial part of the wave function \( \psi(r_1, r_2, r_3) \). The recent measurement on the \( A \)-decay parameters\(^{13}\) gives

\[ \frac{|\rho|^2}{|s|^2 + |\rho|^2} = 0.11 \pm 0.03, \]

from which we obtain \( d_2 = 0.12 \). The analyses of the hyper-triton by several authors\(^5\) have almost confirmed the isotopic spin to be zero, which in our case implies that the isotopic spin part of the wave function \( \psi(1, 2, 3) \) has the form \( |1/2, 1/2 \rangle \). Since the radial part is reasonably assumed to be symmetric with respect to \( r_2 \) and \( r_3 \), the spin part is of the form \( |1/2, 1/2 \rangle \) with \( J = 3/2 \) or \( 1/2 \), of which the latter is favoured.\(^{3,4}\) Then the expectation values \( \langle \tau_1 \cdot \tau_2 \rangle \) and \( \langle \sigma_1 \cdot \sigma_2 \rangle \) is determined to be

\[ \langle \tau_1 \cdot \tau_2 \rangle = 0, \]
\[ \langle \sigma_1 \cdot \sigma_2 \rangle = 1 \quad \text{for} \quad J = \frac{3}{2}, \]
\[ \langle \sigma_1 \cdot \sigma_2 \rangle = -2 \quad \text{for} \quad J = \frac{1}{2}. \]  

For the radial part, several forms have been proposed.\(^5\)\(^6\)\(^7\) We take here the simplest one proposed in reference 5), i.e.

\[ \psi(r_1, r_2, r_3) = N \exp(-\alpha|\mathbf{r}_1 - \mathbf{r}_2| - \alpha|\mathbf{r}_1 - \mathbf{r}_3| - \beta|\mathbf{r}_2 - \mathbf{r}_3|), \]  

with

\[ N^2 = \frac{8}{\pi^3} \frac{\alpha^3 (\alpha + \beta)^3}{(2\alpha + \beta)^3 + \alpha^2 \beta}. \]  

We believe that the use of more detailed wave functions does not change the results so much. From Eq. (3.16), \( f(r) \) and \( g(r) \) are obtained as follows:
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\[
f(r) = 4\pi N^r \frac{1}{4(\beta^2 - \alpha^2)} \{\beta e^{-\alpha r} + \alpha e^{-\beta r} \frac{2\alpha \beta}{\beta^2 - \alpha^2} \frac{1}{r} (e^{-\alpha r} - e^{-\beta r})\},
\]
\[
y(r) = \pi N^z \frac{1}{(\alpha + \beta)^3} e^{-(\alpha + \beta) r} \left\{1 + (\alpha + \beta) r + \frac{1}{3} (\alpha + \beta)^2 r^2 \right\}.
\]

As for the parameters \(\alpha\) and \(\beta\), we take the following two sets of values, of which the first is one of the sets obtained in reference 5) by the variational calculation,

1. \(\alpha = \beta = 0.52 \times 10^{15} \text{cm}^{-1}\),
2. \(\alpha = \frac{\beta}{2} = 0.40 \times 10^{15} \text{cm}^{-1}\).

The numerical results for \(\mathcal{A}\)'s are given in Tables I(a) and I(b) for the case of \(J=3/2\) and \(1/2\), respectively.

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<th>(d_2)</th>
<th>(d_4)</th>
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<td>-0.14</td>
<td>0.13</td>
</tr>
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<td>0.19</td>
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Table I(b). Numerical results for \(\mathcal{A}\)'s in the case of \(J=1/2\).

<table>
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<th>(d_4)</th>
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<td>0.19</td>
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</table>

From these results, we may conclude that the calculated decay rate of hyper-triton \(\Gamma_x(\mathcal{L}^3) = (1 + d_0 + d_1 + d_2 + d_4) \Gamma(\mathcal{A})\) agrees with the observed one \(\Gamma_x(\mathcal{L}^3) = (1.9 \pm 0.5) \Gamma(\mathcal{A})\) within the experimental error for the case \(J=1/2\), and that the rescattering correction is as important as the effects of the Pauli principle. The weak dependence of our results on the choice of parameters \(\alpha\) and \(\beta\), together with the similarity of our \(\mathcal{A}\) with those of references 2), 3) and 4) seems to confirm our expectation that the use of the more detailed wave function does not change the matter. It must also be noted that the rescattering correction \(d_1 + d_4\) is much the same as that for the case of nuclear matter. The difference of the spin dependence of \(d_4\) from that of \(d_1\) and \(d_4\) makes it possible to distinguish the rescattering correction by investigating the pionic decay rate of the other hyper-nuclei which will be discussed in the next section.
4. Concluding remarks

In the last section, we have restricted ourselves to the decay of the hypertriton. It is easy to generalize the expressions (3.7), (3.8), (3.10) and (3.11) for $\Gamma_s^{(i)}$, so as to be applied to the pionic decays of the other hyper-nuclei. $\Gamma_s^{(0)}$ remains the same, while $\Gamma_s^{(1)}, \Gamma_s^{(2)}$ and $\Gamma_s^{(3)}$ are multiplied by the factor $A/2$, if the hyper-nucleus consists of a hyperon and $A$ nucleons. Also, the functions $f(r)$ and $g(r)$ must be re-defined appropriately. Since it is unlikely that these functions should be changed so drastically as to reverse the signs of the integrals involved in the expressions (3.8), (3.10) and (3.11), we may conclude that $\Gamma_s^{(3)}$ is almost always positive because the first term of $B$ in Eq. (3.12) is the dominant term, while the signs of $\Gamma_s^{(1)}$ and $\Gamma_s^{(2)}$ are the same and depend on the expectation values $\langle r_1 \cdot r_2 \rangle$ and $\langle \sigma_1 \cdot \sigma_2 \rangle$.

For $^4\text{He}^+(T=1/2, J=0)$, $\langle r_1 \cdot r_2 \rangle = 1/3$, $\langle \sigma_1 \cdot \sigma_2 \rangle = -1$, and for $^4\text{He}^+(T=1/2, J=0)$, $\langle r_1 \cdot r_2 \rangle = 1/3$, $\langle \sigma_1 \cdot \sigma_2 \rangle = -1$. For both cases, $\Gamma_s^{(1)}$ and $\Gamma_s^{(2)}$ become very small, since the dominant terms proportional to $|s|^2$ are zero. Thus, the enhancement due to $\Gamma_s^{(3)}$ is expected.

For $^4\text{He}^+(T=0, J=1/2)$, $\langle r_1 \cdot r_2 \rangle = 0$, $\langle \sigma_1 \cdot \sigma_2 \rangle = 0$, and $\Gamma_s^{(1)}$ and $\Gamma_s^{(2)}$ become negative.

For larger hyper-nuclei, the expectation values $\langle r_1 \cdot r_2 \rangle$ and $\langle \sigma_1 \cdot \sigma_2 \rangle$ become small (order of $A^{-1}$), and thus $\Gamma_s^{(1)}$ and $\Gamma_s^{(2)}$ are usually negative, though in such cases, the validity of the closure approximation becomes suspicious.

References