Merged binary white dwarf evolution: rapidly accreting carbon–oxygen white dwarfs and the progeny of extreme helium stars

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ABSTRACT

We have examined the evolution of merged low-mass double white dwarfs that become luminous helium stars. We have approximated the merging process by the rapid accretion of matter, consisting mostly of helium, on to a carbon–oxygen (CO) white dwarf. After a certain mass is accumulated, a helium shell flash occurs, the radius and luminosity increase and the star becomes a yellow giant. Mass accretion is stopped artificially when the total mass reaches a pre-determined value. When the mass above the helium-burning shell becomes small enough, the star evolves blueward almost horizontally in the Hertzsprung–Russell diagram. The theoretical models for the merger of a 0.6-M\(_\odot\) CO white dwarf with a 0.3-M\(_\odot\) He white dwarf agree very well with the observed locations of extreme helium stars in the log\(T_{\text{eff}}\)--log\(g\) diagram, with their observed rates of blueward evolution, and with luminosities and masses obtained from their pulsations. Together with predicted merger rates for CO‡He white dwarf pairs, the evolutionary time-scales are roughly consistent with the observed numbers of extreme helium stars. Predicted surface carbon and oxygen abundances can be consistent with the observed values if carbon and oxygen produced in the helium shell during a previous asymptotic giant branch phase are assumed to exist in the helium zone of the initial CO white dwarfs. These results establish the CO‡He white dwarf merger as the best, if not only, viable model for the creation of extreme helium stars and, by association, the majority of R Coronae Borealis stars.

Key words: binaries: close – stars: chemically peculiar – stars: evolution – white dwarfs.

1 INTRODUCTION

The likely outcome of the evolution of two stars in a detached binary is a pair of white dwarfs with masses of ~0.6 M\(_\odot\) or below. Increasing numbers of such binaries are now being discovered. With no nuclear reserves, their evolution will be dominated by orbital decay as a consequence of gravitational radiation or magnetic-wind braking. It is reasonable to suppose that a substantial fraction of the orbits of such binaries will decay completely within a Hubble time. The less massive white dwarf will then fill the common gravitational potential surface that just connects the two stars (Roche lobe) and spill over on to its companion. Tidal forces will take over and disrupt the less massive star, causing it to form a disc from which the more massive companion starts to accrete material. In loose terms, this describes the merger of two white dwarfs; a process that should become increasingly common as the stellar population of a galaxy ages. This paper addresses the question of what happens when a pair of white dwarfs, one with a carbon–oxygen core and the other with a helium core, coalesce. Its motivation is an attempt to explain the evolutionary origin of extreme helium stars (EHes), a rare class of luminous stars with highly processed surfaces.

The task of stellar evolution theory is to explain how these stars acquired their unusual characteristics. Typically, they are rare B- and A-type giant stars with extremely low surface abundances of hydrogen (Jeffery 1996). In most cases they are also characterized by enhancements of CNO-processed, 3\(\alpha\)- and \(\alpha\)-capture products and the majority have log\(L/M\) > 4 (as indicated by their surface gravities), where luminosity, \(L\), and the mass, \(M\), are in solar units. A few have significantly lower \(L/M\) ratios and do not show 3\(\alpha\)- and \(\alpha\)-capture products in their atmospheres (e.g. V652 Her, Jeffery, Hill & Heber 1999). EHes are often considered to be related to the cooler R CrB stars because of their similar surface composition and luminosities (Asplund et al. 2000); the latter will also be considered in this paper.

The major question concerning the evolutionary origin of EHes is whether they are the products of single-star or binary-star evolution. The task has been difficult from the outset because, in
the normal evolution of single stars from the main sequence to the white dwarf phase, it seemed impossible to remove the hydrogen-rich surface. Two principal hypotheses emerged during the 1980s.

The ‘merged binary white dwarf model’ (MBWD: Iben & Tutukov 1984; Webbink 1984) considered the accretion of a white dwarf (WD) secondary on to a white dwarf primary, resulting in the ignition of a helium shell in the accreted envelope, which forces the star to expand to become a cool giant. The subsequent evolution would follow the canonical post-asymptotic giant branch (AGB) contraction to the white dwarf track, in the case of a CO WD merger, or contraction to the helium main sequence – possibly giving a subdwarf B star – in the case of He WD merger (Iben 1990).

The ‘late thermal pulse’ model (LTP: 1 Iben et al. 1983) considered what would happen when the helium layer remaining near the surface of a star at the end of AGB evolution was of such a mass that a final thermal pulse would occur after the star had become a white dwarf, also forcing the star to expand rapidly. Again, the subsequent evolution would resemble the canonical post-AGB sequence.

The LTP model has been studied extensively in recent years (Iben & MacDonald 1995; Blöcker & Schönberner 1997; Herwig et al. 1999) and used to discuss the origins of various hydrogen-deficient stars. Part of the success of LTP models has been caused by the large degree of freedom allowed in reproducing a wide range of surface compositions, from s-process elements in some R CrB stars (Bond, Luck & Newman 1979; Lambert & Rao 1994) to very high C and O concentrations in PG11159 and [WC] stars (Werner, Heber & Hunger 1991; Leuenhagen, Hamann & Jeffery 1996). The LTP model has also been supported by the rapid evolution from WD to cool giant observed in V605 Aql (Pollacco et al. 1992), FG Sge (Herbig & Boyarchuk 1968) and V4334 Sgr (Duerbeck & Benetti 1996), all of which are hydrogen-deficient to some extent. However, another part of the success of the LTP model may have been caused by the absence of detailed numerical MBWD models.

In particular, the LTP model fails to account for all EHe stars of two reasons, namely: (i) a degenerate CO core with a helium-burning shell cannot account for the low-luminosity EHe (Saio & Jeffery 2000) and (ii) correctly computed LTP models predict surface abundances of carbon and oxygen that are an order of magnitude higher than observed in EHe stars (Herwig et al. 1999). Pandey et al. (2001) have emphatically rejected the LTP model for the origin of extreme helium stars on the basis of this argument.

In contrast, the merger of two helium white dwarfs has described successfully the physical dimensions and pulsation properties of one low-$L/M$ EHe with remarkable precision (Saio & Jeffery 2000). It has also been established that no EHe stars are members of binary systems (Jeffery, Drilling & Heber 1987), forcing the conclusion that their evolution must have included the destruction of a binary system. Any putative single-star origin implies that some fraction would continue to have observable binary companions.

Methods to study the consequences of MBWD systems have been developed by considering evolutionary models for accreting white dwarfs (Saio & Nomoto 1998; Saio & Jeffery 2000). Whilst the general properties of such models for both CO and He WDs have been considered (Saio & Nomoto 1998; Iben 1990), it is necessary to examine their detailed properties alongside a number of observational constraints. The first constraint is the distribution of EHe stars in the effective temperature ($T_{\text{eff}}$) versus surface gravity ($\log g$) plane. The latter is a proxy for the luminosity-to-mass ($L/M$) ratio. The second is the surface composition of the EHe stars compared with that given as a consequence of nucleosynthesis and mixing in the evolutionary models. A third is provided by the secular timescales which, in the case of some rapidly evolving EHe stars, have recently been measured (Jeffery et al. 2001a). Finally, pulsational instability is ubiquitous amongst EHe stars (Saio & Jeffery 1988) and, ultimately, direct mass measurements using Baade’s method (Jeffery et al. 2001a) or a comparison with non-linear hydrodynamic models.

It is the purpose of this paper to present evolution models for accreting carbon–oxygen white dwarfs with final masses in the range 0.6–0.9 $M_\odot$. These models, including their pulsation properties, will be compared with the observational constraints summarized above.

2 EVOLUTION MODELS

2.1 Assumptions and initial models

We have calculated evolutionary models starting with white dwarfs rapidly accreting helium-rich material ($Y = 0.98$, $Z = 0.02$). In order to see the effect of a trace amount of hydrogen, we have also considered accretion of matter containing a small fraction of hydrogen ($X = 0.001$). For the initial accretion phase, which is considered as a rough approximation of the merging process of a double white dwarf system, we have adopted an accretion rate of $1 \times 10^{-5} M_\odot$ yr$^{-1}$, which is approximately one-half of the Eddington limit accretion rate for white dwarfs. Initial masses ($M_1$) of white dwarfs considered are 0.6 and 0.5 $M_\odot$. The accretion was stopped when the total mass increased to a pre-determined final mass. Considering the range of masses reported for extreme helium stars in the literature, we have adopted final masses of $M_1 = 0.9, 0.8$ and 0.7 for $M_1 = 0.6 M_\odot$, and $M_1 = 0.9, 0.8, 0.7$ and 0.6 $M_\odot$, for $M_1 = 0.5 M_\odot$. Models will occasionally be referred to in the form ($M_1(M_2)$). Thus $0.9(0.6)$ $M_\odot$ means the model having $M_1 = 0.6 M_\odot$ and $M_2 = 0.9 M_\odot$. The computational method is the same as that used in Saio & Nomoto (1998) and Saio & Jeffery (2000).

Our models assume that the helium white dwarf destroyed in the merger has a mass in the range 0.3–0.4 $M_\odot$. The choice of final mass is between 0.1 and 0.4 $M_\odot$ greater than the initial mass. This range implies a wide range for the assumed efficiency of the merger process.

Initial white dwarf models have been obtained by evolving helium stars of 0.6 and 0.5 $M_\odot$ from the zero-age helium main sequence to the white dwarf sequence. The 0.6–$M_\odot$ white dwarf model has a degenerate C–O core of 0.58 $M_\odot$, while the 0.5–$M_\odot$ model has $M_{\text{CO}} = 0.46 M_\odot$. The luminosity and the central temperature are $\log L/\odot = -3.03$ and $-2.64$ and $T_c = 6.774$ and 6.999 for the 0.6– and 0.5–$M_\odot$ models, respectively. These values for $T_c$ were originally chosen so that the shell temperature in the accretion phase should be low enough to avoid hydrogen ignition (when present). They correspond to evolved white dwarfs with ages of $6 \times 10^3$ yr.

To see the dependence of the results on the initial $T_c$, we have computed additional model sequence starting with a 0.6–$M_\odot$ white dwarf having log $T_c = 7.072$ (log $L/\odot = -2.39$). We have found that the results are insensitive to the choice of the initial $T_c$ except for the early accretion phase of evolution.
2.2 Evolution to a yellow giant

Figs 1 and 2 show evolutionary tracks after starting accretion. Solid lines are for cases starting with the 0.6-M\(_\odot\) white dwarf accreting matter without hydrogen (Y = 0.98, Z = 0.02) and the dotted line is for the case starting with the same white dwarf model but accreting matter containing a small fraction of hydrogen (X = 0.001). Dashed lines represent evolutionary tracks starting with the 0.5-M\(_\odot\) white dwarf accreting matter without hydrogen.

First, we discuss the case starting with the 0.6-M\(_\odot\) white dwarf accreting mostly helium without hydrogen (solid lines in Figs 1 and 2). With the start of accretion the luminosity increases to \(\log L/L_\odot \approx -2.6\) before a helium shell flash is ignited at \(M_r = 0.604\,M_\odot\) (Fig. 3). At ignition a mass of 0.029\,M\(_\odot\) has been accumulated. After ignition, the luminosity increases initially up to \(\log L/L_\odot = 0\) along a white dwarf sequence (i.e. with little increase in radius) then the star expands a little; \(\Delta \log R = 0.8\) (\(\Delta \log T_{\text{eff}} = -0.2\)). With this expansion the first shell flash ends. After the first flash, the helium-burning shell moves inward repeating very mild flashes and the luminosity increases up to \(\log L/L_\odot = 3.5\) on a track nearly parallel to the original white dwarf sequence. Then it turns right on the Hertzsprung–Russell (HR) diagram to become a yellow giant. The transition from a white dwarf to a yellow giant takes \(-200\,\text{yr}\) (Fig. 3).

Evolution sequences starting with the 0.5-M\(_\odot\) white dwarf are shown by dashed lines in Figs 1 and 2. With the start of accretion the luminosity increases along the white dwarf sequence. The first shell flash occurs when \(\log L/L_\odot = 3.5\) and \(\log T_{\text{eff}} \approx 5.2\). The ignition shell is located at \(M_r = 0.5003\,M_\odot\) when 0.013\,M\(_\odot\) is accreted (Fig. 4). The accreted mass before the ignition is smaller than the above 0.6\,M\(_\odot\) case. The earlier ignition is probably caused

Figure 1. Evolutionary tracks starting with CO white dwarfs. The solid line represents the evolution starting with a 0.6-M\(_\odot\) CO white dwarf accreting mostly helium and no hydrogen (Y = 0.98, Z = 0.02). The accretion was stopped when the total mass became 0.7\,M\(_\odot\) nearly at the coolest part of each track. The dotted line is for the same model but accreting gas containing a small fraction of hydrogen (X = 0.001). The dashed line represents evolution starting with a 0.5-M\(_\odot\) CO white dwarf accreting mostly helium and no hydrogen. The accretion was stopped when the total mass became 0.7\,M\(_\odot\).

Figure 2. Expanded section of Fig. 1 with additional models. Initial models are for a 0.6-M\(_\odot\) CO white dwarf accreting mostly helium (solid line), and for a 0.5-M\(_\odot\) CO white dwarf accreting mostly helium (dashed line). (The model sequence that accretes matter with hydrogen is not shown to avoid confusion.) Evolution was stopped when the total mass became 0.7, 0.8 and 0.9\,M\(_\odot\) for \(M_i = 0.6\,M_\odot\), and 0.6, 0.7, 0.8 and 0.9\,M\(_\odot\) for \(M_i = 0.5\,M_\odot\). The tracks after the termination of accretion are shown as thick lines. The dimensions of two extreme helium stars taken from Jeffery et al. (2001a) are shown for comparison. The masses of PV Tel and V2244 Oph were measured as 0.94 ± 0.68 and 0.79 ± 0.46\,M\(_\odot\), respectively.

Figure 3. Evolution of the model with \(M_i = 0.6\,M_\odot\) showing the behaviour of the nuclear luminosity \((L_n)\), total luminosity \(L\), effective temperature \(T_{\text{eff}}\), the position of the helium-burning shell and convective zones as a function of time. Note the rapid expansion (<100\,yr) to low \(T_{\text{eff}}\) following the first He-shell flash. Initially the He-shell/CO-core boundary (where the helium abundance is zero) is located at the point of deepest ingress of the helium-burning shell; thereafter it follows the helium-burning shell.
by the fact that the initial core temperature of the 0.5-\(M_{\odot}\) model is higher than that of the 0.6-\(M_{\odot}\) model. Following shell ignition, the luminosity drops to \(\log L/\dot{L}_{\odot} = 2.84\). Thereafter, the helium-burning shell moves inward repeating very mild flashes and the envelope expands to become a yellow giant.

A jump in the location of the helium-burning shell appears to occur after the giant phase is well established (cf. Figs 3 and 4). This arises because the distribution of the energy generation has two peaks within the shell; the inner peak has a higher temperature but a very small helium abundance and the outer peak has a slightly lower temperature but a larger helium abundance. The inner peak weakens as the helium abundance decreases with time. The jump occurs when the energy generation rate of the outer peak exceeds that of the inner peak.

For the 0.6-\(M_{\odot}\) white dwarf accreting matter containing a small fraction of hydrogen (the dotted line in Fig. 1), a hydrogen shell flash occurs close to the original white dwarf surface (\(M_r = 0.600 M_{\odot}\)) when 0.002\(M_{\odot}\) has been accreted (Fig. 5). The luminosity increases along the white dwarf sequence up to \(\log L/\dot{L}_{\odot} = 3.7\), consuming most of the accreted hydrogen. After \(\sim 600\) yr the shell burning ignites at \(M_r = 0.602 M_{\odot}\), the surface luminosity begins to decrease rapidly and the envelope is transformed to a structure supported by the helium-burning shell. After a rapid decrease in luminosity, which lasts only \(\sim 20\) yr, evolution governed by the helium-burning shell begins and the luminosity begins to increase. Following the helium shell flash, the expansion of the envelope extinguishes the hydrogen-burning shell. Since the value of \(M_r\) at the helium-burning shell is larger than the case without hydrogen, the evolutionary path to the yellow giant region is bluer and more luminous. However, the evolution after the yellow giant phase is very similar to that without hydrogen for a given mass (0.7-\(M_{\odot}\) track is shown in Fig. 1), because the helium-burning shell governs the envelope structure, even considering the weak nuclear-burning shell that is re-ignited after the heliurn-burning shell becomes fully established (Fig. 5). The hydrogen abundance is too small to modify the opacity significantly.

The location and extent of convection zones in the envelope of the yellow giant are important because they can play an important role in dredging-up newly processed material to the stellar surface (Fig. 5). The first helium-shell flash is associated with a convection zone that reaches just to the surface of the star. The carbon and oxygen abundance in the convective shell at that phase are \(\sim 0.1\) and \(\sim 0.0002\), respectively. As the star becomes a giant, a surface convection zone extends inwards to below the position of the first helium flash. The helium-burning shell has moved inwards, leaving material mildly enhanced in carbon. This is dredged up to the surface to increase the surface carbon and oxygen abundances by up to \(\sim 0.1\) and \(\sim 2.5 \times 10^{-4}\), respectively. However, these abundances are diluted considerably as mass accretion proceeds.

In the model with hydrogen, protons are carried downwards into helium-rich layers so that the very weak hydrogen-burning shell is shifted inwards to a layer between the helium-burning shell and the base of the convection zone.

2.3 Blueward evolution from a yellow giant

Since the accretion rate \(10^{-5} M_{\odot} \text{yr}^{-1}\) is high, the envelope stays inflated and the star remains a yellow giant as long as the accretion continues. As can be seen from Fig. 2, the effective temperature of each track increases slightly in the late phase of accretion. This is caused by a decrease in the envelope opacity, which occurs because the carbon abundance in the convection zone decreases with accretion.

The mass accretion is stopped artificially when the total mass reaches a pre-determined value, notionally representing the amount available from the merger process. Further evolution is determined by whether the mass above the helium-burning shell is smaller than

[Figure 4](#) As in Fig. 3 for the model with \(M_l = 0.5 M_{\odot}\). The expansion rate following helium-shell ignition is significantly slower than for \(M_l = 0.6 M_{\odot}\).

[Figure 5](#) The location of the helium (solid line) and hydrogen-burning (broken line) shells and convective zones are shown as a function of time and mass (\(M_{\odot}\)). The concurrent values of nuclear luminosity, surface luminosity and effective temperature (K) are shown in the upper three panels.
the equilibrium value corresponding to the core mass at the current effective temperature. If this is the case, the star starts a blueward evolution immediately toward the equilibrium position (on the HR diagram) corresponding to the envelope and core masses. Such evolution occurs for the case of 0.6(0.5) \text{M}_\odot.

If the envelope mass is sufficiently high at the termination of accretion, the star evolves slowly as a yellow giant for a while. The luminosity increases as the mass of the CO core increases owing to an outward progress of the helium-burning shell. When the mass above the helium-burning shell becomes sufficiently small, the surface temperature begins to increase and the star evolves blueward nearly horizontally on the HR diagram.

The cause for the difference between the models with the same mass but with \( M_0 = 0.5 \) and 0.6 \text{M}_\odot arises from a difference in the relation between the radius and mass of the C–O core. Because the high-temperature (say \( T > 10^4 \text{K} \)) zone in the C–O core is thicker in the model from \( M_0 = 0.5 \text{M}_\odot \) than in the model from \( M_0 = 0.6 \text{M}_\odot \) for a given core mass, the radius at the helium-burning shell in the model from \( M_0 = 0.5 \text{M}_\odot \) is larger because of the relative importance of thermal pressure. A larger radius at the helium-burning shell yields a lower temperature (because of the lower gravity there) and hence a lower luminosity, thus each initial mass is associated with a different core-mass shell-luminosity relation. In turn, the lower luminosity increases the envelope mass required to sustain a given effective temperature. Therefore, for a given total mass and effective temperature the model from \( M_0 = 0.5 \text{M}_\odot \) has a larger envelope mass and is less luminous than the model from \( M_0 = 0.6 \text{M}_\odot \). The difference is smaller for a larger total mass because the ratio of thermal to degenerate pressures is smaller in the larger C–O core and hence the thickness of the high-temperature zone is less important.

3 \( L/M \) RATIOS

Fig. 6 shows evolution tracks during the contraction phase and the observed positions of extreme helium stars in the log \( g \)–log \( T_{\text{eff}} \) diagram. The observational data and their sources are given in Table 1. Solid and dashed lines indicate evolution tracks for models obtained in this paper starting with 0.6- and 0.5-M_\odot CO white dwarfs, respectively. The total masses are 0.7, 0.8 and 0.9 M_\odot for solid lines (\( M_0 = 0.6 \text{M}_\odot \)) and 0.6, 0.7, 0.8 and 0.9 M_\odot for dashed lines (\( M_0 = 0.5 \text{M}_\odot \)). The upper lines correspond to the higher-mass models for a given \( M_0 \).

Since the evolution on the HR diagram is roughly horizontal (Fig. 2), the luminosity-to-mass ratio, \( L/M \), is nearly constant during the contraction evolution. The values of \( L/M \) are higher for higher-mass models ranging from log \( L/M \) ~ 4.22 for 0.7(0.6) \text{M}_\odot to ~ 4.47 for 0.9(0.6) \text{M}_\odot (solid lines) and ~ 3.77 for 0.6(0.5) \text{M}_\odot to ~ 4.46 for 0.9(0.5) \text{M}_\odot (dashed lines). The observed positions of high-luminosity extreme helium stars seem to be consistent with our CO white dwarf merger models having total masses of between ~0.7 and ~1 \text{M}_\odot.

Low-luminosity extreme helium stars such as V652 Her and HD 144941 are close to the dotted line, which is an evolutionary sequence obtained starting with a 0.4-M_\odot helium white dwarf by Saio & Jeffery (2000). The low-luminosity extreme helium star LSS 3184 is considerably above the dotted curve, but the surface gravity may not be accurate (Woolf & Jeffery 2000).

Two stars with log \( T_{\text{eff}} \) ~ 4.2 and log \( g \) = 2.5 along the dashed line for 0.6(0.5) \text{M}_\odot are BD +10°2179 and HD 124448. These stars have \( L/M \) values intermediate between those of high- and low-luminosity extreme helium stars, and seem to be consistent with the evolution model of ~0.6 \text{M}_\odot started with a ~0.5-M_\odot C–O white dwarf.

4 CONTRACTION RATES

Fig. 7 shows rates of the effective temperature change caused by contraction against the effective temperature for various models. Numbers in parentheses indicate the initial white dwarf mass (\( M_0 \)).

The contraction rates in most models are governed by the outward progression of the helium-burning shell so that the faster consumption of helium in more massive models yields a faster contraction. For a given total mass the contraction rate is slower for the model from the \( M_0 = 0.5 – \text{M}_\odot \) white dwarf, because the radius of the helium-burning shell is larger and hence the luminosity is lower.

Comparing dotted and solid lines for 0.7(0.6) \text{M}_\odot in Fig. 7 indicates clearly that the existence of a small amount of hydrogen in the envelope hardly affects the contraction rate.

Saio (1988) estimated contraction rates of extreme helium stars from equilibrium models, which consist of an isothermal degenerate C–O core, helium-burning shell, and a helium-rich envelope. These rates are similar to those of our evolution models obtained with \( M_0 = 0.6 \text{M}_\odot \). For comparison the contraction rate for 0.8 M_\odot obtained from equilibrium models is shown by a dash-dotted line.

Another model proposed to produce luminous helium stars is a...

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\(^2\) Recall that, for shell-burning stars in equilibrium, the shell luminosity is governed by the core mass and the effective temperature is governed by the core mass and the envelope mass (cf. Saio 1988; Iben & Tutukov 1989).

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**Figure 6.** Evolutionary tracks for the blueward evolution from the yellow giant phase on the log \( g \)–log \( T_{\text{eff}} \) plane. Solid and dashed lines are for models obtained starting from 0.6- and 0.5-M_\odot CO white dwarfs, respectively. The upper lines correspond to higher total masses. The masses are 0.7, 0.8 and 0.9 M_\odot for solid lines and 0.6, 0.7, 0.8 and 0.9 M_\odot for dashed lines. The dotted line denotes a 0.7-M_\odot track for low-luminosity extreme helium stars computed in Saio & Jeffery (2000), which was started with a 0.4-M_\odot helium white dwarf. Open squares show observed positions of some extreme helium stars (see Table 1).
late (or final) thermal pulse model, which passes the helium star region on the HR diagram contracting from its born-again AGB stage. From evolutionary tracks shown in Bloécker & Schönberner (1997) we can estimate the log[dT_eff/dt] as 1.6 and 3.3 for 0.625- and 0.836-M_☉ models, respectively. These contraction rates are much faster than those for our models having similar masses. The models of Bloécker & Schönberner (1997) still have substantially hydrogen-rich atmospheres. More recent models with hydrogen-poor surfaces have effectively similar contraction rates (Blöcker 2001). This difference comes from the fact that the envelope mass in the late thermal pulse model is much smaller than that of the equilibrium value for a given effective temperature ranging from ~10,000 to ~40,000 K.

Jeffery et al. (2001a) measured the rates of temperature variations of the extreme helium stars HD 160641, HD 168476, BD−9°4338, and BD−1°3438 (see Table 1). Their results are plotted in Fig. 7 as open squares. It is remarkable that three of the four stars are located along the tracks for the 0.9-M_☉ models. The exception (HD 168476) is consistent with the rate for a model between 0.8(0.6) and 0.9 M_☉. We note, however, that the positions in this figure are not always consistent with the positions in the log T_eff−log g plane (Fig. 6). In particular, BD−1°3438 is located close to the line for 0.9 M_☉ in Fig. 7 but close to the track for low-luminosity extreme helium star models in Fig. 6. The observed contraction rates (Jeffery et al. 2001a) would be consistent with the late thermal pulse model if the mass were ~0.6 M_☉, while our merger model requires ~0.8−0.9 M_☉.

Arguing from the standpoint of observed numbers and predicted birth rates, Iben, Tutukov & Yungelson (1996) arrived at a similar conclusion. Accurate estimates of the masses and the luminosities of extreme helium stars are therefore essential to determine which is the correct model.

### 5 RADIAL PULSATIONS

In previous studies of extreme helium stars, two proxies have been used for estimating the stellar mass. One is the stellar surface gravity, which provides a direct measurement of the stellar luminosity-to-mass ratio. When combined with an appropriate core-mass shell-luminosity (M_☉−L_☉) relation (e.g. Saio 1988; Jeffery 1988), this gives what has been referred to as a...
spectroscopic mass \((M_s)\). The second is provided, for those helium stars that pulsate, by the pulsation period which, together with a theoretical relation derived from linear pulsation theory, provides an independent measure of the luminosity-to-mass ratio (Sao & Jeffery 1988). Again, a suitable \(M_s - L_s\) relation provides a ‘pulsation’ mass \((M_p)\). Sao & Jeffery (1988) showed that, assuming a standard \(M_s - L_s\) relation for shell-helium-burning stars, the masses \(M_p\) for five pulsating extreme helium stars lie in the range 0.77–0.95 \(M_\odot\). Using more recent spectroscopic analyses, Jeffery et al. (2001a) showed \(M_s\) to lie in the range 0.5–1.0 \(M_\odot\) (14 stars), with eight stars having \(M > 0.7 \ M_\odot\).

The weak link in these arguments is the surface gravity, which is frequently measured to an accuracy of ±0.3 dex, and is sensitive to physics in the model atmospheres. Therefore, it is important to measure extreme helium star luminosities directly. This can be achieved for pulsating stars using Baade’s method to measure the stellar radius and luminosity. Again, surface gravity is required to resolve the mass. In the case of two short-period pulsators (V652 Her and LSS3184 = BX Cir), masses of 0.5–0.7 and 0.4 \(M_\odot\) have been measured (Lynas-Gray et al. 1984; Jeffery, Woolf & Pollacco 2001b; Woolf & Jeffery 2000). These are consistent with the low masses inferred from their positions in Fig. 6.

For the more luminous helium stars, Jeffery et al. (2001a) have measured masses of 0.79 ± 0.46 and 0.94 ± 0.68 \(M_\odot\) for LS IV–1\(°2\) = V2244 Oph and HD 168476 = PV Tel, respectively. [We have adopted the value for LS IV–1\(°2\) based on the lower gravity estimate of Jeffery et al. (2001a).] The large errors in mass are primarily caused by the measurement error in \(\log g\). The corresponding radii and luminosities are more precisely measured as 31 ± 3 and 34 ± 4 \(R_\odot\) and \(\log L/L_\odot = 4.28 ± 0.06\) and 4.40 ± 0.06, respectively.

The luminosity data place these two stars directly on the tracks for 0.8(0.6) and 0.9(0.6,0.5)–\(M_\odot\) models in Fig. 2 and in remarkable coincidence with the corresponding mass measurements (0.79 and 0.94 \(M_\odot\)).

HD 168476 has both a direct mass and a contraction rate measurement that fit the models of 0.9(0.6,0.5)–\(M_\odot\) merged binary (CO + He) white dwarf evolution and contradict late thermal pulse models. While more could be done to improve the measurements, these observations are the strongest evidence yet available that extreme helium stars are the result of binary white dwarf mergers and not of evolution following a late thermal pulse in a single white dwarf.

### 6 SURFACE COMPOSITIONS

#### 6.1 Predicted abundances

In addition to the gross stellar dimensions of mass, luminosity, radius and contraction rates, a second window on to the history of highly evolved stars is provided by their surface composition. The chemicals exposed at the stellar surface effectively provide a fossil record of previous evolution, dredged to or deposited on the surface by a variety of processes. The hydrogen deficiency of extreme helium stars is the primary indicator of their extremely processed atmospheres. Previous authors have reviewed and discussed the general abundance patterns at length (Heber 1986; Jeffery 1996; Pandey et al. 2001). Principal among these are: (i) a nitrogen abundance equivalent to the combined carbon, nitrogen and oxygen abundances in any progenitor as estimated from proxies for the stellar metallicity; (ii) a high carbon abundance and occasionally a high oxygen abundance indicative of mixing from a zone affected by helium burning into the stellar envelope; and (iii) traces of hydrogen, representing a relic outer layer mixed downwards into the star. A successful model for the evolution of extreme helium stars must be able to reproduce these observables.

The theoretical models presented in this paper contain a number of assumptions, so we can only demonstrate that they are qualitatively consistent with the observations. The first assumption is that the accreted gas is the fully mixed remnant of a donor helium white dwarf. The donor is assumed to consist predominantly of helium with a mass of less than 0.4 \(M_\odot\). On its surface would have been a hydrogen layer of mass of \(\sim 10^{-3}\)–\(10^{-4}\) \(M_\odot\) (Driebe et al. 1998). All of the original carbon and oxygen in the donor would have been converted to nitrogen through the CNO cycle. From these considerations we have adopted \(Y = 0.98\) and \(X = 0.011\) for the helium and nitrogen abundances. The nitrogen abundance is the sum of the original CNO abundances. The solar abundance scaled to \(Z = 0.02\) has been adopted for the remaining heavy elements. In some models a small fraction of hydrogen \((X = 0.001)\) is included in the accreted gas.

The second set of assumptions concerns the structure of the accreting CO white dwarf. The CO core is assumed to have a mass of \(\sim 0.5–0.6 \ M_\odot\); we have shown how mass affects the subsequent evolution. Exterior to the CO core there exists a helium-rich layer (CNO-processed) with a of mass \(\sim 0.02–0.04 \ M_\odot\). We have neglected a thin hydrogen layer with a mass of some \(\sim 0.0001 \ M_\odot\), which would have surrounded the helium-rich layer. Within the helium layer there would exist considerable amounts of carbon and oxygen created during thermal pulses when the star was on the AGB (e.g. Herwig 2000). However, our initial white dwarf model has a helium zone without carbon and oxygen. In order to see the effect of carbon and oxygen in the helium zone, we have calculated some model sequences starting with initial white dwarf models with a helium zone, the composition of which is modified to include some carbon and oxygen.

First we discuss chemical compositions of evolution models started with a white dwarf model having no carbon/oxygen in the helium zone. Some carbon and little oxygen were produced during the first helium flash. During the flash, the shell convection zone extended through almost all of the envelope above the ignition zone. For the case of \(M_i = 0.6 \ M_\odot\) without hydrogen, the mass involved in the shell convection zone was \(\sim 0.025 \ M_\odot\), and the mass fractions of carbon and oxygen in the convective shell increased up to 0.12 and 2.5 \(\times 10^{-4}\), respectively, before the flash diminished. For the case with hydrogen, the mass in the shell convection zone was \(\sim 0.006 \ M_\odot\) and the maximum carbon and oxygen abundances in the shell were \(\sim 0.06\) and 2 \(\times 10^{-4}\), respectively. For the case of \(M_i = 0.5 \ M_\odot\), the corresponding quantities were \(\sim 0.01 \ M_\odot\), \(\sim 0.03\) and \(2 \times 10^{-5}\), respectively.

The smaller extent of the shell convection zone and the smaller carbon and oxygen abundances for \(M_i = 0.5 \ M_\odot\), are attributed to the weaker helium flash.

The envelope expands after the helium flash and the bottom of the convective envelope becomes deeper and dredges up matter affected by the helium burning as seen in Fig. 5, bringing carbon and oxygen to the surface. At the deepest penetration the surface carbon abundance is as high as \(\sim 0.1\), but it will be diluted considerably by the matter accreted afterwards.

Fig. 8 shows the distribution of elements within the 0.7(0.6)–\(M_\odot\) model with hydrogen. The surface carbon and oxygen abundances of this model are \(5.1 \times 10^{-3}\) and \(1.9 \times 10^{-5}\) in mass, respectively.
shows that the abundance of $^{12}\text{C}$ is lower in a range of the convection envelope and mixed with the accreted matter. Fig. 8 after the first helium flash. The processed matter was included into envelope were produced by helium burning that occurred at and close to those of the accreted matter. The carbon and oxygen in the surface hydrogen, helium and nitrogen abundances are very small amount of hydrogen.

Table 2. Surface carbon and oxygen abundances of various models.

<table>
<thead>
<tr>
<th>$M(M_{\odot})$</th>
<th>$(X_C, X_O)$ in the initial He zone</th>
<th>$(0.0, 0.0)$</th>
<th>$(0.2, 0.05)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7(0.6)</td>
<td>2.0E−2 5.2E−5 4.2E−2 3.7E−3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8(0.6)</td>
<td>8.3E−3 2.1E−5 1.7E−2 1.5E−3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9(0.6)</td>
<td>4.9E−3 1.2E−5 1.0E−2 8.8E−4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7(0.6)H</td>
<td>5.1E−3 1.9E−5 1.2E−2 1.7E−3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8(0.6)H</td>
<td>2.0E−3 7.5E−6 4.7E−3 6.4E−4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9(0.6)H</td>
<td>1.2E−3 4.4E−6 2.8E−3 3.8E−4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6(0.5)</td>
<td>4.6E−3 8.3E−6 1.4E−2 2.2E−3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7(0.5)</td>
<td>7.7E−4 1.4E−6 2.6E−3 4.0E−4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8(0.5)</td>
<td>3.2E−4 5.7E−7 1.1E−3 1.7E−4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9(0.5)</td>
<td>1.8E−4 3.3E−7 6.3E−4 9.7E−5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The symbol H indicates a model with hydrogen.

The surface hydrogen, helium and nitrogen abundances are very close to those of the accreted matter. The carbon and oxygen in the envelope were produced by helium burning that occurred at and after the first helium flash. The processed material was included into the convection envelope and mixed with the accreted matter. Fig. 8 shows that the abundance of $^{12}\text{C}$ is lower in a range of 0.675 ≤ $M_r$ ≤ 0.69. This is owing to the CNO cycle caused by the small amount of hydrogen.

The surface carbon and oxygen abundances for other cases are given (by mass fraction) in the second and the third columns of Table 2, respectively. The carbon and oxygen abundances are smaller for a larger total mass for a given initial mass $M_i$. It is explained by a dilution effect because appreciable mass accumulation occurs when the bottom of the convective envelope is receding (Figs 3–5). For the same reason the surface abundance does not change after the accretion was terminated. For a given accreted mass, the model from $M_i = 0.5 M_{\odot}$ has lower carbon and oxygen abundances, which is attributed to weaker helium burning.

The surface carbon and oxygen abundances for the cases of $M_i = 0.6 M_{\odot}$ are higher than the solar value, but the oxygen abundances are much lower. For the cases of $M_i = 0.5 M_{\odot}$, both of the surface carbon and oxygen abundances are smaller than the solar values. These surface abundances are not consistent with the carbon and oxygen abundances of the extreme helium stars and the RCrB stars. We need additional carbon and in particular oxygen.

Recent thermally pulsing AGB models with overshooting by Herwig (2000) show that carbon and oxygen abundances in the intershell zone can be as high as 0.5 and 0.2 in mass, respectively, after a thermal pulse. Although the chemical composition in the helium zone of a white dwarf depends on how soon it left the AGB after the latter thermal pulse and the chemical diffusion during the cooling phase, it is probable that a considerable carbon and oxygen enrichment exists in the helium zone. In order to see the effect of additional carbon and oxygen we have calculated models starting with white dwarf models in which carbon and oxygen abundances in the helium zone are artificially modified to $(X_C, X_O) = (0.2, 0.05)$. This value was chosen in part to be conservative, but also because models with higher carbon abundances did not converge. The important point is that theoretical models of AGB evolution predict there to be ample carbon available in the helium shell of the CO white dwarf for subsequent mixing in the merger model.

The resulting surface carbon and oxygen abundances are given in the fourth and fifth columns of Table 2. Fig. 9 shows the distribution of elements within the 0.7(0.6)-$M_{\odot}$ model. The existence of carbon and oxygen in the helium zone of the initial dwarf increases surface carbon and oxygen abundances at the surface of the merged star. In particular, it is necessary to have a reasonable oxygen abundance at the surface. Obtained surface carbon and oxygen abundances for the cases of $M_i = 0.6 M_{\odot}$ are comparable with the observed abundances of luminous extreme helium stars and RCrB stars (Table 4 below). Although the difference in $M_i$ did not appreciably change the evolution track in
the HR diagram and the rate of contraction as long as \( M \geq 0.8 \, M_\odot \), models with \( M_\ast = 0.5 \, M_\odot \) have considerably smaller carbon and oxygen abundances compared with the cases of \( M_\ast = 0.6 \, M_\odot \). From the results of our models we may conclude that to have surface abundances consistent with the luminous extreme helium stars and RCHB stars, the initial CO white dwarf should be \( \sim 0.6 \, M_\odot \) and considerable carbon and oxygen should exist in the helium zone of the initial CO white dwarf. The latter requirement is consistent with recent AGB models (Herwig 2000).

Some additional carbon and oxygen could be brought from the outer part of the CO white dwarf during the violent merger process. Hydrodynamic simulations (cf. Benz et al. 1990) will be required to study the merger event and accretion process in detail to determine how much carbon will be dredged up and what additional nucleosynthesis may occur. For example, the mixing of protons, helium and carbon nuclei at very high temperatures could result in the production of new nitrogen, neon, s-process and other exotic elements.

### 6.2 A simple recipe

Since our theoretical models do not treat the complete sequence of mixing events envisaged in the CO + He white dwarf merger, a simple recipe similar to that described by Pandey et al. (2001) is useful. The recipe model is a mixture of several layers that are supposed to contribute to the surface composition of the merger product.

In the conservative case for the 0.9(0.6)-M\(_\odot\) model, the donor white dwarf would have to have been predominantly helium with a mass \( m_{\text{He:He}} = 0.3 \, M_\odot \). On its surface would have been a hydrogen-rich envelope of mass \( m_{\text{He:H}} \sim 10^{-3} - 10^{-4} \, M_\odot \) (Driebe et al. 1998), reduced to this value by Roche-lobe overflow during its first ascent of the giant branch. All of the original carbon and oxygen in the helium core of the donor would have been converted to nitrogen through the CNO cycle. Here and in the following the notation WD:layer has been used to identify the various layers of the original white dwarfs. For example \( m_{\text{He:H}} \) refers to the mass of the hydrogen-rich layer in the helium white dwarf.

The outer part of the accreting white dwarf with a mass of \( 0.5 - 0.6 \, M_\odot \) would be mixed during the violent merger process or by a deep convection envelope. According to canonical post-AGB evolution models, superimposed on the CO core will be a CNO-processed hydrogen-rich layer (with a mass \( m_{\text{CO:CO}} \sim 0.02 \, M_\odot \)), and a hydrogen-rich layer with a mass of some \( m_{\text{CO:He}} \sim 0.001 \, M_\odot \). Within the helium layer will be a rich mixture of ‘old’ s-process elements, carbon and oxygen, created during thermal pulses when the star was on the AGB (e.g. Herwig 2000).

Thus the ingredients of our recipe model comprise five layers with mass

\[
m_{\text{He:He}} + m_{\text{He:H}} + m_{\text{CO:He}} + m_{\text{CO:CO}}.
\]

Table 3. Surface abundances predicted from a simple mixing recipe for merged CO + He white dwarfs. Two models are illustrated: (1) assumes no CO enrichment in the He-rich layer of the CO white dwarf, (2) assumes the same enrichment as the evolution models and no mixing with the CO core.

<table>
<thead>
<tr>
<th>layer</th>
<th>He:H</th>
<th>He:He</th>
<th>CO:H</th>
<th>CO:He</th>
<th>CO:CO</th>
<th>mix</th>
<th>(1) CO:He</th>
<th>CO:CO</th>
<th>mix</th>
<th>(2) CO:He</th>
<th>CO:CO</th>
<th>mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m/M_\odot )</td>
<td>0.00002</td>
<td>0.3</td>
<td>0.00002</td>
<td>0.02</td>
<td>0.007</td>
<td>0.327</td>
<td>0.03</td>
<td>0</td>
<td>0.00009</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>( \beta(\text{H}) )</td>
<td>0.710</td>
<td>0</td>
<td>0.71</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00009</td>
<td>0</td>
<td>0</td>
<td>0.00009</td>
</tr>
<tr>
<td>( \beta(\text{He}) )</td>
<td>0.280</td>
<td>1.00</td>
<td>0.28</td>
<td>0.996</td>
<td>0</td>
<td>0.975</td>
<td>0.71</td>
<td>0</td>
<td>0.970</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta(\text{C}) )</td>
<td>0.0015</td>
<td>0</td>
<td>0.0015</td>
<td>0</td>
<td>0.80</td>
<td>0.017</td>
<td>0.20</td>
<td>0.80</td>
<td>0.0183</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta(\text{N}) )</td>
<td>0.0005</td>
<td>0.0020</td>
<td>0.0005</td>
<td>0.0020</td>
<td>0</td>
<td>0.0019</td>
<td>0.0020</td>
<td>0</td>
<td>0.0020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta(\text{O}) )</td>
<td>0.0059</td>
<td>0</td>
<td>0.0059</td>
<td>0</td>
<td>0.20</td>
<td>0.0043</td>
<td>0.05</td>
<td>0.20</td>
<td>0.0046</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta(\text{Ne}) )</td>
<td>0.0009</td>
<td>0</td>
<td>0.0009</td>
<td>0</td>
<td>0.0029</td>
<td>0.0010</td>
<td>0.0349</td>
<td>0.0029</td>
<td>0.0041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta(\text{Fe}) )</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0006</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values for \( \beta \) following hydrogen burning by the CN and CNO cycles and helium burning by \( 3\alpha \) and \( C-\alpha \) captures can be written in roughly linear form, and can be extended to include other processes (e.g. pp burning, s processing, further proton and \( \alpha \) captures) as required. Within this simple recipe, the relative contribution of the CNO cycle to the combined CN and CNO cycles during hydrogen burning is specified by \( f_{\text{CNO}}(f_{\text{CN}} + f_{\text{CNO}}) = 1 \). The carbon-to-oxygen ratio following \( 3\alpha \) and \( ^{12}\text{C}(\alpha,\gamma)^{16}\text{O} \) is given by \( f_{\text{CNO}}(f_{\text{CN}} + f_{\text{CNO}}) = 0.8 \). A further parameter \( \beta_{\text{C:AGB}} \) specifies the carbon enrichment of the He shell of the CO white dwarf and the oxygen abundance follows the carbon enrichment as above. We have adopted two test values for \( \beta_{\text{C:AGB}} = 0 \) and 0.2, as adopted in the evolutionary calculations. Finally, a parameter \( \beta_{\text{old}}/\beta_{\text{old}}(\sim 10) \) indicates the level of s-process production during AGB evolution.

### 6.3 Observed abundances

The recipe parameters have been adjusted to obtain \( \beta \) reasonably consistent with observations of surface abundances in real extreme helium stars. With the adopted parameters, the composition of each layer is given in Table 3. Table 4 compares the result with observational data from Pandey et al. (2001), and the results from the theoretical models.

Neither the numerical models nor the simple recipe treat the CO:He shell well, particularly with regard to the dredge-up of the production of carbon, oxygen and s-process elements in AGB stars. Secondly, the models and recipes do not make any allowance for nucleosynthesis during the merger event. The simple model leads
to the ingestion of protons and carbon into the same hot material, with the potential to make nitrogen. Fresh nitrogen in a helium-rich environment is capable of producing additional neon, which is enhanced in the helium shell during AGB evolution.

An important result is that the observed carbon abundances require that either $m_{\text{CO,CO}} \sim 0.004–0.012\,M_\odot$ be dislodged from the surface of the CO white dwarf or the observed surface carbon was produced by thermal pulses in the AGB stage. In the latter case the corresponding carbon abundance in the helium zone (CO:He) should be $0.1–0.3$, which is consistent with the AGB models by Herwig (2000), Pandey et al. (2001) note the remarkable consistency of C/He ratios in extreme helium stars that all lie between 0.3 and 1 per cent (by number; 0.9 and 3 per cent by mass).

With $m_{\text{CO,He}} \ll m_{\text{He,He}}$, old s-process elements do not contribute significantly to the new stellar surface. Pandey et al. (2001) report that two cool extreme helium stars and the related RCrB stars show mild to undetectable enhancements of yttrium and barium.

The simple recipe has difficulty in accounting quantitatively for three important observations: the extremely low hydrogen abundance, the high neon abundance and the large star-to-star variation in oxygen.

To account for the observed hydrogen abundance, $m_{\text{He,He}} + m_{\text{CO,He}} \ll 5 \times 10^{-2}\,M_\odot$. However, the hydrogen-rich envelope of a helium white dwarf is thought to be an order of magnitude larger than this (Driebe et al. 1998). Either hydrogen is ejected from the binary before the merger or it is destroyed by proton capture during the merger.

There is a possibility that the high neon abundance is an observational problem; this is discussed by Jeffery & Heber (1993), Jeffery (1998) and Pandey et al. (2001), but more work is necessary. Herwig et al. (1999) show that $\alpha$-captures on $^{14}$N can produce a mass fraction of 3.5 per cent of neon in the helium-rich layer of an AGB star. Incorporating this in our simple recipe (2) for the merger model yields a surface abundance of neon comparable with that seen in the majority of RCrB stars (see Table 4).

The large variation ($\approx 2$ dex) in oxygen abundance from one star to another is difficult to explain while at the same time keeping the fraction of carbon within the tight constraints already discussed. A possible explanation is provided by the observation that the C/O ratio in the helium layer depends on the number of thermal pulses during previous evolution on the asymptotic giant branch (Herwig 2000). The wide range of C/O ratios in EHe stars may simply point to a range of progenitor AGB star masses.

The cooler RCrB stars have been putatively identified as the direct antecedents of extreme helium stars (Schönberner 1977), although the connection remains contentious. Asplund et al. (2000) discuss the surface abundances of RCrB stars. Pandey et al. (2001) compare the surface abundances of extreme helium stars and RCrBs and find them to be broadly similar. It is not clear whether the significant differences, notably in nitrogen and neon, have an observational or astrophysical origin. The models (and recipe) described in this paper apply equally to RCrB stars and to extreme helium stars, if they are to be considered as CO + He white dwarf mergers. It is clear that the models need further refinement in order to explain the detailed surface compositions of both groups. However, it is equally clear that they do a very good job in accounting for the general properties of the surface composition of these stars.

### 7 SOME STATISTICS

In order to pursue the thesis that extreme helium stars are the progeny of mergers between a CO and a He white dwarf, there must be reasonable evidence that such mergers take place with the necessary frequency. The question of close binary white dwarf frequency has been examined extensively (Iben et al. 1996; Han 1998; Nelemans et al. 2001). Most recently, Nelemans et al. (2001) deduced that 20 per cent of all white dwarf pairs consist of a CO and He white dwarf and that the current merger rate for all white dwarf pairs is $2.2 \times 10^{-2}\,\text{yr}^{-1}$ in the Galaxy, or $4.4 \times 10^{-3}\,\text{yr}^{-1}$ in the Galaxy for CO + He mergers (Iben et al. 1996 give $2.32 \times 10^{-3}\,\text{yr}^{-1}$).

Both theoretical models and empirical evidence suggest that the heating rate for extreme helium stars with $T_{\text{eff}}$ between 10 000 and 40 000 K ranges from 10 to 100 Kyr$^{-1}$, depending on mass. This gives evolutionary time-scales of between 3000 and 300 yr. Combining the merger rate and evolutionary time-scale gives an estimate for the number of extreme helium stars in the Galaxy within this temperature range of between 13 and 1.3.

Jeffery et al. (1996) catalogue all known extreme helium stars. Excluding the low-luminosity stars V652 Her and HD 144941 which may be He + He mergers, but including the hot RCrBs MV Sgr and DY Cen, there are 17 extreme helium stars with $T_{\text{eff}}$ between 10 000 and 40 000 K. Four more lie outside this temperature range. Galactic surveys for extreme helium stars are essentially complete (Drilling 1986); they have a bulge distribution so that only a handful are likely to remain undetected in the Galactic plane.
According to these statistics, there appear to be too many extreme helium stars. However, given the number of approximations made in the derivation of merger rates and the small numbers involved, the situation is not irredeemable.

For stars cooler than 10 000 K, theoretical evolution time-scales (Fig. 5) are $\sim 10^3$ yr, so there should be $\sim 30-300$ times as many cool CO + He merger products. There are estimated to be 200–1000 RCrB stars and non-variable cool Hdc stars (Lawson et al. 1990), although only 33 RCrBs and Hdc are actually known in the Galaxy (Jeffery et al. 1996). The merger rate for these would then be $\sim 2 \times 10^{-5} - 10^{-2}$ yr$^{-1}$. The value of Nelemans et al. (2001) is at the low end of this range; the upper value would result in 3–30 extreme helium stars in the Galaxy.

8 CONCLUSION

We have calculated the evolution of a star formed by the accretion of helium-rich material on to a carbon–oxygen white dwarf. After shell-helium ignition the star expands to become a yellow giant, where it will stay for $\sim 10^3$ yr if 0.1–0.4 M$_\odot$ is available for accretion. When the helium is exhausted, blueward evolution occurs at high luminosity. The models were calculated in an effort to simulate evolution following merger of a helium white dwarf with a carbon–oxygen white dwarf, and have been compared with observations of extreme helium stars and, to a lesser extent, R Coronae Borealis stars.

The distribution of most extreme helium stars in the log g–T$_{\text{eff}}$ diagram agrees well with merger models having CO white dwarf masses of 0.6 M$_\odot$ (or 0.5 M$_\odot$ and He white dwarf masses of 0.2–0.3 M$_\odot$ (0.3–0.4 M$_\odot$). A small number of helium stars have lower L/M ratios and may be the products of He + He white dwarf mergers or of lower-mass CO white dwarf mergers. Luminosities, secular contraction rates and masses have been measured directly for a few extreme helium stars and are in excellent agreement with our model for a 0.6 M$_\odot$(CO) + 0.3 M$_\odot$(He) [or 0.5 M$_\odot$(CO) + 0.4 M$_\odot$(He)] white dwarf merger. They are too luminous and too massive to be, for example, products of a late thermal pulse evolution.

Since the models make ab initio assumptions concerning the composition of the accreted matter, they are not completely informative concerning the expected surface compositions of merger products; detailed hydrodynamical calculations will be necessary to probe the physics and outcome of the merger process itself. Meanwhile, we have shown both quantitatively and qualitatively that a CO + He merger can account for most of the surface composition characteristics of extreme helium stars and RCrBs. We have demonstrated that the enhanced carbon and oxygen abundances are probably derived from CO enrichment in the helium layer of the CO white dwarf. This enrichment would have been produced by thermal pulses during the original AGB evolution of the progenitor (Herwig 2000).

Statistically, the product of predicted CO + He merger rates and the evolutionary lifetimes of extreme helium stars are compatible with observed numbers.

While some details need to be refined, our models for the accretion of helium by CO white dwarfs can reproduce qualitatively nearly all of the observables of extreme helium stars. No other evolution model can currently do this. We conclude that the CO + He white dwarf merger provides the most complete and satisfactory model for the creation of the majority of extreme helium stars and, by association, R Coronae Borealis stars.

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