Research Article

Structure Partition and Reasonable Width Determination of Waterproof Coal Pillar in Strip Mining

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Retaining a waterproof coal pillar is the most effective water conservation method for a roadway close to the gob, and determining a reasonable width of the waterproof coal pillar has been a common problem among mining scholars for a considerably long time. Based on the mining of the 15208 mining face in Xinjing Coal Mine, the structure of waterproof coal pillar is divided into a mine-pressure-influenced plastic zone, an effective waterproof elastic core zone, and a water pressure failure zone. The mine-pressure-influenced plastic zone width is determined by using the limit equilibrium theory, the parabolic strength theory, and the separation variable method. The effective waterproof elastic core zone width is determined by the semi-inverse solution method, and the water pressure failure zone width is determined by considering the infiltration and softening of water. After that, combined with the previous theoretical analysis of engineering examples, the theoretical value of waterproof coal pillar width is obtained. In addition, the physical shape distribution of the waterproof coal pillar is measured by ultrasonic detection technology. The results are consistent with the field measured results. The correctness of the model is verified. Finally, the rationality of the model is verified by comparing with the previous classical models. The research results are applied to the design of the waterproof coal pillar in Xinjing Coal Mine, which could provide a theoretical basis for determining the width of the waterproof coal pillar located close to a gob.

1. Introduction

Due to its effective control of surface subsidence, the strip mining method is one of the most commonly used mining methods in ‘three-body’ coal mining [1]. Therefore, strip mining is widely used at coal mines in China. In case of coal seam mining, the stresses of the surrounding rocks are redistributed, causing loosening deformation and failure fractures [2–5]. Because of the different geological and hydrogeological conditions, water inrush accidents rarely happened in some major coal mining countries in the world, such as the USA, Russia, Australia, India, South Africa, Poland, Germany, Canada, and Indonesia [6, 7]. In contrast, water inrush accidents often occur in coal mines in China [8]. After more than half a century of mining, the shallow coal resources in China have been exhausted. Most of the coalmines have entered deep mining. The geological and hydrogeological conditions are more complex, and mine water inrush accidents often occur [9, 10]. In China, from 2004 to 2015, the number of casualties resulted by water accounted for 8.16% in the whole coal mine accidents. And the number of deaths reached 2833 [11], see Figure 1. In 2008, 707 deaths occurred in coal mines in China, while 135 deaths caused by mine water inrush accidents, accounting for 19.1% of the total, were second only to those caused by gas accidents. According to incomplete statistics [12], there were 167 water accidents in coal mines in China from 2010 to 2016, and 710 died. The massive water accidents indicated that controlling the roadway stable has
already become the urgent problem [13, 14]. When the water conducting fractures connect with a water body, such as water in goaf, the water would flow into the roadway, resulting in water inrush accidents at mines [15–17]. Currently, retaining a waterproof coal pillar is considered to be the most effective method for strip mining [18]. Therefore, a reasonable design of a waterproof coal pillar has long been an important research topic in China [19, 20].

To achieve this objective, the main requirement is to maintain the stability of waterproof coal pillars. Many methods to analyze the stability of coal pillar have been proposed. These methods can be divided into three types. The first type of method includes numerical simulation methods [13, 14, 21–25]. In this method, numerical simulation software is used to simulate the coal pillar and its surrounding rock mass, and the stability of the coal pillar is then analyzed. Depending on the numerical simulation method, it is difficult to program the relationship between the coal pillar and quantitative data [26–28]. The second type is based on empirical engineering. Using the English design formula [29, 30], the Pennsylvanian empirical formula [31, 32], or the Chinese regulation for mining under a waterbody [33] produce various empirical formulas for the same geological and hydrogeological conditions. If the coal pillars were too small, water inrush can occur, with serious consequences, such as mine inundation, endangering the lives of miners, and surface collapse. If the coal pillars were too large, coal resources were lost [34]. At present, the existing calculation formula of the width of the waterproof coal pillar is the empirical formula in the regulations of coal mine water prevention and control, but its failure effect on coal pillar caused by coal pressure is not taken into account [35]. On the one hand, the coal pillar size designed according to the experience method is not scientific and the theoretical basis is insufficient. On the other hand, although the safety factor is taken into account, the accidents of waterproof coal pillar destruction and water source communication still occur. In addition, because there are many factors affecting the width of waterproof coal pillars, and the traditional empirical formulas are relatively single, they cannot effectively conform to the actual situations of the mines. Therefore, in actual production, it is very likely that the safety width of the waterproof coal pillar is not enough, resulting in mine fault water inrush and other disasters. The third type includes several theoretical methods. Scholars have researched the determination of the width of the waterproof coal pillar in the cases of coal seam mining under an aquifer and adjacent to water-conducting fault and that of the section coal pillar, which are listed in Table 1 [18]. In these research studies, the failure characteristics of the surrounding rocks that can be attributed to coal seam mining were considered; furthermore, the mine-pressure-influenced plastic zone and effective waterproof elastic core zone were established on the side of a coal pillar near the roadway, and the failure range caused by high water pressure in deep-buried strata or fault was considered on the side of the influence sources [36–39]. Of the theoretical methods, the limiting equilibrium method is the most suitable and has been widely studied. However, most studies have not considered the nature of the coal and have only assumed the coal pillars be a flat model. Therefore, further studies are necessary.

In mine engineering, the failure form of the waterproof coal pillar is more complex than that of general coal pillar. If the waterproof coal pillar is simplified as simply supported beam model under uniform load like the general coal pillar and the width of the waterproof coal pillar is obtained, the simplified process of the model is still open to question [40]. These theoretical methods have contributed to more effective waterproof coal pillars and greater mine safety. However, existing theory and methods were unable to fully accommodate many real-world scenarios, so that the appropriate size of waterproof coal pillars along water in goaf is still a viable, multidisciplinary, research topic.

Therefore, we used a 15# coal seam in the Xinjing mine as the study project and studied the failure characteristics of the overlying strata during the mining process using theoretical analysis. Furthermore, we analyzed the variations in stress, displacement, and pore water pressure during mining and established a reasonable waterproof coal pillar width. Our

Figure 1: The number of casualties in Chinese coal mines from 2004 to 2015 [11].
Table 1: Determination methods of the widths of waterproof coal pillars in different conditions [18].

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results could provide a theoretical basis for determining the widths of the waterproof coal pillar close to a gob, eliminating the hidden trouble of the mine and realizing the safety production of the coal mine.

2. Engineering Background

The Xinjing Coal Mine is located in Yangguan City, Shanxi Province (Figure 2(a)). This coal mine mainly exploits coal seam 15#. The mean depth is 460 m, and the thickness is 6.66 m. The dip angle of this coal seam is 8 degrees. Longwall face 15028 is located in the Northeast of District One. The north is the gob 15027 and the south is longwall face 15029 that is going to be exploited. The longwall face layout is shown in Figure 2(b). The 15028 tailgate (width 5.2 m, height 3.1 m and length 1350 m) is excavated along the gob 15027, and the pillar width is 20 m. The roadway is arranged along the coal seam roof. The comprehensive strata diagram is shown in Figure 2(c). The immediate roof is sandy mudstone with a thickness of 2.40 m, and the main roof is limestone with a thickness of 10.07 m. Between these two rock mass layers, there is a coal seam with a thickness of 0.21 m. The immediate floor is sandy mudstone with a thickness of 4.98 m, and the main floor is medium sandstone with a thickness of 7.54 m.

However, the mining work is often affected by the water damage in the five major mine hazards (the water source is mostly Quaternary Ordovician ash confined water layer), which makes the production of mining area and the safety of personnel life greatly threatened. The coal quality is medium hardness (compressive strength 6.99 MPa, tensile strength 0.44 MPa), and the water pressure of water inrush in reserved coal pillar face is 1.0 ~ 2.2. There are no other communication water source channels near the working face, and the top and bottom plates completely accord with the model conditions in this paper.

In practical engineering, the stability of a waterproof coal pillar is determined by its width. Consequently, analyzing the width of the waterproof coal pillar is the main objective. It is worth noting that to ensure the accuracy of the waterproof coal pillar width, the coal sample of the corresponding mine should first be measured and its own characteristics mastered. The compression testing of coal specimens with water content based on stiffness experiments and in situ tests [41] shows that the deformation and destruction of coal are part of a complicated and progressive process. Consequently, its stress-strain curve can be represented simply by an ideal elastoplastic strain softening model, which is shown in Figure 3.

3. Theoretical Analysis of the Width of the Waterproof Coal Pillar

As for the division of the failure zone of the coal pillar, the Willson two-zone constraint theory divides the coal pillar into two plastic failure zones and the central elastic core zone according to the different mechanical properties of the actual coal pillar [42]. However, for the reasonable width design of the waterproof coal pillar, a special factor should be considered, that is, the role of water, in case of a coal pillar located close to a gob with water (Figure 4). Therefore, a waterproof coal pillar can be divided into a mine-pressure influenced plastic zone \( r_p \), an effective waterproof elastic core zone \( r_e \), and a water pressure failure zone \( r_w \). In order to solve the problem of reasonable width of waterproof coal pillar, it is necessary to model and solve different damage zones, respectively, and finally determine the calculation equation of overall width of waterproof coal pillar.

In the strip mining, the overlying strata are basically in the form of integral movement [43]. Therefore, the rock mass can be approximately regarded as a continuous homogeneous elastic body. If studying one individual mining strip alone, it can be described by the model of the infinite laminated anisotropic plate with a rectangular hole. The previous studies show that [44] the stress at the tip of the rectangular
hole is similar to that of the ellipse hole when the ratio of the mining strip width to its height and the distance of the computational point to the tip of the hole are all large enough. Moreover, the effect of anisotropy on the stress at the tip of the ellipse hole is very little, especially when the mining strip height is small. Therefore, the model of the infinite laminated anisotropic plate with a rectangular hole can be simplified to the model of the infinite isotropic plate with an ellipse hole, which is as shown in Figure 5.

The simplified mechanical model can be solved by the complex function method of the elastic mechanics. The expressions of stress at any point along the elliptic hole edge of the x-axis are [45]

\[
\sigma_x = \frac{q}{2(mk^2 - 1)} \left\{ \frac{1}{(m \xi^2 - 1)} \left[ \frac{1}{(m \xi^2 - 1)} \left( m - \xi^2 \right) \right] \right. \\
\left. \cdot \left( \frac{1}{(m \xi^2 - 1)} \left( m - \xi^2 \right) \right) \right\},
\]

(1a)

\[
\sigma_z = \frac{q}{2(mk^2 - 1)} \left\{ \frac{1}{(m \xi^2 - 1)} \left[ \frac{1}{(m \xi^2 - 1)} \left( m - \xi^2 \right) \right] \right. \\
\left. \cdot \left( \frac{1}{(m \xi^2 - 1)} \left( m - \xi^2 \right) \right) \right\} + (2 + m) \xi^2 + 1,
\]

(1b)
where $a$ is the half-width of the mining strip, $r$ is the distance of the computational point to the tip of the hole, and $q$ is the vertical overburden pressure.

In this paper, the similar mechanical model is established for the actual mining and occurrence of strip mining, and the stability of the strip coal pillar is analyzed theoretically. In order to show that the final research results of this paper have some scientific research, the rationality of a series of approximate simplified mechanical models carried out in the process of the previous research has been further verified in the following paper. As a result, the model is no longer concerned about the error caused by its simplified results.

In order to further illustrate the accuracy of the results obtained by the Equations (2a) and (2b) for the vertical stress of the coal pillar, the difference ratio, $\eta$, between the original theoretical Equations (1a) and (1b) and the simplified Equations (2a) and (2b). The error rate, $\eta = (\sigma_{2(1b)} - \sigma_{2(2b)})/\sigma_{2(1b)}$, is calculated.

As can be seen from Figure 6, the higher the values of $a/b$ and $r$, the smaller the difference rate calculated by the two Equations. In addition, in order to further explain that the rectangular goaf in the actual strip mining is simplified to the ellipse seam in this paper, it has little effect on the calculation results of vertical stress on the edge of coal pillar. In this paper, the vertical stress at the edge of at the edge of the rectangular hole which is close to the shape of the goaf is calculated by the boundary element method and compared with Equations (2a) and (2b), respectively. The variation trend of the difference of the result is shown in Figure 7. We can see that even if the $a/b$ and $r$ values are small, the difference ratio of the two calculated results is still small.

In conclusion, in combination with the variation trend of the difference rate of Figures 6 and 7, the vertical stress of the coal pillar is calculated by using the simplified Equations (2a) and (2b), and the result can meet the actual accuracy requirement of the project.

Because there are usually many coal mining faces in strip mining engineering, it can be assumed to be an inclined colinear crack with a certain distance between infinite plates. According to this, the strip mining area is composed of $N$ cracks, considering the mutual influence between adjacent working faces, according to the concept of strength factor of fracture mechanics theory. The expression of the edge stress of the working face studied should be the product of the edge stress of a single crack multiplied by the stress increase factor $F$ related to the stress intensity factor. Based on the analysis and calculation of the actual data of strip mining, the stress increase factor $F$ ($1 < F < 2$) can be expressed as follows:

$$ F = \left( \frac{0.595 + 0.875L}{L + B} \right) (0.9831 + 0.0106N). $$ (3)

When Equation (3) is multiplied by Equations (2a) and (2b), respectively, the expression of the edge stress of the working
face studied in the presence of multiple mining areas is obtained.

\[ \sigma_x = \frac{F_q [1 - (a + r)]}{\sqrt{r(r + 2a)}}, \quad (4a) \]
\[ \sigma_z = \frac{F_q (a + r)}{\sqrt{r(r + 2a)}}, \quad (4b) \]

3.1. Establishment and Solution of Mine-Pressure-Influenced Plastic Zone. The compression testing of coal specimens based on stiffness experiments and in situ tests shows that the deformation and destruction of coal are part of a complicated and progressive process. Site tests have shown that a waterproof coal pillar will deform under the pressure caused by mining. Generally, for the high stress at the edge of the waterproof coal pillar, the waterproof coal pillar will enter the plastic state. And thus, the stress at the edge of the waterproof coal pillar will decrease. The reduction of stress will extend from the edge to the inner of waterproof coal pillar, until the stress can be borne by the waterproof coal pillar. When the both sides of waterproof coal pillar were all mined out, the distribution of the vertical stress in both sides of the waterproof coal pillar were as shown in Figure 8, where \( \sigma^*_{c} \) is the residual single compression strength of the coal material.

Based on this analysis, the mine-pressure-influenced plastic zone is related to the plastic failure zone in the coal seam around the roadway. According to the engineering experience and previous study [36], most of the nonelastic zone for the waterproof coal pillar is in the plastic rheological state; thus, the stress in this zone will not increase. In other words, the bearing capacity of the waterproof coal pillar in the plastic softening state is limited. Generally, the nonelastic zone which is created is nonstable, because it does not satisfy the limit equilibrium condition. To satisfy the limit equilibrium condition, the nonelastic zone will extend to the inner of the waterproof coal pillar. The coal mass in the mine-pressure-influenced plastic zone is in the limit equilibrium state. Therefore, the width of the limit equilibrium zone

\[ \text{Figure 6: Error rate between the theoretical equation and simplified equation.} \]

\[ \text{Figure 7: Error rate of the rectangular goaf and simplified ellipse goaf.} \]

\[ \text{Figure 8: Distribution of the vertical stress for the waterproof coal pillar in the limit equilibrium state.} \]
should be determined. Considering that the gravity stress \( q \) has no effect on the yield of the waterproof coal pillar, it is obtained by Equation (4b):

\[
\sigma_{zz} + q = \frac{Fq(a + r_n)}{\sqrt{r_n(r_n + 2a)}}. \tag{5}
\]

When the limit equilibrium condition is satisfied, the distribution of vertical stress in the coal pillar can be shown as in Figure 8. In Figure 8, \( r_n \) is the ultimate width of the non-elastic zone, and \( r_o \) is the width of the loose zone for the waterproof coal pillar. Those all can be obtained by the limit equilibrium method based on the stress equilibrium theory of loose media and the stress equilibrium Equation.

At this moment, the width of the non-elastic zone for the waterproof coal pillar can be described as follows:

\[
r_n = a \left[ 1/\sqrt{1 - \left( \frac{Fq}{\sigma_{zz} + q} \right)^2} - 1 \right], \tag{6}
\]

where \( a \) is the half-width of the mining strip, \( F \) is the stress increase factor, and \( q \) is the vertical overburden pressure.

In this study, based on the previous studies [46], the ultimate width of the loose zone can be described as follows:

\[
r_o = \left[ \frac{mR_{\mu\gamma HP} + \sqrt{(mR_{\mu\gamma HP})^2 + 4sR_{\gamma}^2(G\gamma HP/2K)^2}}{(2G\gamma HP/K)^2} \right]^{-2} \tag{7}
\]

where \( A = (\lambda \cos^2 \alpha + \sin^2 \alpha) \cos (\theta/2) \), \( B = \sin (\theta/2) \sin (3\theta/2) \), \( D = \cos (\theta/2) \cos (3\theta/2) \), \( K = \gamma H/\sqrt{W_p} \), \( C = (1 - \lambda) \sin \alpha \cos \alpha \sin (\theta/2) \), \( E = (\lambda \cos^2 \alpha + \sin^2 \alpha) \sin (\theta/2) \), \( F = (1 - \lambda) \sin \alpha \cos \alpha \cos (\theta/2) \),

\[
P = \sqrt{\frac{2(W_z + W_p)}{\pi}} \tan \frac{\pi W_z}{2(W_z + W_p)}, \tag{8}
\]

\[
G = \left\{ K(A - C) + K \{ [A \cdot B + C(1 + D)]^2 + [ED + F(1 - B)]^2 \} \right\}^{1/2} - 2\mu K(A - C) \right}^2.
\]

Therefore, the mine-pressure-influenced plastic zone width of the waterproof coal pillar is

\[
r_p = r_n - r_o = a \left[ 1/\sqrt{1 - \left( \frac{Fq}{\sigma_{zz} + q} \right)^2} - 1 \right] - \left[ \frac{mR_{\mu\gamma HP} + \sqrt{(mR_{\mu\gamma HP})^2 + 4sR_{\gamma}^2(G\gamma HP/2K)^2}}{(2G\gamma HP/K)^2} \right]^{-2} \tag{9}
\]

3.2. Establishment and Solution of Effective Waterproof Elastic Core Zone. The effective waterproof elastic core zone not only plays a major role in preventing the water in the goaf flowing into the roadway but also plays the role of bearing the effective mine load in the upper part. According to the theory of stress redistribution, the upper load of the coal pillar changes after the formation of goaf in stope, and most of the overburden load is transferred to the effective bearing area of coal pillar, that is, the elastic core area. At the same time, the three-axis compression state is formed on both sides of the core region. When the vertical stress at the top of the coal pillar reaches \( \sigma_{zz} \) of the plastic rheological strength of the coal body, the plastic failure of the coal pillar occurs, and the horizontal stress is lateral pressure \( \sigma_{xb} \), \( \sigma_{xb} = \lambda \sigma_{zz} \) which is the boundary stress between the limit equilibrium zone and the elastic core zone, also known as the constraint stress. \( I \) is the lateral pressure coefficient. Hence, the confining pressure in the elastic core region is the confined stress \( \sigma_{xb} \), the axial pressure is the equivalent ore load,

\[
p_z = \frac{(1 - (b/1.2H))b + r_f}{r_e} \gamma H. \tag{10}
\]

\( p_z \) is the effective mine load or mine pressure, \( b \) is the width of the working face, \( g \) is the average bulk density of overlying strata, and \( H \) is the buried depth of coal seam.

In order to simplify the solution of the model, the assumption of the elastic core zone in the center of the waterproof coal pillar is as follows (Figure 9):

1. The pillar is a uniform and continuous elastomer.
(2) The upper part of the coal pillar has the uniform distribution of ore pressure \( p_z \) and assumes that the elastic core area bears all the mine loads.

(3) The water pressure varies little along the height of the coal pillar, and it is considered that the soaking side of the elastic core area is subjected to uniform water pressure \( p_z \). In fact, due to the fracture through the water pressure failure area, loose collapse state, when the water catchment to the outside of the elastic core area, the head pressure value is not weakened, the action water pressure is still calculated according to the actual detection value.

(4) Binding \( \sigma_{xb} \) with uniformly distributed action of the elastic nuclear zone on both sides.

For the elastic core region model, the stress function is determined according to the stress distribution characteristics and basic assumptions of the model, and all the stress components of the model are solved according to the stress boundary conditions and symmetry,

\[
\begin{aligned}
\sigma_x &= \sigma_{xb} + \frac{p_x}{r_e} \\
\sigma_z &= p_z \\
\tau_{xz} &= -\frac{p_z}{r_e}
\end{aligned}
\]  

(11)

At the same time, in order to solve the reasonable width of elastic core region, the model failure strength criterion should be introduced. Because the coal body is a SD material with different tensile and compressive properties, and the elastic core is in a three-direction stress state, it is necessary to consider the different characteristics of tensile and compressive strength of the material and the influence of intermediate principal stress on the strength of the material. At the same time, for the convenience of solving the model, the strength criterion should have a linearization form. Taking into account the existing failure criteria, this paper adopts the unified strength criterion under the generalized compression condition, which is as follows:

\[
\sigma_1 - \frac{\alpha}{1 + \beta} (\beta \sigma_2 + \sigma_3) = -\sigma_1,
\]

where \( \alpha \) is the material compression ratio, \( \beta \) is the criterion parameter, and the value range is \( 0 < \beta < 1 \).

\( \varepsilon_x = 0, \gamma_{xz} = 0, \gamma_{xy} = 0 \) for this plane strain model. The principal stress matrix is

\[
\begin{pmatrix}
\sigma_x - \sigma & 0 & \tau_{xz} \\
0 & \sigma_y - \sigma & 0 \\
\tau_{xz} & 0 & \sigma_z - \sigma
\end{pmatrix} = 0.
\]

(13)

Combined with the generalized Hoek theorem, the three principal stresses of the plane strain model in this coordinate system are obtained:

\[
\begin{align*}
\sigma_1 &= \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} \\
\sigma_2 &= \mu(\sigma_x + \sigma_z) \\
\sigma_3 &= \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}
\end{align*}
\]

(14)

where \( \mu \) is Poisson’s ratio of the coal mass. It is generally believed that Poisson’s ratio tends to limit 0.5 in the case of rock mass failure.

In order to obtain the reasonable width of the elastic core region, based on the failure strength criterion of the elastic core region, the stress function of the model is substituted into the equation (12), in which the principal stress is calculated by equation (14), and finally the failure stress criterion of the core region is obtained:

\[
F = D_1 \frac{\sigma_x + \sigma_z}{2} + D_2 \left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2 + \sigma_t
\]

\[
= D_1 \sigma_{xb} + \left(\frac{p_w}{r_e}\right)x + p_z
\]

\[
+ D_2 \left(\frac{\sigma_x + \left(\frac{p_w}{r_e}\right)x - p_z}{2}\right)^2 + \left(\frac{p_w}{r_e}\right)^2 + \sigma_t,
\]

(15)

where \( D_1 = 1 - \alpha, D_2 = (1 + \alpha + \beta)/(1 + \beta), \alpha \) is the material compression ratio, \( \beta \) is the criterion parameter, and the value range is \( 0 \sim 1 \).

3.3. Establishment and Solution of Water Pressure Failure Zone. The water pressure failure zone is related to the width of the waterproof coal pillar with in the saturation line of the gob. Therefore, the major issue is to determine the saturation line of the gob. Dachler introduced a piecewise method, where a dam was divided into an upper wedge section, a middle seepage section, and a lower wedge section, to analyze the seepage of homogeneous soil dams with an impervious foundation. Furthermore, the gob can be divided into an upper wedge section and a main seepage section considering the similarities between the gob seepage and soil dam. Thus, the saturation line in the upper wedge section can be expressed as

\[
y = h - m_1 x,
\]

(16)

where \( m_1 \) is the gob slope ratio, and \( h \) is the gob’s water level.

Based on the empirical formula proposed by Dachler, flow \( q \) can be expressed as

\[
q = k(h - h_0)\left(1.12 + \frac{1.93}{m_1}\right),
\]

(17)
where \( m_1 \) is the gob slope ratio, \( h \) is the gob’s water level, and \( h_0 \) is the height of the saturation line on the border between the upper wedge and main seepage sections.

In the main seepage section, Dupuit’s formulas are satisfied:

\[
\begin{align*}
  y &= \sqrt{\frac{h_0^2 + (h - h_0)\left(x_0 + x_1 - x\right)}{x_1}}, \\
  q &= k \frac{h_0^2 - h_1^2}{2x_1},
\end{align*}
\]

(18)

where \( h_0 \) is the height of the saturation line on the border between the upper wedge and main seepage sections, \( h_1 \) is the hydraulic head of the boundary of the main seepage section in the affected area, \( x_1 \) is the width of the main seepage section in the \( x \) direction, and \( x_0 \) is the abscissa of the saturation line on the border between the upper wedge and main seepage sections.

Because the flow between the saturation line and impermeable line is equal in each cross-section, thus, the water pressure failure zone of the waterproof coal pillar can be expressed as

\[
r_w = m_1(h - h_2) + x_0 + x_1 - \frac{h_2^2}{h_0^2} - \frac{h_1^2}{h_0^2}x_1,
\]

(19)

where \( m_1 \) is the gob slope ratio, \( h \) is the gob’s water level, \( h_0 \) is the height of the saturation line on the border between the upper wedge and main seepage sections, \( h_2 \) is the height of the floor of the coal seam relative to the gob bottom, \( h_3 \) is the hydraulic head of the boundary of the main seepage section in the affected area, \( x_1 \) is the width of the main seepage section in the \( x \) direction, and \( x_0 \) is the abscissa of the saturation line on the border between the upper wedge and main seepage sections.

### 3.4. Determination of the Reasonable Width of Waterproof Coal Pillar

When calculating the width of the waterproof coal pillar, the acquisition of the model parameters is the key. For the parameters of this model, only empirical differences exist in the lateral pressure coefficient \( \lambda \), criterion parameter \( \beta \), and mining influence factors \( d \). Among them, different values of parameter \( \beta \) correspond to different strength criteria. And when the parameter \( \beta = 0 \), unified strength theory degenerates into class M-C strength criterion; and when the parameter \( \beta = 1 \), the uniform strength theory degenerates into a double shear stress strength criterion. To avoid errors due to different lithologies, the selection of the \( b \) value is determined as the intermediate value of its value range, \( b = 0.5 \) [47]. For lateral pressure coefficient \( \lambda \), it has a strong influence on the width of the waterproof coal pillar. And the causes of lateral pressure and the factors affected are numerous and complex. Therefore, the use of empirical values often has a large error. The author suggests that for the lateral pressure coefficient, the inversion method is appropriate, that is, to measure the mine-pressure-influenced plastic zone; then, the lateral pressure coefficient is calculated by a plastic model. If it does not, it can be obtained by analogy with similar coal mines. Finally, for the acquisition of action factors, considering that most coal mines are now fully mechanized, instead of the original style, the structure of the coal seam is disturbed very little. Therefore, the minimum value of 1.5 \( d \) is the mining influence factor [1].

For the calculation of the overall width of the waterproof coal pillar, the flow chart shown in Figure 10 can be used. When sound waves travel in rock mass, if the original state and force are different, the acoustic wave velocity is also different. As the face advances, the stress state of the coal pillar changes due to the influence of mining. The wave velocity changes, the compression speed is fast, low wave velocity during expansion. According to the principle of drilling in different positions of the waterproof coal pillar along the direction of the working face, multiple repeated observations of acoustic velocity compare its depth variation (Figure 11). Thus, the width of each partition of waterproof coal pillar is detected. The results show the width of the mine-pressure-influenced plastic zone of the waterproof coal pillar between the faces is 2.6 m. To get the parameters, the point load test of the compressive strength of coal and rock, the hydraulic cracking test of tensile strength, and the immersion softening test of coal and rock were carried out, respectively. By inversion of the mine-pressure-influenced plastic zone model, the parameter \( \lambda \) is 0.4. At the same time, the parameters are replaced by the water pressure failure zone, and the water pressure failure zone is 3.4 m. In the effective waterproof elastic core zone, the maximum water pressure is 2.2 MPa. The width of elastic core zone is 8.5 m obtained by trial-error method. By adding the width of the three zones, it can be seen that the waterproof coal pillar is 14.5 m. Compared with the
previous coal mine reserved pillar width of 20 m, the theoretical calculation value has considerable saving space.

4. Sensitivity Analysis and Model Validation

4.1. Mine-Pressure-Influenced Plastic Zone. Because the mine-pressure-influenced plastic zone is characterized by support failure (load migration), the determination of its range is not only the basis of determining the width of the effective bearing zone or the effective water barrier, that is, the elastic core zone, but also the important parameters of other related theoretical calculations and technical applications. At present, there are many theoretical models for the determination of plastic zone range. In order to explain the advantages of this model, we combined it with the previous literature for comparative analysis.

Ma and Hou choose the Mohr-Coulomb strength failure criterion with internal friction angle and cohesion force as parameters and adopt the microelement limit equilibrium method as the same as the model in this paper [48]. However, the peak stress of the coal pillar is the average weight of overlying strata multiplied by the stress concentration coefficient.

Wilson also selects the internal friction angle and cohesion as the parameters of the Mohr-Coulomb strength failure criterion. It is considered that the horizontal stress and vertical stress do not change with the height of the coal pillar but are only affected by the horizontal force. It is also considered that the determination of the plastic zone width is based on the vertical stress reaching the peak stress of the pillar, while the peak stress of the pillar is the average heavy overlying strata multiplied by the stress concentration coefficient. The essence of this model is the general form of the Wilson plastic zone width equation. The foundation of this model is to assume that the directional stress in the plastic zone does not change with the direction of coal seam depth, which is reasonable for the design of reserved coal pillar in thin coal seam and careful for the determination of plastic zone width of higher coal pillar. At the same time, the x and z direction stress are treated as the main stress in the corresponding direction naturally. For the model parameter processing, Wilson [42] thought that the coal seam interface and coal body mechanical parameters are the same. In fact, due to the complex physical morphology and stress condition of the upper and lower interface layers in the mine-pressure-influenced plastic zone, there is a great difference between the mechanical parameters of the coal body itself, so attention should be paid to the use of this conclusion. Meanwhile, Wilson take $K_yH$ as the peak stress in the plastic zone, the correctness of this argument is worth thinking about. Because the formation of the mine-pressure-influenced plastic zone lies in the gradual failure of coal wall caused by coal strength failure, which is a continuous process, and the determination of stress concentration coefficient $K$ is a reflection of the degree of secondary disturbance of coal seam subjected to coal pressure. It is the result of the formation of plastic zone of coal pillar, not the reason. Based on the mechanical model of the coal pillar plastic zone established by continuous medium mechanics, when the mechanical system is based on the solution specification and failure criterion, the actual working condition results can only improve or perfect the model condition simplification and parameter correction. However, the solution system relied on by the model cannot be destroyed. Interestingly, some models similar to Wilson are applied with a stress concentration coefficient of 4. On the one hand, the conclusion of the stress concentration coefficient 4 comes from the statistical results of stress detection in British coal seam; on the other hand, the statistical average value of the friction angle in British coal Wilson 37.5” is substituted into the Mohr-Coulomb criterion result. The strength calculation not only considers the actual stress state of coal and rock but also considers the difference of hardness of coal and rock, thus avoiding the confusion of peak stress calculation. Based on this, this paper thinks that this view is more reasonable.

The SMP criterion of intermediate principal stress is considered under plane strain condition. It is also considered that the determination of plastic zone width is based on the vertical stress reaching the peak stress of the pillar, while the peak stress of the pillar is the average heavy overlying strata multiplied by the stress concentration coefficient. The SMP criterion is used to replace the original Mohr-Coulomb criterion in this model to consider the influence of intermediate principal stress, and the corresponding equation of the plastic zone width is derived. Certainly, the approach is an attempt to improve the model. However, the problem is that the necessity of considering the influence of intermediate principal stress in the plastic zone is open to question, because after the coal seam is excavated, the plastic zone is actually exposed on one side, and the vertical stress of the coal body unit is much larger than that of the horizontal stress. In engineering practice, the plastic zone is regarded as the disturbance range of the surrounding rock. In order to prevent its further expansion in subsequent mining activities, anchor rod (cable), grouting, and other technical means are often used to reinforce it. From this point of view, the support pressure exists after the formation of plastic zone and should not be included in its calculation and design process. Therefore, it is reasonable to adopt the failure criterion without considering the influence of intermediate principal stress for the failure of coal and rock in plastic zone.
4.2. Effective Waterproof Elastic Core Zone. For the establishment of the mechanical model of the effective waterproof elastic core zone as an effective water barrier, this paper considers that the effective waterproof elastic core zone is in a three-direction constraint state, and the influence of intermediate principal stress should be investigated. Therefore, the design theory of goaf is improved by using uniform strength criterion as the description of failure condition.

In addition, the combination strength criterion is used to calculate the width of the effective waterproof elastic core

Figure 12: The influence curve of effective waterproof elastic core zone width by the basic parameters.
zone, that is, the complexity of default coal and rock failure, but the maximum principal stress criterion is used to solve the problem in theoretical analysis. This will inevitably lead to large errors in the results. Taking Liu et al. [29, 30] as an example, the internal damage distribution of the nuclear area is shown when the reasonable width is drawn by trial-error method. When the width of the elastic nuclear area is 2.3 m, there is still a large area of damage (negative area) in an elastic nuclear area, that is, the width of nuclear area is unreasonable at this time. To obtain a reasonable width, it is necessary to adjust the width of the elastic core region. By trial-error method, it is found that when the width of the effective waterproof elastic core zone is 5.2 m, there is exactly no negative value, that is, 5.2 m should be a reasonable width value in theory.

In order to further reflect the rationality of the nuclear zone model, the influence of the parameters of the elastic nuclear zone model on the reasonable width of the nuclear zone is analyzed and discussed. First, the model parameters are substituted into the damage criterion equation (16) of the elastic core zone, in which the principal stress is expressed as shown in equation (18) and the stress components in each direction are shown in equation (15). The principal stress is expressed as shown in equation (15). Through the trial-error method, it is found that the calculation point of the reasonable width of the nuclear zone is $(0, \pm M/2)$, the point is substituted into equation (20) and $F$ is 0. Finally, the equation of the reasonable width of the elastic nuclear zone is obtained:

$$r_e = \frac{M_p}{2\sqrt{(((\eta - 1)(2\alpha p - p_c - x_{ab}))/(2(\eta + 1)) + (\zeta/(\eta + 1))^2 + (p_c - x_{ab}))/2}}.$$  

(20)

where $\eta = (\alpha\beta + 2\alpha)/(2 + (1 - \alpha)\beta)$, $\zeta = (2\sigma_r(1 + \beta))/(2 + (1 - \alpha)\beta)$.

The influence parameter curves of the nuclear region are obtained by equation (20), as shown in Figure 11.

1. From Figure 12(a), it can be seen that the width of the effective waterproof elastic core zone increases with the increase of water pressure strength, which reflects the effect of inrush pressure on the safety of waterproof coal pillar.

2. Figure 12(b) shows that the width of the effective waterproof elastic core zone is proportional to the depth of the coal seam and steepens with the increase of the width of the goaf. This conclusion, whether from field observation or empirical analysis, is first revealed and obvious.

3. Figure 12(c) shows that the compressive strength of coal and rock is negatively correlated with the width of the effective waterproof elastic core zone and steeper with the increase of tensile strength, that is, the compressive strength is consistent with the macroscopic strength, and the increase of tensile strength will reduce the macroscopic strength, thus increasing the need of structural size.

4. According to Figure 12(d), the width of the effective waterproof elastic core decreases with the increase of lateral pressure coefficient and increases with the depth of burial. This phenomenon is caused by the “confining pressure effect” formed by lateral pressure on the elastic core region. This conclusion is also recognized by geotechnical engineering circles all the time. The reason is that the confining pressure not only restricts the rock mass structure to prevent the structure from collapsing but also increases the strength of coal and rock body and improves the deformation resistance of the structure.

5. As can be seen from Figure 12(e), the width of the effective waterproof elastic core decreases with the increase of the $b$ of the medium principal stress parameter and decreases with the increase of compressive strength. Among them, the size of parameter $b$ reflects the influence of the medium principal stress on the strength of the structure, and the greater the parameter, the stronger the influence. This phenomenon is also well known to the geotechnical engineering community. Considering the influence of medium principal stress on structural stability, the essence is to give full play to the potential of rock mass to resist failure and deformation. Compared with the external manifestation of lateral pressure, it is the internal manifestation of confining pressure effect. At the same time, compared with the shape of the influence curve under different compressive strength conditions, it is found that the curve should change obviously at lower pressure, that is, the effect of medium principal stress on “soft” rock is obvious than that of “hard” rock.

4.3 Water Pressure Failure Zone. The coal body on one side of adjacent water not only bears mine pressure but also includes water pressure and infiltration. The effect of mine pressure is the same as that of plastic zone, and the influence....

![Figure 13: Influence of the coal-rock weakening coefficient on the width of the water pressure failure zone.](image-url)
of water, on the one hand, forms lateral water pressure; on the other hand, the fissure effect of water is invalid because of the loose collapse state of coal body under mine pressure, where the weakening coefficient is defined as

$$\eta_0 = \frac{\sigma_w}{\sigma_c},$$  \hspace{1cm} (21)$$

where $\sigma_w$ is the compressive strength of saturated coal and rock samples. According to the definition of the coal and rock weakening coefficient, the low coefficient indicates the strong and weak, while the high coefficient shows that the influence of water on the strength of coal and rock is weak.

In this paper, the water pressure failure area is proposed based on the effect of water accumulation in goaf or old klin water on the infiltration and softening of coal and rock on the side of coal pillar. By introducing the immersion weakening coefficient, the plastic zone model is modified to obtain the width of the water pressure failure zone. In order to reflect the rationality of introducing softening coefficient, the influence curve of softening coefficient on the water pressure failure zone as shown in Figure 13.

Figure 13 shows that with the increase of the weakening coefficient of coal and rock, the water pressure failure zone decreases gradually and shows the stage influence—the lower the coefficient is, the stronger the influence is, and the change is gentle when the coefficient is higher than a certain value. Compared with the influence of different compressive strength on the width of water pressure failure zone under the same coefficient, it can be seen that the higher the compressive strength, the smaller the width of water pressure failure zone. In the relationship of width, the influence of compressive strength is not difficult to understand, and the influence of weakening coefficient can be explained by its definition—low coefficient indicates strong and weak, while high coefficient shows that water has weak influence on coal and rock strength.

5. Conclusions

To determine a reasonable waterproof coal pillar width for the mining of a shallow coal seam located close to gob, we adopted the method of theoretical analysis. The saturation line was deduced by simplifying the saturation line of the gob into an upper wedge section and a main seepage section. Based on the failure characteristics of the coal seam roadway, the waterproof coal pillar was divided into a mine-pressure-influenced plastic zone, an effective waterproof elastic core zone, and a water pressure failure zone. Furthermore, a theoretical method was proposed for determining the width of the waterproof coal pillar. The theoretical result was verified using simulation analysis, indicating that the theoretical method can be used to determine the width of the waterproof coal pillar. According to the results of the research in Xinjing Coal Mine, the prevention compared with the empirical width, the waterproof coal pillar has a better width potential. However, in order to better verify the correctness of the model, more practical tests to be done.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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