Echoes in X-ray binaries

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ABSTRACT

We present a method of analysing the correlated X-ray and optical/UV variability in X-ray binaries, using the observed time delays between the X-ray driving light curves and their reprocessed optical echoes. This allows us to determine the distribution of reprocessing sites within the binary. We model the time-delay transfer functions by simulating the distribution of reprocessing regions, using geometrical and binary parameters. We construct best-fitting time-delay transfer functions, showing the regions in the binary responsible for the reprocessing of X-rays. We have applied this model to observations of the soft X-ray transient GRO J1655-40. We find that the optical variability lags the X-ray variability with a mean time delay of 19\(^{+2}_{-2}\) s. This means that the outer regions of the accretion disc are the dominant reprocessing site in this system. On fitting the data to a simple geometric model, we derive a best-fitting disc half-opening angle of 13\(^{+2}_{-1}\)\(^{+2}_{-8}\), which is similar to that observed after the previous outburst by Orosz & Bailyn. This disc thickening has the effect of almost entirely shielding the companion star from irradiation at this stage of the outburst.


1 INTRODUCTION

X-ray binaries (XRBs) are close binaries that contain a relatively unevolved donor star and a neutron star or black hole that is thought to be accreting material through Roche-lobe overflow. Material passing through the inner Lagrangian point moves along a ballistic trajectory until impacting on to the outer regions of an accretion disc. This material spirals through the disc, losing angular momentum, until it accretes on to the central compact object. X-rays are emitted from inner disc regions via thermal bremsstrahlung with an effective temperature \(\sim 10^8\) K. The X-ray flux depends on the mass transfer rate, which in turn depends on the structure of the disc and its ability to transport angular momentum. In XRBs the structure of the accretion disc is governed by irradiation.

Much of the optical emission in XRBs arises from reprocessing of X-rays by material in regions around the central compact object. The disc is highly ionized and outshines the donor star. Light travel times within the system are of the order of tens of seconds. Optical variability may thus be delayed in time relative to the X-ray driving variability by an amount characteristic of the position of the reprocessing region in the binary and the geometry of the binary. The optical emission may be modelled as a convolution of the light curve of the X-ray emission with a time-delay transfer function.

This time delay is the basis of an indirect imaging technique, known as echo tomography, to probe the structure of accretion flows on scales that cannot be imaged directly, even with current interferometric techniques. Echo mapping has already been developed to interpret light curves of active galactic nuclei (AGN), where time delays are used to resolve photoionized emission-line regions near the compact variable source of ionizing radiation in the nucleus. In AGN the time-scale of detectable variations is days to weeks, giving a resolution in the transfer functions of 1–10 light-days (Krolik et al. 1991; Horne Welsh & Peterson 1991). In XRBs the binary separation is light-seconds rather than light-days, requiring high-speed optical and X-ray light curves to probe the components of the binary in detail. The detectable X-ray and optical variations in the light curves of such systems are also suitably fast. Extreme examples of this rapid variability are X-ray bursts, for example from Cygnus X-2 (Kuulkers, van der Klis & van Paradijs 1995), where the X-ray flux can increase by \(\sim 50\) per cent with a rise time of 2–3 s and a duration of \(\sim 5\) s. Time-delayed optical bursts have been seen clearly in the object 4U/MXB 1636-53 (Pedersen et al. 1982; Lawrence et al. 1983; Matsuoka et al. 1984). Pulsed X-ray

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emission from Hercules X-1 produces faint optical pulsations that are thought to be echoes from the irradiated companion star (Middlefield & Nelson 1976).

Recently, Hynes et al. (1998b) found correlated time-delayed X-ray and UV variability in the light curves of the Soft X-ray transient GRO J1655-40, using XTE and HST. The data, centred around binary phase 0.4, shows a mean time-delay of 14 s and an RMS delay of 10.5 s. The mean time delay is consistent with reprocessing in the outer regions of the accretion disc, while the relatively large range of delays suggests that there is relatively little companion star reprocessing. This may occur if a thick outer accretion disc shields the companion star from the X-ray source.

In this paper we present a simple geometric model for the time-delay transfer functions of XRBs, using a synthetic binary code. We analyse correlated X-ray and UV variability in GRO J1655-40, using our computed transfer functions, to constrain the size, thickness and geometric shape of the accretion disc.

Section 2 describes the features seen in typical optical light curves of low-mass X-ray binaries (LMXBs) and how these affect our analysis. Section 3 describes the reprocessing of X-rays in XRBs in more detail, including the reasons for the time delays between the X-ray and optical variability. Section 4 describes the model created to describe the time delays found, while Section 5 describes an alternative method that uses Gaussian transfer functions. Section 6 shows the results of this analysis. This is followed by a discussion of the results in Section 7.

2 OPTICAL LIGHT CURVES

The light curves of X-ray binaries contain many temporal and spectral features (see van Paradijs & McClintock 1995 for a review of many of these), some periodic and others purely random or quasi-periodic in nature (see van der Klis 2000 for a review). The periodic features, such as eclipses and the minima and maxima of light curves, are caused by the orbital motion of the components of the binary around the centre of mass of the system. These periodic features are relatively easy to model and much success has been made of such models in determining the geometric parameters of binaries using the information in multicolour light curves. Refinements have been made to include effects such as limb darkening, caused by an decreasing source function with projected radius from the stellar core, and gravity darkening, which causes a change in temperature of the distorted shape of the roche-lobe filling star (e.g. Orosz & Bailyn 1997).

In the case of X-ray binaries the intense X-ray flux from the regions surrounding the compact object also affect both the periodic and non-periodic features of the observed optical light curves. In the standard model of reprocessing, X-rays are emitted by material in the deep potential well of the compact object. These photoionize and heat the surrounding regions of gas, which later recombine and cool, producing lower energy photons. Hard X-rays that penetrate below the photosphere emerge as continuum photons, with an energy distribution characteristic of the temperature of the photosphere. Soft X-rays that are absorbed above the photosphere emerge as emission line photons from a temperature inversion layer near the surface of the disc. Such irradiation will have the effect of changing the overall form of the orbital light curve as the aspect of the hot, irradiated regions changes with the binary orbit, this effect is very noticeable with the changing irradiation of the inner face of the companion star in Hercules X-1, during the 35-d precession phase of the tilted accretion disc (Boynton et al. 1973). While these features are all vital in interpreting the long time-scale orbital light curves of X-ray binaries, many of them are not important in this work, which deals with a very small range of binary phases (less than 1 per cent).

The short duration of the observations is also important when considering the effects of other transient features, such as star spots, the lifetime of which is much longer than the duration of our observation and whilst it will undoubtedly make a small effect on our model fits, the uncertainties involved in including them far outweigh any benefits to the model from including them.

In our model, we have assumed that all the optical variability is in fact caused by the reprocessing of X-ray irradiation. Furthermore, we have assumed that the thermal component will dominate the reprocessed emission in our broad-band observations. These assumptions are clearly not strictly true, observations of cataclysmic variables and other interacting binaries, where irradiation is no longer the dominant source of optical emission, also show non-periodic optical variability superimposed on the periodic orbital light curves (e.g. the Nova-like AE Aquarri: Welsh, Horne & Oke 1993). However, while such non-correlated features will introduce noise into the analysis, thus increasing the absolute value of the badness-of-fit of our models, they will have little effect on the relative values, which have contributions from all points in the light curve. Similarly, the optically thin emission is only a small fraction of the total broad-band emission and for this reason is assumed to vary in phase with the thermal emission.

3 REPROCESSING OF X-RAYS

The reprocessed, optical emission seen by a distant observer is delayed in time of arrival relative to the X-rays by two mechanisms. The first is a finite reprocessing time for the X-ray photons and the second is the light travel times between the X-ray source and the reprocessing sites within the binary system.

3.1 Reprocessing times

The average reprocessing time for line photons is given by the average recombination time (Hummer & Seaton 1963),

$$\tau_{rec} = \left( \frac{n_e}{10^{15} \text{cm}^{-3}} \right)^{-1}.$$  \hspace{1cm} (1)

In the accretion disc the high electron density, $n_e \sim 10^{15} \text{cm}^{-3}$, ensures that the time-scale for reprocessing of line photons is short compared with the overall time delay.

The continuum photons which scatter from deeper within the accretion disc undergo a ‘random walk’ before escaping through the photosphere, leading to a longer reprocessing time. The exact determination of this continuum reprocessing time is complicated, requiring detailed model atmosphere calculations, which is outside the scope of this paper. However, the short time delays between X-ray and optical bursts (Pedersen et al. 1982) and the observations of 1.24 second optical pulsations from Hercules X-1 (Chester 1979), imply that a significant fraction of the reprocessed optical photons emerge from the reprocessing site within $\sim 0.6$ s of the absorption of the incident X-ray photons. This delay is smaller than the measured uncertainty in the mean delay for the systems and so we have treated the reprocessing as instantaneous. We have also treated the reprocessing of X-rays as ‘passive’ reprocessing, where the absorbed X-rays do not affect the structure of the material in the binary.
3.2 Light travel times

The light travel times arise from the time of difference for photons that are observed directly and those that are reprocessed and re-emitted before travelling to the observer. These delays can be up to twice the binary separation, obtained from Kepler’s third law,

\[
a/c = 9.76 \left( \frac{M_x + M_e}{M_\odot} \right)^{1/2} \left( \frac{P}{d} \right)^{1/2},
\]

where \( a \) is the binary separation, \( M_x \) and \( M_e \) are the masses of the compact object and donor star, \( P \) is the orbital period. In LMXBs, the binary separation is of the order of several light seconds.

The time delay \( \tau \) at binary phase \( \phi \) for a reprocessing site with cylindrical coordinates \((R, \theta, Z)\) is

\[
\tau(x, \phi) = \frac{\sqrt{R^2 + Z^2}}{c} \left[ 1 + \sin i \cos(\phi - \theta) \right] - \frac{Z}{c} \cos i
\]

where \( i \) is the inclination of the system and \( c \) is the speed of light. This can also be expressed using the position vector, \( x \) and the unit vector, \( e(\phi) \), pointing toward the Earth,

\[
\tau(x, \phi) = \frac{|x| - e(\phi) \cdot x}{c}.
\]

The X-ray driving light curve, \( f_1(t) \), is described as the sum of a constant and a variable component,

\[
f_1(t) = \bar{f}_e + \Delta f_1(t).
\]

The reprocessed light curve, \( f_2(t) \), is similarly divided into two components. The relationship between the two is given by

\[
f_2(t) = \bar{f}_r + \int \Psi_\tau(\lambda, \tau, \phi)(f_1(t - \tau) - \bar{f}_e) d\tau,
\]

where \( \Psi_\tau(\lambda, \tau, \phi) \) is the time delay transfer function. This transfer function is the strength of the reprocessed variability delayed by \( \tau \) relative to the X-ray variability.

The dynamic response function is found by considering how a change in X-ray flux drives a change in the reprocessed flux. We can define the dynamic time delay transfer function to be

\[
\Psi_\tau(\lambda, \tau, \phi) = \left\{ \frac{\delta f_2(x, \lambda) - \delta f_1(x, \lambda)}{\delta f_1(x, \lambda)} \right\} d\Omega(x, \phi) - d\tau - \tau(x, \phi),
\]

where \( \tau(x, \phi) \) is the geometric time delay of a reprocessing site at position \( x \), see equation (3). In the next section we describe the model X-ray binary code we have been using and how we have used this to find the reprocessed flux.

4 MODEL X-RAY BINARY CODE

We have developed a code to model time delay transfer functions based on determining the contributions from different regions in the binary. In this section we describe the models used to construct the individual regions of the binary; the donor star, the accretion stream and the accretion disc. The code uses distances scaled to the binary separation in a right-handed Cartesian coordinate system corotating with the binary. The X-direction is along the line of centres for the binary, the Y-direction is perpendicular to this in the orbital plane of the binary, so that the X-ray source is at \((0,0,0)\) and the centre of mass of the donor star is at \((1,0,0)\). Each surface element is a triangles, characterized by its area \( d\Omega \), orientation \( n \), position \( x \) and temperature \( T \).

We calculate the total monochromatic flux by summing up contributions from all visible elements. The area \( d\Omega \) and the normal vector \( n \) for the triangular panel are calculated and then the projected area of the panel is calculated using the projected earth vector \( e(\phi, \iota) \). The effects of occultations by regions in the binary are also considered. Therefore the observability, \( O(x, \phi) \), is given by

\[
O(x, \phi) = d\Omega(x, \phi) e(\phi),
\]

which is the foreshortened area of pixel \( x \), observable at phase \( \phi \). This is related to the solid angle of the observed pixel, \( d\Omega \), by the relation

\[
d\Omega(x, \phi) = \frac{O(x, \phi)}{D^2},
\]

where \( D \) is the distance to the source. If \( d\Omega(x, \phi) < 0 \) then the panel is not visible to the observer and does not contribute to the flux. The monochromatic intensity is calculated using the Planck function,

\[
B(\lambda, T) = \frac{2hc}{\lambda^3} \exp \left( \frac{hc}{\lambda kT} \right) - 1
\]

This is scaled using the projected area of the panel \( O(x, \phi) \), as seen by the observer. The standard linear limb-darkening law is assumed,

\[
L(u, \alpha) = \frac{1 - u + u \cos(\alpha)}{1 + u^{3/2}}
\]

where \( u \) is the linear limb-darkening coefficient, assumed to be 0.6, and \( \alpha \) is the angle between the normal vector \( n \) and the earth vector \( e(\phi, \iota) \),

\[
\cos \alpha = n \cdot e.
\]

The response curve for a given detector, \( P(\lambda) \), is combined with the limb-darkened Planck function to create the synthetic reprocessed flux from each visible triangular element. The total detected flux due to reprocessed X-rays from a single panel, \( F_\tau(\lambda, \iota) \), is given by

\[
F_\tau(\lambda, T) = \int B(\lambda, T) P(\lambda) L(u, \alpha) O(x, \phi) d\lambda,
\]

where \( L(u, \alpha) \) and \( d\Omega(x, \phi) \) are the expressions for the limb-darkening and the solid angle of the exposed panel, see equations (11) and (9) respectively. In the next sections we describe the geometric model used to calculate the time delay for a given panel and the effects of irradiation which are used to determine the contribution of each panel to the final transfer function.

4.1 Donor star

The donor star is modelled assuming it fills its critical Roche potential, so that mass transfer occurs via Roche lobe overflow through the inner Lagrangian point. Optically thick panels are placed over the surface of the Roche potential. The panels are triangular so that the curved surfaces of the binary are mapped more accurately than is possible using four-sided shapes (Rutten & Dhillon 1994). These panels are equally spaced in longitude and latitude across the surface of the star. In order to specify the temperature of each panel correctly on the face of the tidally distorted star, one must take into account the degree of gravity darkening. Using von Zeipel’s theorem (Zeipel 1924) for the relationship between the local gravity and the local emergent flux,
one finds that the relationship between the local temperature, $T_e$, and the gravity, $g$, is

$$T_e^4(x) \propto g(x).$$

(14)

As a consequence the temperature at any point on the star is given by

$$\frac{T(x)}{T_{\text{pole}}} = \left(\frac{g(x)}{g_{\text{pole}}}\right)^\beta,$$

(15)

where $T_{\text{pole}}$ and $g_{\text{pole}}$ are the temperature and gravity of the pole of the star. The ‘gravity darkening exponent’ $\beta$ is 0.25 for stars with fully radiative envelopes (Zeipel 1924; as is the case in our models) and 0.08 for stars with fully convective envelopes (Lucy 1966). $T_{\text{pole}}$ is taken to be the effective temperature of a field star with a similar spectral type to the donor star.

### 4.2 Accretion stream

The accretion stream is modelled by following the ballistic trajectories of four test particles. The thickness ($w$) of the stream defines the initial positions of the test particles. These test particles determine the ‘width’ of the stream (its deviation from the line of centres of the binary, in the plane of the binary, or $y$-direction) and the ‘height’ of the stream (its extent in the direction normal to the plane of the binary, the $z$-direction), assuming the stream is symmetric about the $x$-$y$ plane.

The particles start at the L1 point with a small velocity in the direction of the compact object ($-\varepsilon(0,0)$, from positions $(RL1),(0,0)$, $(RL1),w(0)$, $(RL1),w(0)$ and $(RL1),w(w)$. The trajectory is cut into discrete steps, with the step size as a parameter of the code. The velocity and position of each particle are determined from the Roche potential after each step. The stream is curtailed when one of two criteria are reached; (1) the stream has collapsed vertically, or (2) the core trajectory is moving outwards, i.e. the stream has passed the compact object without collapsing vertically.

The unirradiated accretion stream is assumed to have a constant temperature $T_i$, along its length and the effects of irradiation are considered in the same way as those of the donor star in Section 4.4.

### 4.3 Accretion disc

The disc thickness is assumed to increase with radius from 0 at $R = R_{\text{in}}$, to $H_{\text{out}}$ at $R = R_{\text{out}}$, with the form

$$H = R_{\text{out}} \left(\frac{H_{\text{out}}}{R_{\text{out}}} \left(\frac{R - R_{\text{in}}}{R_{\text{out}} - R_{\text{in}}}\right)^\beta, \right.$$ \n
(16)

where the parameters are the inner and outer disc radii, $R_{\text{in}}$ and $R_{\text{out}}$ in units of $(L1)$, the half thickness of the outer disc $(H/R)_{\text{out}}$ and the exponent $\beta$ which describes the overall shape of the disc. The temperature structure of the unirradiated disc is that of a steady state disc, in the absence of irradiation,

$$T_{\text{disc}}(R) = T_{\text{out}} \left(\frac{R}{R_{\text{out}}}\right)^{-\frac{1}{2}},$$

(17)

where $T_{\text{out}}$ is the temperature at the outer disc and $T_{\text{in}}$ is the temperature of the inner disc.

The disc is divided radially and azimuthally, into $N_R$ and $N_{\phi}$ sections. The monochromatic intensity is again calculated using the Planck function, corrected for limb-darkening using a linear limb-darkening law, with a constant coefficient. The intensity is again scaled using the projected area of the panel, for the given values of binary phase $\phi$ and inclination $i$.

### 4.4 Irradiation model

The effective temperature of a region at a distance $R$ from the X-ray source, assumed in our model to be a point source located at the centre of the accretion disc, is found from the accretion luminosity for a typical LMXB,

$$T_i^4 = \frac{L_x(1 - A)}{4\pi\sigma R^2}$$

(18)

and

$$L_x = \frac{GM_iM}{R_{\text{ns}}^2},$$

(19)

where $T_i$ is the temperature, $A$ is the albedo, $\eta$ is the efficiency, $M_i$ the mass of the compact object, $M$ the accretion rate on to the compact object, $R_{\text{ns}}$ is the size of the compact object and $R$ is the distance between the compact object and the irradiated element. This is normalized using the binary separation $a$, the distance between the centres of mass of the stars, as is the coordinate system for the binary. In the case of Scorpius X-1, this gives $T_i \sim 10^5$ K for a 1.4-M$_\odot$ neutron star $(R_{\text{NS}} \sim 10$ km) accreting $10^{-9}$ M$_\odot$ yr$^{-1}$, with an efficiency $\eta = 0.1$, an albedo of 0.5, at a distance equal to the binary separation of $3.4 \times 10^{11}$ cm.

The irradiation of the binary takes place in three stages. The first stage is to calculate the temperature structure of the binary in the absence of any irradiation. This is done with characteristic temperatures for the donor star (from its spectral type) and the accretion stream and disc. The temperature structure of the disc is assumed to that for an unirradiated disc as given in equation (17). The surface elements of the binary exposed to X-rays are determined by projecting the binary surfaces onto the spherical polar representation of the sky, as it appears from the X-ray source. Each triangular element is mapped to the sky starting with the one furthest from the source and ending with the triangle closest. Those elements remaining visible and unocculted on the sky map are irradiated. The change in effective temperature of an element is scaled by the projected area with respect to the X-ray source at a distance $R$ from the source. Hence the temperature after irradiation is given by

$$T^4 = T_i^4 \cos \theta \left(\frac{a}{R}\right)^2 + T_{\text{eff}}^4$$

(20)

where $T$ is the temperature of the panel, $\theta$ the angle between the line of sight from the central source and the normal to the surface of the element and $T_{\text{eff}}$ is the unirradiated effective temperature of the panel.

Since we are interested in finding the correlation between the variable component of the X-ray and reprocessed fluxes, we split the X-ray flux into constant and time-dependent components. These components of the flux are converted into components of temperature,

$$T_i(t) = T_i + \Delta T_i(t),$$

(21)

where

$$\frac{\Delta T_i(t)}{T_i} = \frac{4\Delta T_i(t)}{T_i}$$

(22)

The second stage is to irradiate the binary with the constant component of the X-ray flux. This component of the X-ray flux is...
equated to the mean effective temperature of the X-ray source, as given in equation (20), where \( T_s = T_x \). The third and final stage is to repeat stage two with \( T_s = T(t) \), which represents irradiating the binary with a time varying component. The difference between stages two and three represents the temperature change of the elements due to the time varying component of the X-ray flux alone, \( \Delta f_x(t) \).

Thus the response of a panel to the variable component of the irradiating X-ray flux is given by

\[
I_x[\lambda, x, \Delta f_x(t)] = \int \{ B_x[\lambda, T_x(t)] - B_x(\lambda, T_s) \} \times P(\lambda) I(u, \alpha) d\Omega(x, \phi) d\lambda.
\]  

(23)

**Figure 1.** Left: model X-ray binaries, based on typical binary parameters for an LMXB, showing isodelay surfaces projected on to the irradiated surfaces of the binary. Right: the associated time delay transfer functions, showing the relative contributions from the regions highlighted in the model X-ray binaries.
This response is substituted into the expression for the dynamic response given in equation (7).

### 4.5 Transfer functions

In order to transform the reprocessed flux into a time delay transfer function, we define isodelay surfaces. These surfaces are nested paraboloids around the line of sight to the X-ray source, defined by the earth vector, \( e(\phi, i) \). The parabolic surfaces have a mean time delay \( \tau \) and a width \( \delta \). The mean time delay \( \tau \) in seconds between the directly observed X-ray flux from the central source and the reprocessed signal from a point with cylindrical coordinates \((R, \theta, Z)\) is

\[
\tau = \frac{\sqrt{R^2 + Z^2}}{c} \left[ 1 + \sin i \cos(\phi - \theta) \right] - \frac{Z}{c} \cos i, \tag{24}
\]

where \( i \) is the inclination of the system, \( \phi \) is the binary phase and \( \theta \) is the angle from the line of centres of the binary.

We consider models with reprocessing taking place in the accretion disc, accretion stream and companion star. Each of these regions makes a contribution to the transfer function for the system, see Fig. 1 for a diagram of the results of our code and the calculated transfer functions.

The phase dependence of \( \tau \) allows us to create time-delay transfer functions as a function of binary phase, allowing us to produce phase-delay diagrams for the system, see Fig. 2. X-rays reprocessed at the companion star have a time delay that varies sinusoidally in phase with semi-amplitude \((a/c) \sin i\) around a mean value \(a/c\). X-rays reprocessed by a circular disc appear as a phase independent contribution to the delay distribution \( \delta(i, \tau, \phi) \) between the inner and outer radii of the disc. The accretion stream shows up as a non-symmetric contribution that varies roughly sinusoidally with the orbital motion of the companion star and can be seen faintly in Fig. 2 as the contribution near superior conjunction between the outer rim of the disc and the inner face of the companion star.

The relative intensities of the contributions, represented by the area under the transfer function, constrain the geometric parameters of the system, especially the contribution from the accretion disc, which is probably the most important region for reprocessing of X-rays in interacting binaries.

Thus the shape of the isodelay surfaces when projected on to the plane of the binary depends on the inclination of the system. For a face-on disc \((i = 0^\circ)\) the projected isodelay surfaces are simple circles on the disc. As the inclination of the system increases these surfaces become elongated along the line of sight to the X-ray source, until finally they form parabolae if viewed from edge-on \((i = 90^\circ)\). This inclination dependence is shown in the transfer functions in Fig. 3.

The peak in these transfer functions is also a function of inclination, with the peak occurring with the time delay of the largest isodelay surface that is contained entirely within the disc,

\[
\tau = \frac{R}{c} [1 - \sin (i - \delta)], \tag{25}
\]

where \( \delta \) is the opening angle at the edge of the disc, \( \delta = \tan^{-1}(H/R) \).

The disc shape, characterized by the exponent \( \beta \), also affects the position and shape of the peak in the transfer function. The contribution is scaled with the projected area of the surface element, therefore as \( \beta \) increases the peak should move to longer time delay, as the edge of the disc becomes steeper and the projected area decreases. This can be seen in Fig. 4, where time-delay transfer functions are plotted as a function of \( \beta \) with values between 1.01 and 3. (Note: when \( \beta = 1 \), the surface of the disc is flat, and its thickness goes to zero at the position of the compact object, therefore the projected area of the disc as seen from the compact object is zero, except on the inner edge. For this reason \( \beta \)

![Figure 2](https://example.com/fig2.png)

**Figure 2.** A plot of time-delay transfer functions as a function of binary phase, based on the typical binary parameters for an LMXB. The accretion disc has constant time delays in the region 0–4 s, whereas the time delays from the companion star are seen to vary sinusoidally with binary phase between 2 and 10 s.

![Figure 3](https://example.com/fig3.png)

**Figure 3.** Model time-delay transfer functions for an accretion disc as a function of binary inclination. The solid line is for a binary inclination of 0°, the dashed for 35° and the dot–dashed for 70°.

![Figure 4](https://example.com/fig4.png)

**Figure 4.** Model time-delay transfer functions as a function of \( \beta \), the disc exponent for \( \beta = 1.01 \) (solid line), 1.5 (dashed line), 2 (dot–dashed line), 2.5 (dotted line) and 3 (triple dot–dashed line).
is given a value just greater than 1 to show the limit as $\beta$ tends towards the case of an unflared disc.)

5 GAUSSIAN TRANSFER FUNCTIONS

In Hynes et al. (1998), hereafter Paper I, Gaussian time delay transfer functions were used to create synthetic reprocessed light curves for the SXT GRO J1655-40. This is a relatively simple form of transfer function as it assumes nothing about the geometry of the system. These transfer functions have the form

$$\Psi(t) = \frac{\Psi}{\sqrt{2\pi}\Delta t} \exp\left[-\frac{1}{2}\left(\frac{t - t_0}{\Delta t}\right)^2\right].$$

Thus the transfer functions have three parameters, the mean time delay $t_0$, its variance $\Delta t$ and the strength of the response $\Psi$. These transfer functions were convolved with the X-ray driving light curve in the same way as the model transfer function and the badness of fit between the synthetic and real reprocessed light curves calculated.

The equivalent parameters $t_0$ and $\Delta t$ can be determined for any transfer function, $\Psi(t)$, by calculating the first and second moments.

$$t_0 = \frac{\int_{-\infty}^{\infty} \Psi(t) d\tau}{\int_{-\infty}^{\infty} \Psi(t) d\tau}.$$  

$$\Delta t = \left[\frac{\int_{-\infty}^{\infty} \Psi(t)(t - t_0)^2 d\tau}{\int_{-\infty}^{\infty} \Psi(t) d\tau}\right]^{1/2}.$$  

6 OBSERVATION AND DATA REDUCTION

The soft X-ray transient (SXT) GRO J1655-40 was discovered in 1994 July when the Burst and Transient Source Experiment (BATSE) on GRO observed it in outburst at a level of 1.1 Crab in the 20–200 keV energy band (Harmon et al. 1995). After a period of apparent quiescence from late 1995 to early 1996, GRO J1655-40 went into outburst again in late 1996 April (Remillard et al. 1996), and remained active until 1997 August. During the early stages of this outburst we carried out a series of simultaneous HST and XTE visits. One of the primary goals of this project was to search for correlated variability in the two wavebands. The long-term evolution of this outburst argued against significant reprocessing, as the seemingly anticorrelated optical and X-ray fluxes observed (Hynes et al. 1998a) are not to be expected if the optical flux is reprocessed X-rays. Nonetheless, significant short term correlations were detected. In our previous paper (Hynes et al. 1998b) we have analysed these correlations using both acausal and causal Gaussian transfer functions. We were able to put constraints on the possible regions responsible for X-ray reprocessing. In this paper we summarize this previous analysis and develop it further, using our synthetic binary model to create more physical transfer functions.

6.1 Observations

The HST and RXTE observations used in this paper were first presented, along with similar exposures, in (Hynes et al. 1998b). In

Paper I we fitted causal and acausal Gaussian transfer functions to four similar exposures in a similar analysis as used in the previous section. The time-delay distribution based on all the observations is 14.6 ± 1.4 s, with a dispersion of 10.5 ± 1.9 s.

6.1.1 HST

The HST observations are shown in fig. 1(b) of Paper I, along with a detailed description of the data reduction techniques used. In this paper we have used one of the observations, referred to as exposure 6, shown in fig. 1(b) in (Hynes et al. 1998b) and reproduced in this paper as Fig. 5. This observation took place during the 1996 June 8 visits, using The Faint Object Spectrograph in RAPID mode with the PRISM and blue detectors (PRISM/BL), covering the spectral range ~2000–9000 Å. The resulting light curve has a time resolution of ~3 s, and the absolute time accuracy of this data is limited to 0.255 s.

6.1.2 RXTE

The X-ray data was taken with the PCA onboard RXTE on 1996 June 8, simultaneous with the HST data. The light curve was created from the standard-1 EDS mode data, using the sextend task in the FTOOLS software package. This mode has a maximum time-resolution of 0.125 s, but with no spectral resolution. It is desirable for echo-mapping for the driving X-ray light curve to have a higher time resolution than the reprocessed one, so that the time delay transfer function can have a time resolution greater than the time resolution of the reprocessed light curve. A light curve with a time resolution of 1 s and an absolute timing accuracy of about 8 µs was extracted. For a full description of the data, see Paper I.

6.2 Results of modelling

6.2.1 Gaussian transfer functions

The results for the fits to the acausal Gaussian transfer functions are described in depth in Paper I and summarized in this paper in
Table 1. Best-fitting values for the acausal Gaussian time delay transfer function fitting to the X-ray and optical light curves for GRO J1655-40. The label ‘exp. 6’ refers to exposure 6 in the original paper, Paper I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
<th>( \chi^2 ) (N – 3)</th>
<th>( \Delta \theta )</th>
<th>( \Psi/10^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td></td>
<td></td>
<td>0.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td>782</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \chi^2 = \frac{1}{N-m} \sum_{i=1}^{N} (y_i - y_{\text{model}})^2 \]

Table 2. Parameters used in our model of an X-ray binary; the values are taken from Orosz & Bailyn (1997).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td></td>
<td>kpc</td>
<td>3.2</td>
</tr>
<tr>
<td>Period</td>
<td></td>
<td>d</td>
<td>2.6</td>
</tr>
<tr>
<td>Inclination</td>
<td></td>
<td>( \circ )</td>
<td>69.5</td>
</tr>
<tr>
<td>Mass ratio</td>
<td></td>
<td>( q )</td>
<td>0.3344</td>
</tr>
<tr>
<td>Primary mass</td>
<td></td>
<td>( M_x )</td>
<td>( M_2 )</td>
</tr>
<tr>
<td>Binary sep.</td>
<td></td>
<td>( a )</td>
<td>12</td>
</tr>
<tr>
<td>Star Temp</td>
<td></td>
<td>( T_{\text{pole}} )</td>
<td>6500</td>
</tr>
<tr>
<td>Stream Temp</td>
<td></td>
<td>( T_s )</td>
<td>5000</td>
</tr>
<tr>
<td>Outer Disk</td>
<td></td>
<td>( T_{\text{out}} )</td>
<td>6000</td>
</tr>
<tr>
<td>Inner Disk</td>
<td></td>
<td>( T_{\text{in}} )</td>
<td>100000</td>
</tr>
<tr>
<td>Irrad Temp</td>
<td></td>
<td>( T_x )</td>
<td>10^5</td>
</tr>
<tr>
<td>Inner radius</td>
<td></td>
<td>( R_{\text{in}} )</td>
<td>( R(L_{1}) )</td>
</tr>
<tr>
<td>Outer radius</td>
<td></td>
<td>( R_{\text{out}} )</td>
<td>( R(L_{1}) )</td>
</tr>
<tr>
<td>Disk thickness</td>
<td></td>
<td>( H/R )</td>
<td>free</td>
</tr>
<tr>
<td>disc exponent</td>
<td></td>
<td>( \beta )</td>
<td>free</td>
</tr>
</tbody>
</table>

Table 3. Best-fitting values for the synthetic X-ray Binary model time delay transfer function fitting to the RXTE and HST light curves for GRO J1655-40.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data set</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \phi )</td>
<td></td>
<td></td>
<td>0.42</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td>782</td>
</tr>
</tbody>
</table>

\[ \chi^2(N - 4) = 1.230 \]

\[ R_{\text{out}} = 0.65^{+0.06}_{-0.06} \]

\[ H/R = 0.24^{+0.04}_{-0.03} \]

\[ \beta = 4.7^{+2.0}_{-2.0} \]

6.2.2 Synthetic binary transfer functions

The transfer functions created by our X-ray binary model, using the binary parameters determined by (Orosz & Bailyn 1997) and summarized in Table 2 were used to create synthetic HST light curves by convolving them with the RXTE light curves. The badness of fit between this synthetic reprocessed light curve and the real reprocessed light curve from the HST observations was calculated. The fitting again contained three parameters, which were optimized as before. The best-fitting parameters, together with their one-parameter 1σ confidence regions are given in Table 3. We find that the disc extends to 67 per cent of the distance to the inner Lagrangian point, with an opening angle of 14 degrees and an exponent of 3.8.

6.2.3 Comparison of the two models

This best-fitting solution from our modelling is shown along with the best-fitting solution from the acausal Gaussian fitting in Fig. 6. The best-fitting solution for this modelling shows a similar \( \chi^2 \) to the Gaussian fitting, which shows that our modelling of the binary system and simple Gaussian fits are both good fits to the data. The first and second moments from the model transfer function are, as expected, similar to the best-fitting parameters from the Gaussian fitting. The opening angle for the disc from our synthetic transfer functions, whilst being surprisingly large is comparable with that derived by (Orosz & Bailyn 1997) from light curve fitting shortly after the previous outburst in 1995 March. They found that the opening angle of the disc was 11° (see table 6 of Orosz & Bailyn 1997) compared with our value of 14°.

7 DISCUSSION

We have used the correlated X-ray and optical variability seen in low-mass X-ray binaries to determine geometric parameters for the binary system GRO J1655-40. These parameters are principally the size and shape of the accretion disc. This is inferred from the
relative contributions to the time delay transfer function of the different regions of the binary. We have used time delay transfer functions, along with the known binary parameters to find best-fitting solutions to the data along with their corresponding confidence regions.

There is evidence for a larger fraction of disc reprocessing. The geometric parameters determined from our fit give an opening angle of 19 degrees and a very flared geometric shape, implying that most of the reprocessing is taking place in the outer regions of the disc. This means that the companion star is almost entirely shielded from X-rays, which in turn reduces the mass accretion rate, which is driven by irradiation. The observations of GRO J1655-40 took place during outburst, which could explain the flared shape of the outer disc. Another possible interpretation is that there is a localized region of enhanced reprocessing in the outer disc that is non-axisymmetric that is adding a large component from the outer disc region to the transfer function but is difficult to distinguish from reprocessing from a thick disc. This would explain the high value of $b$ observed, as the model attempts to move all the reprocessing to the outer disc. The most likely site for this reprocessing would be at the disc-stream interaction point, where the rim of the disc swells greatly. This inhomogeneity is observed in GRO J1655-40 as X-ray dips around binary phase 0.8. This is the site proposed for the reprocessing of the X-ray bursts seen in 4U/MXB 1636-53 (Matsuoka et al. 1984).

The effect of reprocessing time-scales in different regions of the binary also needs to be studied in more detail, with detailed radiative transfer models. The incidence angle of the X-rays to the atmosphere may cause large variations in the time-scale. Normally incident X-ray photons may be reprocessed deeper within the companion star and hence take longer to diffuse to the surface, than those with a grazing incidence angle. This would also affect the ratio of disc to companion star reprocessing, as the incidence angles for the disc will be predominantly grazing.

In order to distinguish between these scenarios and determine the importance of reprocessing time-scales it is necessary to have data with better phase coverage to observe a change in the time delay of this enhanced region as a function of binary phase and any reprocessing time-scale effects. It is clear from our analysis that the companion star is responsible for a small fraction of the instantaneously reprocessed flux, see Fig. 6.

These are the first data showing correlated X-ray and optical variability with sufficient time resolution and a long enough baseline to perform this form of echo-tomography. We have used the time delayed optical variability observed from GRO J1655-40 to constrain the binary parameters of these systems and find that both the accretion disc and companion star can be important regions for reprocessing in XRBs.

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