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discussion

Elasto-Plastic Indentation of a Layered Medium[1]

H. A. FRANCIS.[2] The development of numerical finite-element models is undoubtedly the most fruitful theoretical approach for predicting and analyzing the complex strain field produced by plastic indentation. The authors' model for spherical indentation of an elastic-plastic semi-infinite layered body has yielded many interesting results. It is felt that some of these results can be enhanced by clarification of several points.

The results have been presented as functions of the independent geometric variable D/H (indentation depth/layer thickness). However, the ratio D/H alone does not completely define the contact. The deformation mechanics must also depend on the geometry of the spherical segment of material displaced by the indenter (radius R), as specified by the ratio D/R. For Fig. 9, $R/H = 20$; was this value used for the other results as well?

Fig. 8, which purports to show that the shape of the plastic boundary under load is strongly dependent on Y_2/Y_1, in fact represents a contradiction. As long as (1) the substrate is not softer than the layer ($Y_2 \geq Y_1$), (2) the stress invariant J_2 decreases with depth z (which seems intuitively reasonable, considering the geometry of the plastic boundary), and (3) the plastic boundary does not extend to the layer-substrate interface, the solution must be independent of the substrate yield stress Y_2. It can therefore be concluded solely from the loaded plastic boundary for $Y_2/Y_1 = 4.75$ in Fig. 8 that during loading up to this load the solution is the same as for a homogeneous body ($E_1 = E_2$, $Y_1 = Y_2$). Thus, the two plastic boundaries shown for $Y_2/Y_1 = 1.0, 4.75$ are mutually inconsistent.

The surface profiles shown in Fig. 9 demonstrate that the deformation depends on the rigidity of the substrate and the frictional characteristics of the layer-substrate interface. It is well known that the nature of the irreversible changes in surface shape outside the contact also depends on the strain-hardening rate E_T. For plastic spherical indentation, the residual normal surface displacement $s(r)$ (measured positive outward) always has a positive maximum which for low E_T is near or at $r = a$ and whose distance from $r = a$ in general increases with E_T [1, 2].[3] For low E_T, $s(a) > 0$ ("piling up"), and for high E_T, $s(a) < 0$ ("sinking in") [3, 4, 5]. Thus the residual displacement field is strongly dependent on the flow stress gradient in the plastic zone. For $Y/E = 0.00084$, the value $E_T/E = 0.244$ given in Fig. 3 gives, practically speaking, a very high strain-hardening rate: at a uniaxial plastic strain of 0.01, the uniaxial flow stress is $4.8Y$. Assuming $E_T/E = 0.244$ for Fig. 9, it is, therefore, realistic that the unloaded contact perimeter lies below the original surface level. Did the authors investigate the non-strain-hardening case $E_T = 0$, and, if so, did it give the expected "piling up" behavior ($s(a) > 0$)? Were any other results of the model found to be dependent on E_T/E? For example, the dimensions of the plastic boundary have been found to increase with E_T [4, 5].

References

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2 Ichihara, M., "A Contribution to Brinell Ball Hardness Tests," *Technology Reports Tohoku Imp. Univ.*, Vol. 10, 1931, pp. 25–41.

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5 Francis, H. A., "Phenomenological Analysis of Plastic Spherical Indentation," To be published.

Authors' Closure

The authors wish to thank the discusser for his pertinent comments on their paper. Several points raised in the discussion require further clarification.

It is indeed true, as the discusser noted, that the geometry of the indenter affects the deformation in the indented body. In order to neglect the influence of this factor, the indenter geometry was kept the same for all reported cases, with the ratio of indenter radius R/layer thickness H remaining constant at 19.7. This ratio was inadvertently not reported in the paper.

The discusser has, evidently, misinterpreted the results shown in Fig. 8 of the paper. The results, which are not at all contradictory, indicate that when a given layer of thickness H is indented to a depth D the region of plastic deformation in the layer may be affected by the yield stress of the substrate. In the case shown, when the substrate was exactly the same material as the layer ($E_2/E_1 = \nu_2/\nu_1 = Y_2/Y_1 = 1.0$) the elastic-plastic boundary extended below the layer-substrate interface (dotted curve in Fig. 8). When the substrate was replaced by another of higher yield stress ($Y_2/Y_1 = 4.75$) but otherwise unchanged properties ($E_2/E_1 = \nu_2/\nu_1 = 1.0$), the same indentation produced a different plastic zone (solid curve in Fig. 8). Although the stresses below the layer-substrate interface had been high enough to cause yielding in the first case, the higher yield stress of the second substrate was not exceeded and that substrate remained elastic, necessitating a greater amount of plastic deformation in the layer for the second case. Therefore the substrate yield stress does influence the deformation in the indented layer as well as in the substrate.

The third point raised by the discusser, concerning the effect of strain hardening rate on deformation, is a valid one. Although this effect was not investigated in the reported study, it was found that there is a greater tendency for sinking in to occur for larger values of E_T/E. As shown in Fig. 9 of the paper, the only time piling up was found to occur upon indentation of a strain hardening layer was when the thin layer was adhered to a rigid substrate. The case $E_T = 0$ was not investigated in any detail, but there were indications that piling up behavior does occur in such a case. Incidentally, the strain hardening rate reported in Fig. 3 is in error and should be $E_T/E = 0.0244$ instead of 0.244. The authors thank the discusser for bringing the error to their attention.

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[1] By F. E. Kennedy and F. F. Ling, published in the April, 1974, issue of the JOURNAL OF ENGINEERING MATERIALS AND TECHNOLOGY, TRANS. ASME, Series H, Apr. 1974, pp. 97–103.

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[3] Numbers in brackets designate References at end of discussion.