An investigation of gravitational lens determinations of $H_0$ in quintessence cosmologies

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ABSTRACT

There is growing evidence that the majority of the energy density of the Universe is not baryonic or dark matter, but rather it resides in an exotic component with negative pressure. The nature of this ‘quintessence’ influences our view of the Universe, modifying angular diameter and luminosity distances. Here, we examine the influence of a quintessence component upon gravitational lens time-delays. As well as a static quintessence component, an evolving equation of state is also considered. It is found that the equation of state of the quintessence component and its evolution influence the value of the Hubble constant derived from gravitational lenses. However, the differences between evolving and non-evolving cosmologies are relatively small. We undertake a suite of Monte Carlo simulations to examine the potential constraints that can be placed on the universal equation of state from the monitoring of gravitational lens systems, and demonstrate that at least an order of magnitude more lenses than currently known will have to be discovered and analysed to accurately probe any quintessence component.

Key words: gravitational lensing – cosmological parameters – cosmology: theory.

1 INTRODUCTION

The searches for supernovae at cosmological distances have proved very successful, providing evidence that, while topologically flat, the majority of energy in the Universe is in the form of an exotic component with negative pressure (Riess et al. 1998; Perlmutter et al. 1999). The recent identification of a supernova at z = 1.7 (Riess et al. 2001) has provided further weight to these claims (Turner & Riess 2002), which suggest that this component may differ from the classical cosmological constant $\Lambda$. Termed ‘quintessence’, or more colloquially ‘dark energy’, this has an equation of state of $w < 0$ equating to non-relativistic matter (dust), $w = 1/3$, being radiation and $w = -1$, a classical cosmological constant. More exotic components are: massless scalar fields $w = 1$, cosmic string networks $w = -1/3$, and two-dimensional topological defects $w = -2/3$. As well as the supernova programs, other approaches, such as gravitational lens statistics (Cooray & Huterer 1999), geometrical probes of the Ly$\alpha$ forest (Hui, Stebbins & Burles 1999) and galaxy distributions (Yamamoto & Nishioka 2001), and classical angular-size redshift tests (Lima & Alcaniz 2000), will provide complementary probes of the universal equation of state.

The value of the quintessence component, $w$, influences our view of the Universe, modifying the various distances used in mapping the cosmos. This paper concerns itself with the influence of $w$ on angular diameter distances, especially in relation to the determination of the Hubble constant from the measurement of time-delays in gravitational lens systems. Unlike local determinations of Hubble’s constant (e.g. Freedman et al. 2001), the cosmological nature of gravitational lenses means that they are more sensitive to the underlying cosmological parameters. Section 2 briefly covers the basic formulae for generalized angular diameter distances in quintessence cosmologies, while in Section 3 we consider the influence of $w$ on the determination of $H_0$ from lensed systems. Section 4 extends this analysis to simple models of an evolving quintessence component. In Section 5 a series of Monte Carlo simulations are undertaken to estimate the efficacy of this approach in probing the cosmological equation of state, while in Section 6 we speculate on the possibility that current observations of gravitational lens systems may suggest that $w < -1$. The conclusions of this study are presented in Section 7.

2 GENERALIZED ANGULAR DIAMETER DISTANCES

While there has been a resurgence in quintessence cosmology, the generalized cosmological equations for such universes were
presented more than a decade ago by Linder (1988a,b, including a
generalized form of the Dyer–Roeder equation for the evolution of
a bundle of rays travelling from a distant source (Dyer & Roeder
1973). Expressing the angular diameter distance as \(D = (c/H_0)r\),
the generalized beam equation is given by
\[
\ddot{r} + \left( \frac{3 + q(z)}{1 + z} \right) \dot{r} + \sum_w \frac{3r(1 + w)\Omega_w(z)\Omega_w(\dot{z})}{2(1 + z)^2} = 0,
\]
(1)
where \(\Omega_w(z)\) is the density, in units of the critical density \(\rho_c(z)\), of
an equation of state parameter and is given by
\[
\Omega_w(z) \equiv \frac{\rho_w(z)}{\rho_c(z)} = \Omega_w(0)(1 + z)^{3(1 + w)} - \frac{H_0^2}{H(z)^2},
\]
(2)
where \(\Omega_w(0)\) is the contribution of this component to the present
equation-density budget at the present epoch, and the critical density
is given by \(\rho_c(z) = 3H(1)^2/8\pi G\). Here, \(H(z)\) is a generalized form
of the Hubble parameter and is given by
\[
H(z) = H_0 \left[ \sum_w \Omega_w(0)(1 + z)^{3(1 + w)} - K(1 + z)^2 \right]^{1/2},
\]
(3)
where \(K = \Omega_m - 1\), and \(\Omega_m \equiv \Omega(0) = \sum_w \Omega_w(0)\), are related to
the overall curvature of the Universe. \(q(z)\) is a generalized form
of the deceleration parameter and is given by
\[
q(z) = \frac{1}{2} \sum_w \Omega_w(0)(1 + 3w)(1 + z)^{1 + 3w} - K.
\]
(4)
Finally, equation (1) also contains the parameter \(\alpha_w(z)\), which
represents how much of the fluid lies in the beam and influences the
evolution of a ray bundle. For a universe containing matter, the
solution to equation (1) with \(\alpha_w(z) = 1\) represents the classic Dyer–
Roeder ‘filled beam’ distance, while \(\alpha_w(z) = 0\) is the ‘empty beam’
distance (Dyer & Roeder 1973).

When solving equation (1), the boundary conditions need to be
defined. These are
\[
r(z_0, z_0) = 0,
\]
(5)
\[
\frac{dr(z_0, z)}{dz} \bigg|_{z=z_0} = (1 + z_0)^{-1} \left[ \frac{H_0}{H(z_0)} \right].
\]
(6)
Equation (1) was integrated using a Runge–Kutta scheme (the
RKSUITE package from http://www.netlib.org) and compared to both
the analytic results and the minimum angular extent redshifts in
quintessence cosmologies as tabulated in Lima & Alcaniz (2000);
excellent agreement was found. Throughout this work, filled-beam
distances \(\alpha_w(z) = 1\) are employed.

3 GRAVITATIONAL LENS TIME-DELAYS

Refsdal (1964) was the first to note that cosmological parameters
could be determined from the measurement of a time-delay between
the relative paths taken by light though a gravitational lens system.
Since the discovery of multiply-imaged quasars, this has become
the goal of a number of monitoring campaigns (e.g. Cohen et al.
2000; Oscoz et al. 2001; Patnaik & Narasimha 2001), although
these analyses are frustrated by degeneracies in the derived mass
models. The cosmological model simply enters the determination
of Hubble’s constant;
\[
H_0 \Delta \tau \propto \frac{r_{a}r_{o}}{r_b},
\]
(7)
where \(r_{ij}\) are the normalized angular diameter distances between
and observer (o), lens (l) and source (s), and \(\Delta \tau\) is the measured
time-delay between an image pair.

Giovi & Amendola (2001) examined the influence on a quint-
nessence component on gravitational lens time-delays and the
determination of Hubble’s constant. Their analysis, however, was mainly
centered with the influence of the clumping of material and the
dependence on \(H_0\) of whether distances are empty-beam or full-
beam. Here, a different approach is considered; solving equation (1),
equation (7) is evaluated for a range of quintessence components.

For the study, the redshifts of the lens and source in seven gravita-
tional lens systems that are favourable for time-delay measures were
considered [table 2 in Giovi & Amendola (2001), Q0957+561 and
B1608+656], plus two fiducial redshift pairs of \((z_l = 1.5, z_s = 2.5)\)
and \((z_l = 1.0, z_s = 2.0)\). While there are currently no lensed systems
with established time-delays at these particular redshifts, there are
several potential systems; e.g. the quadruple lens H1413+117 at a
redshift of 2.55, with a lens redshift, established from prominent
absorption features, at \(\sim 1.5\) and HE1104–1805 at a redshift of 2.31
and an estimated lens redshift of \(z \sim 1\). It is assumed throughout
that the Universe is flat, \(\Omega_m + \Omega_w = 1\).

Fig. 1 presents the results of this analysis; six panels are presented,
each for a different combination of \(\Omega_m\) and \(\Omega_w\). A grey-scale-coded
line for each redshift pair is presented. The abscissa presents the
equation of state parameter, \(w\), while the ordinate presents the rel-
ative time-delay; this represents the change in the time-delay for a
fixed Hubble’s constant. Conversely, this is the relative value of \(H_0\)
for a system with a fixed time-delay. Each curve is normalized to
the minimum value of the time-delay. The relative time-delay de-

deps quite strongly on the value of \(w\), with the \((z_l = 1.5, z_s \approx 2.5)\)
possessing changes of \(\sim 40\) per cent at \(w = 1\) as compared to \(w = 0\),
although, for the observed lensing systems, the same range in \(w\) pro-
duces a change of \(\sim 10\) per cent in the same quantity. Interestingly,
for a fixed combination of \(\Omega_m\) and \(\Omega_w\), all the curves, irrespective of
the redshift of the lens and source possess a minimum at the same
value of \(w\). The location of the minimum is tabulated in Table 1.

At present, the general analytic solution to equation (1) is quite
complex (Giovi & Amendola 2001) and it is difficult to further
analyze the minimum seen in Fig. 1. For one case, where \(\Omega_m = 0.0\)
and \(\Omega_w = 1.0\), however, analytic solutions for the relative angular
distance are straight forward. As the overall curvature is flat, the angular diameter distance between us and a distant source is
\[
D_A(z) = \frac{D(z)}{1 + z}
\]
(8)
where \(D(z)\) is the comoving distance and is given by
\[
D(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}
\]
(9)

Table 1. The minima for the curves in Fig. 1 for the various cosmologies under consideration.

<table>
<thead>
<tr>
<th>(\Omega_m)</th>
<th>(\Omega_w)</th>
<th>(w_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>-0.33</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>-0.22</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>-0.15</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>-0.12</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>-0.10</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
The redshift of the lens-source pair is given in brackets. Each panel presents a different combination of $\Omega_m$ and $\Omega_w$; note that $w = 0$ corresponds to a universe composed entirely of matter in all cases.

where $E(z) = \sqrt{2}\Omega_0(1 + z)^{\frac{2}{3}(1+w)}$. Again, as the overall cosmology is flat, the angular diameter distance between the source and the lens is given by:

$$D_L(z_1, z_2) = \frac{1}{1 + z_2} \left[ D_L(z_2) - D_L(z_1) \right]$$

(10)

With this, the relative time-delays for differing values of $w$ are seen to be:

$$H_0 [z_1, z_2] \propto \frac{2}{1 + z_1} \frac{2}{1 + 3w} \left[ \frac{1 - (1 + z_1)^f}{(1 + z_1)^f - (1 + z_2)^f} \right]$$

(11)

where $f = -(1/2)(1 + 3w)$. This function possesses a minimum at $w = -1/3$, which is independent for the redshift pairs under consideration, as seen in Fig. 1.

The preceding section has considered arbitrary combinations of cosmological parameters. Here, the impact of the choice of $w$ on the estimation of $H_0$ in the favoured cosmological model with $\Omega_m = 0.3$ and $\Omega_w = 0.7$ (top right panel in Fig. 1) is examined in more detail.

Often, a cosmological model where $w < -1$ results in $H_0$ values that are $\sim 5$ per cent different to the all matter case ($w = 0$). For several combinations of redshifts, however, (including the chosen fiducial points) more substantial differences of $\sim 12–15$ per cent in the determined value of $H_0$. Considering $w < -1$ results in more significant discrepancies, a point that will be returned to later.

### 4 EVOLVING QUINTESSENCE

One interesting aspect of quintessence cosmology is that, unlike classical $\Omega_m$, $\Omega_w$ cosmologies, it is possible that the equation of state $w$ may vary over the cosmic history of the Universe. Typically, a linear model where $w(z) \sim w_0 + w_1 z$ has been adopted (e.g. Goliath et al. 2001). In the following analysis, it is assumed that $\Omega_m = 0.3$ and $\Omega_w = 0.7$, which is consistent with the recent supernova experiments (Riess et al. 1999; Perlmutter et al. 1999) results. Four models for the evolution of $w$ are adopted (taken from Linder 2001), namely, $(w_0, w_1) = (-0.7, 0.2), (-0.7, 0.4), (-1.0, 0.2)$ and $(-1.0, 0.4)$. As the cosmology is flat, we follow the approach outlined in equations (8), (9) and (10), with

$E(z) = \sqrt{(1 + z)^3 \Omega_m + f_w(z) \Omega_w}$

(12)

and

$f_w(z) = \exp \left[ 3 \int_0^z \frac{1 + w(z')}{1 + z'} \, dz' \right]$.

(13)
where diameter distance in a non-evolving cosmology, with a constant and one where \( w / \Delta z_1 \) where above, for the lens systems in Section 3. The largest changes are measured Hubble constant for the linear evolution models described angular diameter distances. Table 2 presents the relative values in the 'asymptote seen.

With the values under consideration in Fig. 3, this is precisely the what counter-intuitive, given the convergence of distances for an Fig. 3, they converge to a non-zero value. This seems some-
what interesting, ensuring that for each a time-delay could be determined to a specified accuracy. For real lensed systems, additional uncertainty is introduced from the lens modelling. Here, however, it is assumed that the lens modelling introduces no systematic uncertainty and that the random error is taken into account as an additional source of uncertainty in the time-delay.

6. The lower panel presents the percentage difference between the evolving models for an observer at \( z = 0 \) and \( z = 1 \), respectively. It is interesting to note that for an observer at redshifts greater than zero the fractional difference between the angular diameter distances for an evolving and a non-evolving quintessence component does not converge to zero as \( z \to z_1 \), rather, as seen in the lower box of Fig. 3, they converge to a non-zero value. This seems somewhat.

Figs 2 and 3 present the angular diameter distance in these evolving models for an observer at \( z = 0 \) and \( z = 1 \), respectively. It is interesting to note that for an observer at redshifts greater than zero the fractional difference between the angular diameter distances for an evolving and a non-evolving quintessence component does not converge to zero as \( z \to z_1 \), rather, as seen in the lower box of Fig. 3, they converge to a non-zero value. This seems somewhat-interesting, given the convergence of distances for an evolving and non-evolving quintessence component does not differ from non-evolving models: this difference is not substantial, being typically less than a few per cent for evolution models with \( w_1 = 0.4 \).

5 MONTE CARLO SIMULATIONS

How many lensed systems are required before firm limits can be made? To address this question a number of Monte Carlo simulations were undertaken with a population of lensed sources, assuming that for each a time-delay could be determined to a specified accuracy. For real lensed systems, additional uncertainty is introduced from the lens modelling. Here, however, it is assumed that the lens modelling introduces no systematic uncertainty and that the random error is taken into account as an additional source of uncertainty in the time-delay.

A number of programs have examined the relative statistics of gravitational lensing (e.g. Kochanek 1993a,b; Williams 1998; Falco, Kochanek & Munoz 1998; Helbig et al. 1999). The complex analyses in these studies go beyond this paper and a more straightforward approach was adopted; this was to examine the distribution of source and lens redshift pairs in the CASTLES data base (Muñoz et al. 1998). This is seen to be a scatter plot of sources between redshifts 1 and 4, with lenses between 0.2 and 1.1. Source (\( z_s \)) and lens (\( z_l \)) redshifts were, therefore, selected randomly in these ranges, but ensuring that \( z_s > 4z_l - 2.2 \). This cut ensures that lens and sources are always reasonably separated in redshift space.

\[
D_{\Lambda}(z, z_1 + \Delta z) \sim \frac{\Delta z}{1 + z_1} \frac{dD_{\Lambda}(z)}{dz} \bigg|_{z = 1}, \tag{14}
\]

where \( \Delta z \ll 1 + z_1 \). Therefore, the asymptotic ratio of the angular diameter distance in a non-evolving cosmology, with a constant \( w_0 \), and one where \( w(z) = w_0 + zw_1 \), as \( z_2 \to z_1 \), simply becomes

\[
\frac{D_{\Lambda}(z_1, w(z))}{D_{\Lambda}(z_1, w_0)} = \sqrt{\frac{\Omega_m(1 + z_1)^3 + \Omega_k(1 + z_1)^3 + \Omega_\Lambda g(z_1)}{\Omega_m(1 + z_1)^3 + \Omega_k(1 + z_1)^3 + \Omega_\Lambda g(z_1)}}, \tag{15}
\]

where

\[
g(z) = \exp \left[ 3zw_1 + (1 + w_0 - w_1) \log(1 + z) \right]. \tag{16}
\]

With the values under consideration in Fig. 3, this is precisely the asymptote seen.

As seen from equation (7), the cosmological component of the measure of Hubble’s constant is dependent upon a combination of angular diameter distances. Table 2 presents the relative values in the measured Hubble constant for the linear evolution models described above, for the lens systems in Section 3. The largest changes are seen in the first two rows which represent fiducial models; these possess the highest lens redshifts in the sample. For the observed lens system, it is clear that the evolving quintessence models do differ from non-evolving models: this difference is not substantial, being typically less than a few per cent for evolution models with \( w_1 = 0.4 \).
As for Fig. 2, but for an observer at $z = 1.0$.

Table 2. The ratio of the time-delay (or conversely $H_0$) for the cosmologies with linearly evolving quintessence. The first two columns are the lens and source redshifts, while the next four columns present the time-delays for $(w_0, w_1)$. Those with $w_0 = -1$ are normalized with respect to $(-1.0, 0.0)$, while those with $w_0 = -0.7$ are normalized with respect to $(-0.7, 0.0)$.

<table>
<thead>
<tr>
<th>$z_l$</th>
<th>$z_s$</th>
<th>$(1.0, 0.2)$</th>
<th>$(1.0, 0.4)$</th>
<th>$(0.2, 0.2)$</th>
<th>$(0.2, 0.4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50</td>
<td>2.50</td>
<td>0.997</td>
<td>1.003</td>
<td>1.012</td>
<td>1.043</td>
</tr>
<tr>
<td>1.00</td>
<td>2.00</td>
<td>1.000</td>
<td>1.005</td>
<td>1.010</td>
<td>1.029</td>
</tr>
<tr>
<td>0.68</td>
<td>0.96</td>
<td>1.003</td>
<td>1.007</td>
<td>1.007</td>
<td>1.015</td>
</tr>
<tr>
<td>0.77</td>
<td>2.32</td>
<td>1.001</td>
<td>1.005</td>
<td>1.009</td>
<td>1.026</td>
</tr>
<tr>
<td>0.31</td>
<td>1.72</td>
<td>1.002</td>
<td>1.005</td>
<td>1.005</td>
<td>1.012</td>
</tr>
<tr>
<td>0.42</td>
<td>1.59</td>
<td>1.002</td>
<td>1.006</td>
<td>1.006</td>
<td>1.014</td>
</tr>
<tr>
<td>0.89</td>
<td>2.51</td>
<td>1.000</td>
<td>1.005</td>
<td>1.010</td>
<td>1.030</td>
</tr>
<tr>
<td>0.36</td>
<td>1.41</td>
<td>1.002</td>
<td>1.006</td>
<td>1.005</td>
<td>1.012</td>
</tr>
<tr>
<td>0.63</td>
<td>1.34</td>
<td>1.002</td>
<td>1.006</td>
<td>1.008</td>
<td>1.017</td>
</tr>
</tbody>
</table>

In choosing a background cosmology, a fiducial model with $\Omega_m = 0.3$ and $\Omega_w = 0.7$ with a $w = -1$ and $H_0 = 1$ was employed. In an ideal universe, where there are no measurement errors and where gravitational lens models can be uniquely determined, the analysis of a sample of systems may still produce a distribution of $H_0$, even if the quintessence equation of state $w$, influences differently each system (e.g. Fig. 1). In this ideal universe, however, all one would need to do is to vary the cosmological parameters until all systems yielded the same value for the Hubble constant. In this way, not only would $H_0$ be measured, but the underlying cosmology would be determined as well.

In the real Universe, however, the influence of measurement uncertainties needs to be considered. Instead of obtaining a unique value of Hubble’s constant, one could vary the cosmological parameters such that the dispersion in the resultant $H_0$s are minimized. An example of this for a population of one hundred gravitational lens systems is given in Fig. 4; note that the function that is minimized is $\sigma_{H_0}/(H_0)$ as it is the fractional dispersion of the results that is of interest. The dot-dashed line is the ideal universe case, where there are no sources of uncertainty, which possesses a minimum value.
Figure 5. The results of the Monte Carlo simulations of the determination of the quintessence equation of state and Hubble’s constant from the measurement of samples of gravitational lens time-delays. The black line assumes an uncertainty in the time-delay and lens modelling of 5 per cent, the lighter, grey curve of 10 per cent and the lightest grey curve 15 per cent.

(of zero) at the chosen fiducial value. The solid curves represent varying values of noise in the determination of the time-delay, the black being 5 per cent, grey 10 per cent and light grey 15 per cent noise. Clearly this function broadens as more noise is added, and the minimum is not necessarily at the fiducial value of \( \omega \).

Fig. 5 presents the results of undertaking this procedure for various samples of gravitational lens systems. The left-hand panel presents the probability distribution for the quintessence equation of state \( \omega \) while the right hand panel is the corresponding distribution in Hubble constants; note that all the distributions have been normalized to a peak value of one. The top row is for a sample of 10 gravitational lens systems, akin to the situation today, followed by 50, 100 and 500 systems. 10 000 realizations were undertaken for each sample size. Each line on the plot corresponds to a different level of uncertainty in the gravitational lens time-delay as outlined in the previous paragraph. Clearly today, where there are but a handful of gravitational lens systems for which we have determined the time-delay, with overall uncertainties exceeding 5–10 per cent, then the resultant Hubble’s constant and \( \omega \) that can be derived from the sample are unlikely to accurately represent the underlying values.

Table 3 presents the 95 per cent confidence interval for the estimation of \( \omega \) and \( H_0 \) for the various samples. Increasing the sample size greatly improves the situation, although even with 500 lenses and 15 per cent noise, the values of \( \omega \) and \( H_0 \) are not strongly constrained. One is led to conclude, therefore, that a large sample of lenses with very accurately determined time-delays and lens models is required to significantly determine the underlying cosmological parameters. Given the observational effort in such a task, other approaches to probing the cosmological equation of state are likely to prove more fruitful. Given this, the analysis was not extended to consider the smaller influences of quintessence evolution (see Section 4).

6 DO CURRENT LENS SYSTEMS SUGGEST \( \omega < -1 \)?

Can the current observations of time-delays in gravitational lens systems tell us anything about the equation of state of the quintessence cosmologies?
component? In recent years, dedicated monitoring of a number of lensed systems has provided accurate (∼10–20 per cent) time-delay determinations (e.g. Fassnacht et al. 1999; Koopmans et al. 2000). An examination of the Hubble constant derived from such studies reveals that, typically, it is less than the value determined from local studies, even accounting for standard cosmological differences (e.g. Impey et al. 1998; Koopmans et al. 2001; Winn et al. 2002). This very question was also recently addressed by Kochanek (2002) who suggests that this discrepancy is potentially due to galaxies possessing concentrated dark matter halos with a constant mass-to-light ratios, at odds with expectations from cold dark matter structure models. Here, an alternative solution is considered.

Examining the panel in Fig. 1 corresponding to an \( \Omega_m = 0.3, \Omega_{\Lambda} = 0.7 \) cosmology, it is apparent that choosing \( \Omega_m = 1 \) (equivalent to setting \( w = 0 \)) results in almost the lowest possible determination of \( H_0 \). Considering a cosmology with a classical cosmological constant \( (w = -1) \) increases the determined value of \( H_0 \) by ∼5–20 per cent (dependent upon the source and lens redshifts), but as noted above, the currently determined values still tend to lie below the 72 km s\(^{-1}\) Mpc\(^{-1}\) derived locally (Freedman et al. 2001). Assuming that the gravitational lens models are correct, one way to reach concordance between the two approaches is that \( w < -1 \); such a conclusion is consistent with the recently derived limit of \( w < -0.85 \) from an analysis of a combination of cosmic microwave background, high-redshift supernovae, cluster abundances and large scale structure data (Wang et al. 2000; Bean & Melchiorri 2001). While tantalizing, however, it must be conceded that the current scale structure data (Wang et al. 2000; Bean & Melchiorri 2001). and thanks the Gorillaz for their self-titled album. Terry Bridges is thanked for providing the computational cycles on ODIN. GFL also acknowledges using David W. Hogg’s wonderful ‘Distance measures in cosmology’ cheat sheet (astro-ph/9905116), and thanks Eric Linder for additional comments on the paper.

Several models of evolving quintessence were also examined, consisting of a linear evolution of the equation of state with redshift. The cosmologies resulted in significantly different forms of the angular diameter distance. Hence, our view of the cosmos would be different in the various cosmologies. When considering the specific combination of angular diameter distances that constitute the cosmological contribution to the gravitational lensing determination of Hubble’s constant, it is seen that the resulting variations between cosmologies is very small, a matter of only a few per cent, relative to an unevolving case with the same present-day constitution.

A number of Monte Carlo simulations of the determination of Hubble’s constant and the quintessence equation of state, \( w \), were undertaken to explore the efficacy of this approach. These revealed that the present situation with only a handful of lensed systems does not allow an accurate determination of the cosmic equation of state, and that at least an order of magnitude more lenses are truly required to provide a reasonably robust determination of the underlying cosmology. The next generation of all-sky surveys are presently underway (e.g. Sloan Digital Sky Survey) or being planned (e.g. VISTA, PRIME), and these data sets will greatly increase the number of lensed quasars available for monitoring studies. Cooray & Huterer (1999) estimate that ∼2000 lensed quasars will be identified from the SDSS data base alone, and a much larger number can be expected from the deeper VISTA and PRIME surveys. The number of these sources amenable for follow-up monitoring campaigns will naturally be much smaller, but one can confidently expect a sample of several hundred systems to eventually become available. However, given the effort required to first find such systems, as well as monitor them to determine the time-delays and the modelling procedure, it is likely that \( w \) will be first determined using one of the other various techniques currently being proposed. We conclude, therefore, that gravitational lens time-delays are likely to prove poor probes of the universal equation of state.

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### REFERENCES


### Table 3

<table>
<thead>
<tr>
<th>Component</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 per cent</td>
<td>−3.52,0.14</td>
<td>−3.39,0.78</td>
<td>−3.25,0.89</td>
<td>−3.10,0.92</td>
</tr>
<tr>
<td>10 per cent</td>
<td>−4.41,0.13</td>
<td>−3.85,0.15</td>
<td>−3.70,0.54</td>
<td>−3.23,0.92</td>
</tr>
<tr>
<td>15 per cent</td>
<td>−4.45,0.13</td>
<td>−4.52,0.13</td>
<td>−4.09,0.14</td>
<td>−4.18,0.88</td>
</tr>
<tr>
<td>5 per cent</td>
<td>0.91,1.30</td>
<td>0.97,1.07</td>
<td>0.98,0.04</td>
<td>0.99,1.02</td>
</tr>
<tr>
<td>10 per cent</td>
<td>0.88,1.39</td>
<td>0.92,1.36</td>
<td>0.95,1.13</td>
<td>0.99,1.05</td>
</tr>
<tr>
<td>15 per cent</td>
<td>0.87,1.42</td>
<td>0.92,1.42</td>
<td>0.93,1.39</td>
<td>0.99,1.10</td>
</tr>
</tbody>
</table>
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