Population effects on the red giant clump absolute magnitude: the $K$ band

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ABSTRACT

We present a detailed analysis of the behaviour of the red clump $K$-band absolute magnitude ($M_k^{RC}$) in simple and composite stellar populations, in light of its use as a standard candle for distance determinations. The advantage of using $M_k^{RC}$, following recent empirical calibrations of its value for the solar neighbourhood, arises from its low sensitivity to extinction by interstellar dust. We show that, as in the case of the $V$- and $I$-band results, $M_k^{RC}$ is a complicated function of the stellar metallicity $Z$ and age $t$. In general, $M_k^{RC}$ is more sensitive to $t$ and $Z$ than $M_I^{RC}$, for high $t$ and low $Z$. Moreover, for ages above $\sim$1.5 Gyr, $M_k^{RC}$ decreases with increasing $Z$, i.e. the opposite behaviour with respect to $M_V^{RC}$ and $M_I^{RC}$.

We provide data and equations that allow the determination of the $K$-band population correction $\Delta M_k^{RC}$ (the difference between the red clump brightness in the solar neighbourhood and in the population under scrutiny) for any generic stellar population. These data complement the results presented by Girardi & Salaris for the $V$- and $I$-band. We show how data from galactic open clusters consistently support our predicted $\Delta M_V^{RC}$, $\Delta M_I^{RC}$ and $\Delta M_K^{RC}$ values.

Multiband $VIK$ population corrections for various galaxy systems are provided. They can be used in conjunction with the method devised by Alves et al., in order to derive simultaneously reddening and distance from the use of $VIK$ observations of red clump stars. We have positively tested this technique on the Galactic globular cluster 47 Tuc, for which both an empirical parallax-based main-sequence-fitting distance and reddening estimates exist. We have also studied the case of using only $V$ and $I$ photometry, recovering consistent results for both reddening and distance. Application of this method to an OGLE-II field, and the results by Alves et al., confirm a Large Magellanic Cloud distance modulus of about 18.50, in agreement with the Hubble Space Telescope extragalactic distance scale zero-point.

Key words: Hertzsprung–Russell (HR) diagram – stars: horizontal branch – solar neighbourhood – Magellanic Clouds – galaxies: stellar content.

1 INTRODUCTION

During the last few years a large body of work has been focused on the use of helium-burning red clump (RC) stars as distance indicator; this has mainly been a result of the fact that Hipparcos parallaxes allows an accurate calibration of the average RC brightness in the solar neighbourhood (as a comparison, allows an accurate calibration of the average RC brightness in the solar neighbourhood, arises from its very low sensitivity to extinction by interstellar dust. We show that, as in the case of the $V$- and $I$-band results, $M_k^{RC}$ is a complicated function of the stellar metallicity $Z$ and age $t$. In general, $M_k^{RC}$ is more sensitive to $t$ and $Z$ than $M_I^{RC}$, for high $t$ and low $Z$. Moreover, for ages above $\sim$1.5 Gyr, $M_k^{RC}$ decreases with increasing $Z$, i.e. the opposite behaviour with respect to $M_V^{RC}$ and $M_I^{RC}$.

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determined with an accuracy of hundreds of a magnitude (Paczynski & Stanek 1998; Stanek, Zaritsky & Harris 1998).

Once the mean apparent magnitude of the RC in a given photometric band, $m^{RC}_{I}$, is measured in a nearby galaxy, its absolute distance modulus $\mu_0 = (m - M)_0$ is easily derived by means of

$$\mu_0 = m^{RC}_{I, \text{local}} - A_m + \Delta M^{RC}_{I}. \quad (2)$$

In this equation, $A_m$ is the interstellar extinction to the RC population of an external galaxy and $\Delta M^{RC}_{I} = M^{RC}_{I, \text{local}} - M^{RC}_{I, \text{galaxy}}$ is the population effect, i.e. the difference of the mean RC absolute magnitude between the local and external samples of stars.

After early claims (Paczynski & Stanek 1998; Udalski et al. 1998; Stanek et al. 1998) based mainly on the constancy of $\mu_0 = (m - M)_0$, this is complex and non-monotonic. This implies that accurate RC distances can be determined only if one can estimate SFR and AMR for the stellar population under scrutiny. If there are no determinations of these two key parameters, errors of up to $\sim 0.3$ mag in the derived distance modulus are to be accepted.

In the specific case of the Large Magellanic Cloud (LMC) distance, which sets the zero-point of the extragalactic distance scale, we obtained a population correction in the $I$ band of $\Delta M^{RC}_{I} = +0.20$. Unfortunately, the evaluation of the RC distance to the LMC is plagued by what possibly are uncertainties in the estimate of the interstellar extinction. Romaniello et al. (2000) and Sakai, Zaritsky & Kennicutt (2000) obtain, respectively, $I^{RC}_{0} = 18.12 \pm 0.02$ and $I^{RC}_{0} = 18.06 \pm 0.02$ from Hubble Space Telescope (HST) data, after deriving the extinction to individual stars from multiband colour fitting using theoretical model atmospheres results; these values, used in conjunction with the appropriate evolutionary correction and $M^{RC}_{I} = -2.33 \pm 0.03$ for the local RC, provide $\mu_0 = 18.55 \pm 0.04$ and $\mu_0 = 18.49 \pm 0.04$, respectively. On the other hand, Udalski (2000) has obtained $I^{RC}_{0} = 17.94 \pm 0.05$, using different photometric data and determining the average stellar reddening from the COBE/DIRBE reddening maps (Schlegel, Finkbeiner & Davis 1998), which gives a much shorter value, $\mu_0 = 18.37 \pm 0.06$.

To overcome uncertainties related to the extinction correction, Alves (2000) and Grocholski & Sarajedini (2002) have recently discussed the use of the RC in the $K$ band. The main advantage of working in this wavelength range is a very much reduced sensitivity to the extinction ($A_K \sim 0.2A_I$). Alves (2000) has derived $M^{RC}_{K} = -1.61 \pm 0.03$ for the local RC stars with Hipparcos parallaxes and spectroscopic metallicities, and obtained a distance to the Galactic centre ($\mu_0 = 14.58 \pm 0.11$ or $8.24 \pm 0.42$ kpc) using the observed RC $K$-band magnitude for a sample of Baade’s window RC stars with the same mean metallicity of the local sample. The underlying assumption is that there is no potential age effect on the $K$-band RC level. Grocholski & Sarajedini (2002) have tested the behaviour of $M^{RC}_{K}$ on a sample of single-age, single-metallicity stellar populations, constituted by 14 Galactic open clusters and two globular clusters with independent estimates of reddening and distances, finding a good agreement with the behaviour predicted by the theoretical models by Girardi et al. (2000), which are the same as used in Papers I and II. Moreover, Alves et al. (2002) have used simultaneously $V$, $I$ and $K$ photometry of LMC RC stars to determine both the LMC distance and the extinction for the observed RC population. They have demonstrated the need to apply population corrections to the RC absolute brightness determined from the local clump, otherwise a negative extinction is obtained; their final result $\mu_0 = 18.506 \pm 0.033$ for the LMC ($\mu_0 = 18.493 \pm 0.033 \pm 0.03$ random plus systematic error) is, after correcting for the location of the observed fields with respect to the LMC centre, and including an estimate of the probable systematic errors involved) and $E(B - V) = 0.089 \pm 0.015$ was obtained by making use of evolutionary corrections derived with the techniques and models presented in Paper II.

In light of these results we consider it important to investigate theoretically the properties of the RC in the $K$ band, as well as to assess the simultaneous reliability of the population corrections to the $V$, $I$ and $K$ bands for a given stellar population.

In Section 2 we discuss in detail the properties of the RC in the $K$ band and its dependence on age and metallicity, while in Section 3 we perform further tests to assess the reliability of the population corrections predicted by theory.

In Section 4 we present multiband population corrections for the galaxy systems discussed in Paper II; in addition, we will discuss the technique used by Alves et al. (2002) for their LMC distance determination, and apply it to the case of the Galactic globular cluster 47 Tuc (for which reddening estimates and an accurate empirical main-sequence-fitting distance do exist) and to a field of the LMC with independent reddening determinations.

Our final conclusions are presented in Section 5.

2 THE THEORETICAL RED CLUMP LOCATION IN THE $K$ BAND

As in Papers I and II we will base the discussion of the model behaviour on the set of evolutionary tracks and isochrones from Girardi et al. (2000). We remark that different models in the literature present systematic luminosity differences for the core helium-burning (CHb) stars that have passed through the helium flash – which are the stars belonging to the observed RC populations – owing mainly to the different values of the helium core mass at the flash (see, e.g. Castellani et al. 2000; Salaris, Cassisi & Weiss 2002); however, the variation of their brightness with respect to age and metallicity is much more consistently predicted by theory (Castellani et al. 2000; Salaris et al. 2002). As in Paper II, the main results of this paper will be based on the strictly differential use of the model predictions.

We start our analysis by showing in Fig. 1 the location of RC stars at the beginning of the He-burning phase on a magnitude–effective temperature plane, in simple stellar populations (single-age, single-metallicity) of various ages between 1 and 14 Gyr, and two different metallicities1 ($Z = 0.001$ and a solar $Z = 0.019$); the behaviour of the bolometric luminosity as well as the $V$, $I$- and $K$-band brightnesses are determined with an accuracy of hundreds of a magnitude (Paczynski & Stanek 1998; Stanek, Zaritsky & Harris 1998).

1 In the course of the paper we will also denote the metallicity of a given stellar population with the spectroscopic notation $[M/H] = \log(M/H)_{\text{phot}} - \log(M/H)_{\odot}$, where $M$ is the total metal abundance. $[M/H]$ is related to $Z$ by means of the approximate relation $[M/H] = \log(Z/Z_{\odot})$, where we assume $Z_{\odot} = 0.019$. In case of a scaled solar metal distribution $[M/H] = [\text{Fe/H}].$
at high metallicities. This is a straightforward consequence of the effect for high ages and a reduced one for lower ages, especially of the stellar brightness on the He-core mass. As for the dependence since MK

Lower effective temperatures correspond to higher values of BC K

sensitivity of BC K

V

while the bolometric, brightness is displayed. One striking feature is readily apparent; for giants in the K band, for both Z = 0.019 (dashed line) and Z = 0.0004 (solid line).

is obvious that the expected differential properties of the mean RC brightness will somehow mirror the results shown in Fig. 1.

In Paper II we have discussed in detail how to derive the mean RC level for a given stellar population. The most detailed approach comes from a population synthesis algorithm; a synthetic CMD is produced for a given stellar population model, and then M 3 K

is derived by fitting equation (1) to the synthetic RC data. This is the method we have followed to produce the population corrections presented in Paper II and those we also discuss in this paper. However, it is very instructive to summarize the simpler approach presented in Paper II and to CHeB stars:

\[ \langle M_i(t, Z) \rangle = -2.5 \log \left[ \frac{1}{N_{cl}(t, Z)} \int_{M_i}^{M_{i+1}} \phi(m_i) 10^{-0.4M_i} dm_i \right] \]  

(3)

where \( M_i \) is the absolute magnitude in the passband \( \lambda \), \( m_i \) is the initial mass of the star at each isochrone point, and \( \phi(m_i) \) is the Salpeter initial mass function (IMF) (the number of stars with initial mass in the interval \([m_i, m_i + dm_i])\). \( N_{cl} \) is the number of clump stars (at age \( t \)) per unit mass of stars initially born. It is simply given by the integral of the IMF by number, along the CHeB isochrone section, i.e.

\[ N_{cl}(t, Z) = \int_{M_i}^{M_{i+1}} \phi(m_i) dm_i \]  

(4)

From equation (3), accurate values for the mean clump magnitudes can be determined in the case of a simple stellar population. These mean magnitudes enter straightforwardly in the computation of the mean clump magnitude for a given galaxy model of total age \( T \). In fact, in this case one needs to perform the following integral:

\[ \langle M_i(gal) \rangle = \frac{1}{N_{cl}(gal)} \int_0^T N_{cl}(t, Z) \psi(t) \langle M_i(t, Z) \rangle dt \]  

(5)

Figure 1. The location of RC stars at the beginning of the He-burning phase on an absolute magnitude–effective temperature plane, in simple stellar populations (single-age, single-metallicity) of various ages (ages of 2, 3, 4, 5, 6, 8, 9, 10, 11, 13, 14 Gyr moving clockwise; for each metallicity the oldest RC is also the faintest one) and two different metallicities (Z = 0.001 and 0.019). The age sequences are shown for, from top to bottom, K, I, bolometric, and V magnitude.

Figure 2. Bolometric corrections for giants in the K band, for both Z = 0.019 (dashed line) and Z = 0.0004 (solid line).
\[ N_c(gal) = \int_{t=0}^{T} N_c(t, Z) \psi(t) \, dt. \quad (6) \]

The function \( \psi(t) \) is the SFR (in \( \text{M}_\odot \text{by unit time} \)) at a moment \( t \) in the past, for the galaxy model considered; in addition, also the AMR \( Z_c(t) \) should be specified. One can average the magnitudes in equation (5), instead of luminosities as in the previous equation (3); the reason is, as explained in Paper II, that in this way one obtains a quantity similar to the \( M_{RC} \) derived by means of equation (1). As discussed in Paper II, the population corrections evaluated with this approach are in very good agreement with the results obtained from the population synthesis approach. They are directly evaluated from the average RC brightness of the elementary simple populations, for which properties are highlighted in the following.

Table 1 presents the \( K \)-band mean absolute magnitude of clump stars, \( \langle M_K \rangle \), as a function of age and metallicity, as derived from Girardi et al. (2000) isochrones by means of equation (3), together with the values of \( N_c(t, Z) \). This table can be used to easily derive the population correction in the \( K \) band for any galaxy model. The same information is provided for the \( V \) and \( I \) band in table 1 of Paper II. It is important to note the pronounced maximum of \( N_c \) for ages between 1 and 2 Gyr, which causes these relatively younger clump stars to be very numerous in galaxies with recent star formation. This aspect is thoroughly discussed in Paper II, and in Girardi (1999).

Fig. 3 portrays the behaviour of the mean RC magnitude in simple stellar populations of different metallicities and ages, for the \( V \), \( I \) and \( K \) passbands. In line with our desire to use the models only in a differential way, we have actually plotted the theoretical population correction (not the actual mean RC magnitude) for any given simple population considered. The theoretical counterpart of the local RC has been computed using the SFR and AMR by Rocha-Pinto et al. (2000a,b, see Paper II for further discussions on this issue).

It is evident that the behaviour of the population corrections – and therefore of the mean RC brightness – is strongly dependent on the photometric band used. For ages larger than about 1.5 Gyr, \( \Delta M_{RC}^K \)
increases for increasing metallicity (hence $M_{K}^{RC}$ decreases), which is exactly the opposite of what happens to $\Delta M_{V}^{RC}$ and $\Delta M_{I}^{RC}$. If we consider the entire $[M/H]$ range spanned by our models (from $-1.7$ up to $0.20$), for ages below $\sim 4$ Gyr, $M_{K}^{RC}$ is on average less sensitive to $[M/H]$ than both $M_{V}^{RC}$ and $M_{I}^{RC}$. However, when the age is larger than this limit $M_{K}^{RC}$ is more affected by $[M/H]$ than $M_{V}^{RC}$; for ages larger than $\sim 11$ Gyr, $M_{K}^{RC}$ is also more metallicity-dependent than $M_{V}^{RC}$, at least in the case of $[M/H]$ lower than $\sim -0.7$. In the $[M/H]$ interval between $0.0$ and $\sim -0.4$ (this lower limit corresponds approximately to the average metallicity of the LMC clump stars) $M_{K}^{RC}$ is less affected by the metallicity than both $M_{V}^{RC}$ and $M_{I}^{RC}$, at least up to ages of about $11$ Gyr. At super-solar $[M/H]$ and for ages above $\sim 3$ Gyr, $M_{K}^{RC}$ is more sensitive to metallicity than the $V$ and $I$ brightnesses.

This complex relationship among the RC $V$, $I$ and $K$ brightnesses stems from the different sensitivities to age. In particular, $M_{K}^{RC}$ is more affected by age than both $M_{V}^{RC}$ and $M_{I}^{RC}$, for the lower metallicities and higher ages.

3 TESTING THE THEORETICAL MULTICOLOUR POPULATION CORRECTIONS

Paper II and Grocholski & Sarajedini (2002) have already shown separately the consistency of the theoretical results in the $I$ and $K$ band, when applied to a sample of Galactic open clusters of various ages and metallicities. The very different behaviour of the population corrections in different passbands allows one to perform the following additional tests for the accuracy of the models.

Fig. 4 compares the difference of the population corrections in two different photometric bands, between theory and observations, for a sample of Galactic open clusters with photometry in $V$, $I$ and $K$ bands. The data concerning the clusters $V$ and $I$ magnitudes, as well as ages, [Fe/H] and reddening values are taken from Sarajedini (1999). K-band photometric data are from Grocholski & Sarajedini (2002). The ages of the clusters have been derived with respect to the reference cluster M67, which is assumed to be 4 Gyr old on the basis of previous studies; the age scaling has been obtained from distance modulus difference (derived from the main-sequence fitting) with respect to M67. It is important to note that the difference of the evolutionary corrections in two different photometric bands is independent of the adopted distance modulus (while the cluster age is dependent on the relative main-sequence-fitting distance moduli and the age zero-point). As for the empirical calibration of the local RC we have used the values provided by Alves et al. (2002), namely $M_{K}^{RC} = -1.60 \pm 0.03$, $M_{V}^{RC} = -0.26 \pm 0.03$ and $M_{I}^{RC} = 0.73 \pm 0.03$ (the theoretical values we obtain from our simulation are $M_{K}^{RC} = -1.54$, $M_{V}^{RC} = -0.17$ and $M_{I}^{RC} = 0.84$; we stress again that we do not use individual absolute magnitudes from the stellar models, just magnitude differences). The value $M_{V}^{RC} = -0.26 \pm 0.03$ agrees, within $1\sigma$, with the previous determination by Stanek & Garnavich (1998).

The agreement between theory and observations is satisfactory – also considering the uncertainty of the order of $\sim 0.1$ dex associated with the individual cluster $[M/H]$ estimates – which is another confirmation of the population corrections predicted by the stellar models we employ. In fact, taking into account the very different behaviour of the predicted evolutionary corrections in $V$, $I$ and $K$, it is highly improbable that a combination of errors in the theory and in the parameters for the adopted clusters would produce such a good agreement.

Another interesting test makes use of the available $V$- and $K$-photometry of the few RC stars populating the *Hipparcos* data base for the Hyades and Praesepe clusters. There is a large body of work suggesting that these two clusters share the same age and metallicity [e.g. the discussion and references in van Leeuwen (1999) and Castellani et al. (2002)]. From the *Hipparcos* data the cluster distances are $\mu_{V, Hyades}^{H} = 3.33 \pm 0.01$ and $\mu_{V, Praesepe}^{H} = 6.37 \pm 0.15$ (Perryman et al. 1997; van Leeuwen 1999); a comparison of their
The brightness of the RC has the appropriate mean value obtained after applying the theoretical population corrections (which are of the order of $\sim-0.3$ mag in both passbands) to the local RC absolute magnitude. On the same plot we show the $V$ and $K$ absolute magnitude of the six stars (two in the Hyades and four in Praesepe; open and closed circles, respectively) populating the RC of the composite CMD of these two clusters (see text for details).

For a common age $t = 625 \pm 50 \times 10^6$ yr (higher than the lower limit for the existence of the RC, which is about $500 \times 10^6$ yr) and $[\text{Fe/H}] = 0.14$ (Perryman et al. 1997), we have determined the appropriate population correction and plotted in Fig. 5 the theoretical luminosity function (number of stars per magnitude bin) for RC stars in the $V$ and $K$ bands (the normalization is arbitrary). The brightness of the RC has the appropriate mean value obtained after applying the theoretical population corrections (which are of the order of $\sim-0.3$ mag in both passbands) to the observed local RC absolute magnitude. On the same plot we show the $V$ and $K$ absolute magnitude of the six stars (two in the Hyades and four in Praesepe) populating the RC of the composite CMD of these two clusters. Owing to the small size of the star sample it is not possible to determine a meaningful mean brightness to compare with theory, but one can note that the empirical stellar absolute magnitudes are in good agreement with the location and the width of the RC as predicted by the theoretical population corrections.

As a warning, it is also important to take into account the fact that the values of $\Delta M^\text{RC}_K$ and $\Delta M^\text{RC}_V$ are both a strong function of $t$ in this age range (in the case where Praesepe is younger than Hyades by $\sim100 \times 10^6$ yr, its RC would be on average brighter by $\sim0.2$ mag than the Hyades RC). We note, however, that here is no obvious offset between the two sets of observational points, and between them and the theoretical luminosity function. In conclusion, we regard the agreement between theoretical and empirical data in Fig. 5 as an additional indication of the accuracy of the population corrections predicted by theory.

The generally small values of $\Delta M^\text{RC}_K$ for ages between $\sim1.5$ and $\sim4$ Gyr, as displayed in Fig. 5, raise the question of to what extent one is allowed to disregard the population corrections when considering the $K$-band RC magnitude as a distance indicator (see also the discussion in Grocholski & Sarajedini 2002). In this age range, the lower sensitivity of the $K$-band RC brightness to both age and metallicity may suggest the use of the local $K$-band RC brightness as a perfect standard candle, without taking into account the appropriate $\Delta M^\text{RC}_K$.

To answer this question, Fig. 6 again shows the population corrections to the $K$ band for simple stellar populations, but this time the region in which the population correction is within $\pm0.1$ mag is also marked. If a systematic uncertainty of this order on the RC distance is considered to be acceptable, the allowed range of age and metallicity is easily evaluated. For example, one can obtain distances with this precision, without evaluating the population correction, for $t$ and $Z$ values typical of the sample of Galactic open clusters older than $\sim2.5$ Gyr. However, we cannot neglect the evaluation of $\Delta M^\text{RC}_K$ a priori, owing to the fact that it can reach values as high as $\pm0.5$ mag.

### 4. $V$-, $I$-, $K$-BAND POPULATION CORRECTIONS FOR EXTERNAL GALAXIES

Having assessed the adequacy of the theoretical population corrections for simple stellar populations, in this section we proceed to compute the corrections in the $V$, $I$ and $K$ bands for the composite systems discussed in Paper II, using the appropriate SFR and AMR discussed in Paper II; the results are summarized in Table 2. Full references for the SFR and AMR can be found in table 4 of Paper II.

As a test, we show in Figs 7 and 8 the empirical data for the local RC (from Alves 2000) in the $V$, $I$ and $K$ passbands, and the
Table 2. Population corrections for the clump in nearby galaxy systems.

<table>
<thead>
<tr>
<th>System</th>
<th>$\Delta M^{RC}_{V}$</th>
<th>$\Delta M^{RC}_{I}$</th>
<th>$\Delta M^{RC}_{K}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar neighbourhood (Hipparchos)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Baade’s window (scaled-solar)</td>
<td>-0.21</td>
<td>-0.08</td>
<td>-0.07</td>
</tr>
<tr>
<td>Baade’s window ($\alpha$-enhanced)</td>
<td>-0.06</td>
<td>-0.01</td>
<td>-0.11</td>
</tr>
<tr>
<td>Carina dSph</td>
<td>+0.59</td>
<td>+0.35</td>
<td>-0.17</td>
</tr>
<tr>
<td>SMC</td>
<td>+0.31</td>
<td>+0.29</td>
<td>-0.07</td>
</tr>
<tr>
<td>LMC</td>
<td>+0.26</td>
<td>+0.20</td>
<td>-0.03</td>
</tr>
</tbody>
</table>

Figure 7. Empirical CMDs for the local RC (from Alves 2000) in the $V$, $I$ and $K$ passbands.

Figure 8. The same as Fig. 7, but for a theoretical simulation with a much larger number of stars.

results from the theoretical simulations, respectively. Note the good correspondence between the predicted and observed shape of the RC in the different colour planes. In particular, the RC in the $M_{V}-(V-I)$ plane is basically horizontal [see the in-depth discussion of Paper II on the local RC and its properties in the $M_{V}-(V-I)$ plane], while in the $M_{K}-(V-K)$ plane it is slightly tilted towards higher $M_{K}$ values, and in the $M_{V}-(V-I)$ plane it is tilted towards lower $M_{V}$ values. These properties are well reproduced by the theoretical models. In more detail, the observed slopes $\Delta M_{V}/\Delta(V-I)$ and $\Delta M_{K}/\Delta(V-K)$ are equal to $1.46 \pm 0.27$ and $-0.63 \pm 0.12$, respectively, which compare well with the corresponding theoretical values of $1.18 \pm 0.07$ and $-0.54 \pm 0.02$. In addition, the theoretical simulation predicts no significant correlations between [Fe/H] and either $M_{K}$ or $M_{V}$, in agreement with the empirical data of Alves (2000).

The data in Table 2, in conjunction with the empirical $V$, $I$ and $K$ absolute brightness of the local RC reported in Section 3, can be used to derive simultaneously reddening and distances to nearby galaxies, following the technique employed by Alves et al. (2002). The basic idea is that the apparent distance moduli determined simultaneously in various photometric bands, using the appropriate population corrections, must all provide the same unreddened distance. After assuming a reddening law, one can therefore use this constraint to derive simultaneously reddening and the true distance modulus. Alves et al. (2002) have adopted the following ratios for the reddening law: $A_{V}/E(B-V)=0.35$, $A_{I}/E(B-V)=1.96$, $A_{K}/E(B-V)=3.24$. We will use these same ratios throughout the rest of this section.

The LMC value of $\Delta M^{RC}_{V}$ provided in Table 2 is slightly different from that used by Alves et al. (2002), which we obtained after a preliminary evaluation of this quantity. This small difference (0.04 mag) does not influence appreciably the results obtained by Alves et al. (2002). In fact, we repeated their analysis employing the $\Delta M^{RC}_{V}$ of Table 2 ($\Delta M^{RC}_{I}$ and $\Delta M^{RC}_{K}$ are unchanged), and obtained $\mu_{0}=18.505 \pm 0.045$ and $E(B-V)=0.079 \pm 0.014$, almost coincident with $\mu_{0}=18.506 \pm 0.033$ and $E(B-V)=0.089 \pm 0.015$ obtained by Alves et al. (2002).

We have further tested this method (and the corresponding population corrections) on the Galactic globular cluster 47 Tuc, for which an accurate Hipparcos-based empirical main-sequence-fitting distance $\mu_{0}=13.25 \pm 0.07$ has been provided recently by Percival et al. (2002), and reddening estimates are also available [$E(B-V)=0.04 \pm 0.02$, see the discussion in Percival et al. (2002)]. Spectroscopic analyses give [Fe/H] = $-0.70 \pm 0.1$, together with an enhancement of the $\alpha$-elements typical of the Galactic Halo population. We have taken into account the $\alpha$-element enhancement -- as in Paper II -- using the results of Salasnich et al. (2000). $V$- and $I$-band data come from the photometry employed by Percival et al. (2002), while $K$-band data are taken from the analysis by Grocholski & Sarajedini (2002). We have assumed an a priori age of $11 \pm 2$ Gyr for the cluster, which corresponds to the age range allowed by an uncertainty of more than $\pm 0.1$ mag around the main-sequence-fitting distance. By applying the procedure of Alves et al. (2002) we have obtained $\mu_{0}=13.19 \pm 0.07$ and $E(B-V)=0.055 \pm 0.015$, in good agreement with the independent determinations given previously.

$K$-band data for a large variety of Galactic and extragalactic stellar populations are not yet widely available, while $V$/$I$ data are much
more common in the literature. We have therefore applied the technique of Alves et al. (2002) using only $V$/$I$ data. If, as shown in the previous section, our population corrections are accurate, one would expect to obtain distances consistent with the results from the $VIK$ data, albeit possibly with larger errors owing to the lack of an additional constraint from the K-band magnitudes. In the case of old metal-poor stellar populations, the RC $V$ and $I$ brightness is much less dependent on age than the K magnitude, so that the assumptions concerning the stellar ages are less critical. By repeating the determination of 47 Tuc distance with the $VIK$ data only, we obtained $\mu_0 = 13.26 \pm 0.07$ and $E(B-V) = 0.02 \pm 0.02$, consistent with the result from $VIK$ photometry. The same exercise can be performed on the LMC data from Alves et al. (2002); in this case we derived $\mu_0 = 18.53 \pm 0.07$ and $E(B-V) = 0.071 \pm 0.022$, in very good agreement with the result from the full $VIK$ analysis.

As a final exercise, we have studied the field LMC-SC6 from the OGLE-II data base (Udalski et al. 2000); this field overlaps with the field centred on the eclipsing binary HV982, studied by Larsen, Clausen & Strom (2000). We have corrected the zero-point of the OGLE-II $V$ photometry, in order to place it on the same system as Alves et al. (2002), and employed stars with a random photometric error smaller than 0.04 mag.

We obtain for the RC level in field LMC-SC6 $V^{RC} = 19.21$ and $K^{RC} = 18.17$. By employing the population corrections given in Table 2 we have obtained $\mu_0 = 18.47 \pm 0.04$ and $E(B-V) = 0.085 \pm 0.015$. This distance agrees well within the errors with the results from Alves et al. (2002). The minimum reddening for this field as estimated by Larsen et al. (2000) is $E(B-V) = 0.085 \pm 0.02$. Fitzpatrick et al. (2002) obtain from spectral fitting of the eclipsing binary HV982 a value $E(B-V) = 0.086 \pm 0.005$. Oestreich, Gochermann & Schmidt-Kaler (1995) provides a minimum reddening of 0.05 mag. All of these values are consistent with our reddening estimate for the RC population.

5 CONCLUSIONS

We have presented a detailed analysis of the behaviour of the RC brightness in the $K$ band, together with tests for the accuracy of the predicted population corrections to the local RC $VIK$ brightness, and a discussion of the Alves et al. (2002) method for deriving simultaneously distance and reddening using multiband RC photometric data. In more detail:

(i) We have shown that the $K$-band brightness of the RC is mainly determined by the behaviour of the bolometric correction to the $K$ band. For ages greater than about 1.5 Gyr the trend of the RC brightness with respect to $[M/H]$ is reversed in comparison with the $V$- and $I$-band values; that is, $M^{RC}$ decreases for increasing metallicity. By considering the entire $[M/H]$ range spanned by our models (Girardi et al. 2000), for ages below $\sim 4$ Gyr $M^{RC}$ is on average less sensitive to $[M/H]$ than both $M^{RC}$ and $M^{RC}$ at greater ages $M^{RC}$ is more affected by $[M/H]$ than $M^{RC}$; for ages greater than $\sim 11$ Gyr it is more metallicity-dependent than $M^{RC}$, at least for $[M/H]$ lower than $\sim -0.7$. It is interesting to note that for $[M/H]$ between 0.0 and $\sim -0.4$, $M^{RC}$ is less affected by the metallicity than both $M^{RC}$ and $M^{RC}$, at least up to ages of about 11 Gyr. This complex relationship among the RC $V$, $I$ and $K$ brightness is a result of their different sensitivities to age. In particular, $M^{RC}$ is more affected by age than both $M^{RC}$ and $M^{RC}$, for the lower metallicities and higher ages. Owing to the non-trivial dependence of $M^{RC}$ on both $[M/H]$ and age, it is always necessary – as in the case of $M^{RC}$ and $M^{RC}$ – to determine the appropriate population correction for the stellar population under scrutiny, when using the RC as a standard candle.

(ii) In Table 1 we have provided values of the RC average $K$ brightness for a large range of $[M/H]$ and ages. These data, used in conjunction with equations (5) and (6), and the data in table 1 of Paper II, can be employed to determine multiband population corrections for any given stellar population, once SFR and AMR are prescribed.

(iii) We have simultaneously and positively tested the evolutionary corrections $\Delta M^{RC}$, $\Delta M^{RC}$ and $\Delta M^{RC}$ against Galactic open cluster data.

(iv) We have discussed the method applied by Alves et al. (2002) for determining reddening and distance from the $V$-, $I$- and $K$-band RC brightness; we have tested this technique positively on the Galactic globular cluster 47 Tuc, for which both an empirical parallax-based main-sequence-fitting distance and reddening estimates exist. We have also studied the case of using only $V$ and $I$ photometry, recovering consistent results for both reddening and distance. Application of this method to an OGLE-II field, and the results by Alves et al. (2002), confirm a LMC distance modulus of about 18.50, in agreement with the HST extragalactic distance scale zero-point (Freedman et al. 2001).

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