nal bearings exhibits stiffness and, therefore, influences the rigid body and bending critical speeds of the rotor bearing system. In the case of rigid body criticals, calculate the cylindrical and conical modes as a function of support spring constant. Then calculate the fluid-film stiffness as a function of speed. The intersection of the rotor and bearing characteristics corresponds to the critical speed.

The critical speed of the system may be represented by

$$\omega_c = \sqrt{\frac{k}{m}} \left(1 + \frac{1}{E} \frac{S}{d_f} \right)$$

(14)

The design chart given in Fig. 15 enables one to calculate directly the critical speed of elastic rotor supported by plain cylindrical journal bearings. It combines in one figure the rotor and the bearing data, and the critical speed is found by a trial-and-error process. The left-hand abscissa axis of Fig. 15 is entered with an assumed critical speed a loop is made around the chart. If the loop closes the assumed value was correct, otherwise the process must be repeated with a new critical speed. For example, for

$$L/D = 1, F/\omega_0 = 2, \text{and } \frac{1}{2} = \frac{1}{C} \omega_0 = 5.$$  By trial and error, we find \( \omega_0 = 0.88 \) from Fig. 15. Thus if the rigid bearing critical speed is 5000 rpm then the actual critical speed is 4600 rpm.

(f) Resonant Whip (Reference [4]). This form of instability has been extremely troublesome with flexible rotors (rotors operating at speeds above the first bending critical) operating in hydrodynamic fluid film forces. An analysis of a symmetrical rotor supported on two symmetrical bearings has been carried out (reference [4]) and the regions of instability have been established. Fig. 16 gives the ratio of stability threshold to first system critical as a function of the eccentricity ratio, \( L/D \), and dimensionless number \( E \). The figure indicates that while for low eccentricity ratios the instability occurs at approximately twice the first critical speed, this number increases with increase in eccentricity ratio.

Conclusions

1. The dynamic response of a rotor system is influenced by the fluid film within the bearings which exhibits damping and stiffness.
2. Experimental data of synchronous whirl (under conditions of small angular acceleration and radial velocity) are in complete agreement with the theoretical calculations based on the steady whirl analysis.
3. Graphical methods can be used to determine the shaft locus from the dynamic incompressible plain cylindrical journal bearing results.
4. The experimental results for half-frequency whirl verify the trends predicted by theory. The threshold of instability has a minimum within the range of operating clearances. That is to say that there exists a \( C^* \) for each \( p_p/p_c \)-curve, say \( C_n^* \), where the stability characteristics are the lowest.
5. Hybrid bearings depending upon supply pressure level and bearing geometry can be subjected to hydrodynamic forces which will have a net effect of producing destructive self-sustained whirl at speeds lower than that of onset of half-frequency whirl, if the bearings had been purely self-acting.
6. Effects of increase in load on onset of fractional-frequency whirl are pronounced at low supply pressure when self-acting effects predominate; same, however, become negligible when supply pressures are raised to the point where changes in eccentricity ratio are very small for static load changes and externally pressurized characteristics predominate.
7. Fluid-film bearings provide an important source of attenuation. In order to determine the transmitted force it is necessary to couple the dynamic characteristics of the bearing with the elastic characteristics of the rotor.
8. The fluid film within a bearing possesses stiffness and therefore influences the system critical speeds. The bearing damping influences the amplitude of rotor vibration.
9. The threshold of resonant whip increases with increase in eccentricity ratio. At low eccentricity ratios the instability occurs at approximately twice the first system critical speed.

Recommendations

Dynamic fluid-film forces for other than plain cylindrical journal bearings should be analyzed and these should be coupled with symmetrical and nonsymmetrical rotors to obtain the dynamic response of the rotor-bearing system.

References


W. F. Boudreau

Analytical studies of the wide variety described in Dr. B. Sternlicht's paper are particularly significant now that bearings must be designed to be lubricated with such diverse fluids as liquid metals, or gases, over a wide variety of temperatures and pressures. Such studies can minimize the very high cost of conducting tests under actual design conditions, and can help eliminate the need for redesigning expensive components after an unsuccessful test. These remarks may sound trite, but the writer knows recent instances where manufacturers were forced to spend large sums of money to work themselves out of situations in which injudicious extrapolation rather than careful analysis had been the guide.

The recommendation that "Dynamic fluid film forces for other than plain journal bearings should be analyzed..." is very pertinent. Several unusual geometries have recently been used to solve the half-speed whirl problem for high-speed gas-lubricated bearings, but the limits of application of these systems are poorly known. This is only one example of an area in which further theoretical studies of a variety of bearing geometries could be of material assistance.

N. Van De Verg

The author has made valuable contributions to the understanding of the stiffness and damping properties of the lubricating film in bearings and their important influence on the stability and dynamics of rotors.

DISCUSSION

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The author in this brief summary paper has treated several forms of instability separately in such a way that some readers may believe that the half frequency whirl, the fractional frequency whirl, and the resonant whirl would require entirely different stability criteria. In reality the fundamentals which determine stability with respect to self-excited whirl are the same in all three cases, but the relative magnitude of the influencing parameters differs.

Papers presented by Hagg [10], Poritsky [11], Hori [12], Sterndlicht, Poritsky and Arwas [4], and Gross [13], are associated with the stability of rotors of varying degree of flexibility supported on fluid film bearings which are rigidly held. The threshold of whirl is indicated in each case to occur at a frequency which is some multiple of the first critical speed of the system. Following the mathematical reasoning of Poritsky it can be shown that this multiple is proportional to the ratio of the damping coefficient to the tangential component of force coefficient and a correction depending on the degree of symmetry.

The force coefficients and damping can result from the bearing and/or from other components such as electrical machines or turbines. Sixsmith [14] reports a turbine which whirled when driving pressure gas was admitted even though the shaft was locked against rotation. The bearing characteristics may vary considerably depending on type. The relative magnitudes of the damping coefficient and the tangential force coefficient differ appreciably in the externally pressurized bearing and in the tilting pad bearing from those of the simple self-acting cylindrical bearing.

After the threshold of instability is passed during experiments the instability may be continuous or intermittent, mild or severe, symmetrical or unsymmetrical, half-frequency, fractional frequency, or resonant whirl depending on the relative values of the components of bearing film stiffness, the damping coefficients, and the shaft stiffness.

Hori [12], and Sterndlicht [15], point out that half-frequency whirl may not be very severe. Sterndlicht [15] points out how half-frequency whirl may develop into resonant whirl as the speed is increased. Newkirk and Lewis [16] discuss resonant whirl and show in some cases a large difference between the "quiet" whirl-impeping speed and the bumping speed.

It should be noted that the system critical frequency of the resonant whirl is not that associated with the threshold of whirl but rather that associated with the stiffer nonlinear bearing which results from the relatively large eccentricity of the journal during resonant whirl.

The significant critical speed associated with self-excited whirl is that which is based on the film radial component of force coefficient as indicated by Sterndlicht [4]. Due to the frequency dependence of the film stiffness this critical speed is a function of the operating speed. It has a significant influence on the dynamics of the system because of its proximity to the operating speed. It is not a speed at which a synchronous vibration would occur but an external shake table type of excitation would indicate it to exist physically.

Newkirk [17] states that one simple underlying cause of these disturbances seems to be the tendency of the journal to whirl in the bearing and that the manifestations of this unstable characteristic are so diverse that it is desirable to classify the phenomena. He also pointed out that any complete solution of this problem must take account of the bearing parameters, those of the rotor, those of the structure, and those of any exciting forces. Hagg [10] and Gross [18] also discuss complex system effects and Landzberg [19] has presented a method for extending the stability analysis to rotor systems having distributed mass and flexibility. Schnittger [20] has applied the fundamentals concerning self-excited whirl to obtain a smooth running, double-speed, gas-turbine rotor system.

It is hoped that this discussion helps make it clear that the classification of phenomena based on their manifestations does not imply any fundamental difference in their underlying cause. Theories presented to date on stability criteria have not adequately included the influence of the mass, flexibility, and damping of the support; these theories do not generally predict amplitude of motion; and therefore some applications may appear to dispute the theories.

Additional References


M. Wildmann

The stability and dynamics of rotors supported on fluid film bearings is an important subject that deserves a comprehensive treatment and it seems therefore appropriate to add to the discussion presented here.

First, any rotor system will in general involve a shaft which has a certain degree of flexibility. Mass and bearings will be distributed axially along the shaft. Under dynamic conditions, there is in general an infinite number of resonances, because the shaft is a continuous rather than a lumped mass body. Our terminology must be adequate to represent this real case, as well as representing limiting cases which our systems may approach.

Some of the terminology which is used is based upon simplified models which have either a rigid shaft or rigid bearings. It is in the attempt to extend this model-based terminology to the real case, in which interactions between shaft and bearing stiffness occur, that terminology difficulties arise.

We may classify the dynamic behavior of a rotor system into two types. The first is the passive response of the system to an excitation. This can be described by amplitude and phase characteristics of the vibration. There will be resonances at frequencies determined by the masses, moments of inertia, shaft stiffness, and bearing stiffnesses. If the excitation is rotor imbalance, the oscillation occurs at the frequency of rotation of the shaft and is commonly called synchronous whirl.

The second type is a self-excited vibration, which exists due to hysteresis in the material of the rotating system, or more com-
monly, due to fluid film forces in the support bearings. The self-excited vibration due to fluid film forces involves orbiting of the shaft axis within a bearing at up to, but just under, one-half the rotational speed of the shaft. As a consequence, this self-excited whirl has often been called Half-Frequency Whirl. The self-excited whirl can also occur with orbiting at a fractional frequency of the shaft rotational speed when an externally pressurized bearing is used.

A complete dynamical analysis of a rotor system must necessarily take into account both shaft flexibility and bearing reaction forces. In the case of fluid film bearings, the lubricating film force may not be represented by a set of linear springs and dashpots. The film force for a fluid film bearing has phase and amplitude (in the plane of the bearing) which in general are nonlinear functions of shaft position and velocity in the bearing. The film force is also determined in part by the past history of motion in the case of a gas lubricating film. In special cases of steady-state oscillations, this history effect is, of course, negligible.

It seems preferable not to speak of a rigid body critical in relation to a real system, although it is clearly desirable for a designer, as a preliminary step, to calculate what would be the resonant frequencies of the system if the shaft were rigid and the bearing forces linear. As observed by the author, shaft flexibility will result in resonant frequencies which are less than these approximations. The resonant frequencies of the systems will be modified even more when better approximations of the bearing forces are used.

It is therefore necessary in using the term critical speed; if the term is used to describe an approximation, it should be so stated.

If it is possible to increase the rotational speed of the shaft until the first bending mode of the shaft occurs, we have what was first recognized many years ago, whippings of the shaft, and called "shaft whip." Aside from resonances in the system not due to the bearing or shaft, this will be the lowest resonance when the bearings are stiffer than the shaft. When the system with fluid film bearings reaches a condition such that the rotational speed is approximately twice the first system resonance, there is a positive feedback of energy. The consequence is that resonant whirl, as described by the author, can occur.

In the past, bearing systems have often been described as being unstable when the shaft executed self-excited whirl. Actually, the whirl orbit may be uniformly asymptotically stable to sufficiently small disturbances. A small disturbance will then result in a displacement of the shaft axis, but it will return to its stable orbiting path. It should be noted, however, that a moderate disturbance might be sufficient to cause failure. Of greatest importance is the fact that small increases in speed can result in much larger orbiting paths of the shaft. Hence the importance of the term threshold. Under some conditions, of course, the orbiting whirl path may have an eccentricity ratio high enough that even a small disturbance will cause failure. Technically, self-excited whirl is unstable with respect to the steady no-whirl position. Although self-excited whirl may be technically stable, it is normally an unsatisfactory operating condition.

It is convenient to utilize the concepts of cylindrical and conical whirl recognizing that they each apply to a rigid rotor and that cylindrical whirl is simply a special case of conical whirl, in which the apex of the orbiting shaft is at infinity. For convenience, we commonly speak of purely conical whirl as being that rigid shaft whirl which results in no orbital motion of the shaft axis at a point midway between two supporting bearings. The center of gravity may, of course, not be on this geometrical axis. Since purely cylindrical and conical whirl can only occur with perfectly rigid shafts, these definitions are useful principally from the standpoint of visualization. Unfortunately, orbiting paths may often be neither circular nor conical [21].

Another aid to visualization of shaft motion in a fluid film bearing may be obtained by representing the cylindrical and conical motions in equation form. Under no load conditions, the average eccentricity ratio \( e_0 \) (the bearing being represented is \( j = 2 \)) is zero, and whirl occurs about the bearing axis. However, as the load is increased, the average eccentricity ratio vector increases. Then, in the absence of self-excited whirl, a synchronous whirl whose path does not enclose the bearing axis may occur about the tip of the eccentricity vector. The eccentricity vector in bearings \( j = 2 \) may be represented by (Reynolds and Gross):

\[
\epsilon_j = \epsilon_i + \epsilon_x \exp \left( i(\omega t - \alpha_j) \right) + \epsilon_y \exp \left( i(\omega t - \beta_j) \right) + \epsilon_z \exp \left( i(\Omega t - \gamma_j) \right) \tag{1}
\]

in which \( k = 1 \) for cylindrical whirl and \( k = 2 \) for conical whirl. The space average eccentricity ratio vector \( e_0 \) has magnitude \( e_i \) and an altitude angle \( \phi_i \). The amplitude of the synchronous component is \( e_0 \), and of the self-excited components, \( e_x, e_y, e_z \). The frequency of subharmonic orbiting is \( \omega \), and \( \Omega \) is the frequency of another component \( e_0 \) due to a rotating load. The phase between the cylindrical and conical modes of the components is given by \( \alpha \), \( \beta \), \( \gamma \). In general, all components are present and under some conditions, the trace of the eccentricity vector can be quite strange.

Inasmuch as this particular paper is concerned with terminology, the writer suggests that the term externally pressurized should be used to describe bearings which operate with lubricants supplied by external pressure. This is because all fluid film bearings operate due to the hydrodynamic effect of the fluid. As a consequence, self-acting and externally pressurized bearings work because of hydrodynamic effects.

It is worth noting that the bearing number \( A \), which the author calls a compressibility number, applies appropriately both to liquid and gas lubricated bearings. At small values of the bearing number, both gas and liquid lubricating films behave as if they were incompressible. At higher values of the bearing number, the gas lubricating film takes on compressibility characteristics and the liquid lubricating film cavitates. Because of the applicability to both gas and liquid lubricating films, the writer suggests that the term "bearing number" rather than "compressibility number" be used.

It would be helpful if the author would clearly indicate the three incompressible fluid film bearings selected and the three gas film bearings.

Fig. 10 illustrates the effect of imbalance as has been previously described. These are among the useful references applicable to bearing stability. It is unfortunate that the author chose to list only his own papers. For example, to mention only a few, papers by Whitley and Betts [22], Capriz [23], Ausman [24], and Larson and Richardson [25] are significant. The subject is also reviewed elsewhere.

Finally, the author mentions the linearization procedure consisting of a power series expansion solution of equation (5) and described in reference [2]. This linearization procedure was first used by Ausman [20] to solve this same equation.

Additional References

Author’s Closure

It is most rewarding to see the interest that the discussers have taken in this paper. This is an indication of the importance of the subject matter and the recognition by machinery designers that bearing-rotor systems are an integral part of any successful machinery development. There are a number of points which deserve additional comments and these will be discussed in order.

The discussion offered by Mr. Boudreau is in agreement with the author’s point of view and does not require additional comments.

The discussion by Mr. Van De Verg demands some clarification. While it appears that Mr. Van De Verg is familiar with some of the literature, he seems to confuse the various instabilities and the effects of damping and stiffness on dynamic response.

My sole purpose in including the terminology in this paper was to define the terms used and to assist the reader in differentiating the various phenomena. Apparently I have not been too successful in doing this and I hope that with the aid of Figs. 17(a) and 17(b) some of the confusion will be clarified.

It has been shown [27 and 28] that Half-Frequency Whirl can occur from zero speed on. Balanced vertical rotors in plain cylindrical journal bearings in the absence of external forces will whirl at all speeds. It is for this reason that the most severe stability condition for a rotor is in the zero "g" field. On the other hand, Resonant Whirl is a resonant vibration of a shaft in a fluid-film journal bearing which occurs at speeds equal to or above twice the system bending critical, and at a frequency equal approximately to a natural frequency of the rotor system, regardless of running speed.

Mr. Van De Verg next discusses references [10, 11, 12, 4, and 13]. The first three references deal primarily with Resonant Whirl (Oil Whip, Resonant Whirl, Resonant Fluid Film Whirl). The fourth reference deals with several forms of instability (e.g., Half-Frequency Whirl, Resonant Whirl) and several resonances, while the last reference deals with Fractional-Frequency Whirl. The differences between these references are considerably more than the varying degree of flexibility as Mr. Van De Verg implies. The statement, “Following the mathematical reasoning of Poritsky, it can be shown that this multiple \( \omega = K_0 \omega_r \) is proportional to the ratio of the damping coefficient to the tangential component of force coefficient and a correction depending on the degree of symmetry,” is surprising and has no mathematical rigor or justification.

Reference [14] to which Mr. Van De Verg refers has nothing to do with the forms of instabilities discussed in this paper and should not be confused with them.

With regard to critical speed analysis, damping plays a second order effect on the natural frequency while it plays a major role on the amplitude of vibration [8 and 9]. On the other hand, in stability analyses both damping and stiffness must be considered.

There are two points which are significant in stability analysis. The first deals with the prediction of the threshold of instability while the second deals with the establishment of the rotor trajectory in order to predict whether the instability is amplitude limited. References [10 and 28] deal with both these questions while all the other references mentioned consider the former problem. From good design practice, instability should be avoided within the operating speed range and for this reason not enough attention has been payed to the rotor trajectory.

It is quite true in many systems the bearing mass, inertia, flexibility, and damping should be considered in dynamic system analysis. This is especially true in turbomachinery where weight is important and the structure is relatively flexible, e.g., jet engines, marine power plants, space power plants, etc.

The comments by Mr. Wildmann are most valuable and it is hoped that the few additional points will further add to the clarity of this paper. While in principle it is true that a rotor is continuous rather than made up of lumped masses, any physical rotor can be considered as made up of discreet masses and, therefore, will have a finite number of resonances. In fact, it is this assumption that has provided the analytical tool for the prediction of rotor dynamics. Fig. 18 illustrates these discreet resonances and points out the effect of bearing stiffness on mode shapes.

Note that the first bending critical (sometimes referred to as third critical) with the soft bearing has approximately the same node shape as the first bending critical with the stiff bearing even though the resonance frequency is considerably higher. It is here that the confusion comes about and for this reason it would be considerably clearer if one would define the resonances by mode shapes rather than refer to them as 1st, 2nd, 3rd, etc. The rigid body criticals are sometimes referred to as the zero order criticals and correspond to the cases where the bearing stiffness is low in comparison to the rotor stiffness, and the mode shapes are virtually translatory or conical. Which natural frequency occurs first depends entirely on the mass, inertia, and stiffness distribution.

The comment that externally pressurized bearings work because of hydrodynamic effects is misleading. There are numerous externally pressurized bearings which work purely because of hydrostatic effects, e.g., gimbal bearings in gyros (no rotation). Since we want to differentiate between hybrid bearings (combination of externally pressurized and self-acting) and the purely externally pressurized bearings we have introduced this new terminology of "hybrid." All of the bearings treated in this paper were plain cylindrical and their dimensions are indicated in the respective figures. The first three topics

(a) Synchronous Whirl
(b) Half-Frequency Whirl
(c) Fractional-Frequency Whirl (hybrid bearing with one plain of feeding) deal with compressible fluid lubrication while the last three topics

References

1. [Portitsky, 1937]
2. [Van De Verg, 1963]
Mr. Wildmann is quite right in pointing out that there are many other applicable references. The listed references were only those that were used in the paper and they were not intended to serve as a bibliography. The main reason for the omission was that there are numerous references on each of the topics (e.g., reference [29] contains 88 references dealing solely with the subject of Resonant Whip) and space did not permit including them all. Rather than offend some authors by omission, the decision was made to refer only to the papers from which some of the material was summarized. A person interested in the field can then refer to the original papers and there find considerably more reference material.

In closing, I wish to thank all of the discussers for their valuable comments and it gives me great pleasure in seeing the interest that they and a number of other people have taken in this paper.

**Additional References**

