Evolution of planetesimal discs and planetary migration

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Accepted 2002 October 21. Received 2002 September 29; in original form 2002 July 25

ABSTRACT
In this paper, we further develop the model for the migration of planets introduced by Del Popolo, Gambera & Ercan and extended to time-dependent planetesimal accretion discs by Del Popolo & Eksi. More precisely, the assumption of Del Popolo & Eksi that the surface density in planetesimals is proportional to that of the gas was released. Indeed, the evolution of the radial distribution of solids is governed by many processes: gas–solid coupling, coagulation, sedimentation, evaporation/condensation, so that the distribution of planetesimals emerging from a turbulent disc does not necessarily reflect that of the gas. In order to describe this evolution we use a method developed by Stepinski & Valageas, which, using a series of simplifying assumptions, is able to simultaneously follow the evolution of gas and solid particles for up to 107 yr. This model is based on the premise that the transformation of solids from dust to planetesimals occurs through hierarchical coagulation. Then, the distribution of planetesimals obtained after 107 yr is used to study the migration rate of a giant planet through the migration model introduced by Del Popolo, Gambera & Ercan. This allows us to investigate the dependence of the migration rate on the disc mass, on its time evolution and on the value of the dimensionless viscosity parameter $\alpha$. We find that in the case of discs having a total mass of $10^{-3} - 10^{-1} M_\odot$, and $10^{-4} < \alpha < 10^{-1}$, planets can migrate inward over a large distance while if $M_d < 10^{-3} M_\odot$ the planets remain almost at their initial position for $\alpha > 10^{-3}$ and only in the case where $\alpha < 10^{-3}$ do the planets move to a minimum value of orbital radius of $\sim 2$ au. Moreover, the observed distribution of planets in the period range 0–20 d can be easily obtained from our model. Therefore, dynamical friction between planets and the planetesimal disc provides a good mechanism to explain the properties of observed extrasolar giant planets.

Key words: planets and satellites: general – planetary systems.

1 INTRODUCTION
The discovery of solar-like stars showing evidence for planets orbiting around them (Mayor & Queloz 1995; Marcy, Butler & Vogt 2000; Vogt et al. 2000; Butler et al. 2001) has greatly intensified the interest in understanding the formation and evolution of planetary systems, and the long-standing problem of the origin of the Solar system. At the same time these discoveries have raised a number of questions concerning the formation mechanisms of such systems. Indeed, the extrasolar planets discovered so far are all more massive than Saturn, and most either orbit very close to their stars or travel on much more eccentric paths than any of the major planets in our Solar system. The present small sample can be broadly divided into three groups:

(i) Jupiter analogues, in terms of period $P$ and semi-axis $a$, and with low eccentricities (such as 47 UMa);
(ii) planets with highly eccentric orbits (such as 70 Vir);
(iii) close-in giants (or ‘hot Jupiters’) within 0.1 au the orbits of which are largely circular (such as 51 Peg).

The properties of these planets, most of which are Jupiter-mass objects, are difficult to explain using the standard model for planet formation (Lissauer 1993; Boss 1995). Current theories (Mizuno 1980; Bodenheimer & Pollack 1986) predict that giant planets were formed by gas accretion on to massive ($\sim 15 M_\oplus$) rocky cores, which themselves were the result of the accumulation of a large number of icy planetesimals. The most favourable conditions for this process are found beyond the so-called ‘snow line’ (Hayashi 1981; Sasselov

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Consequently, this standard model predicts nearly circular planetary orbits and giant planets at distances \( \geq 1 \) au from the central star where the temperature in the protostellar nebula is low enough for icy materials to condense (Boss 1995, 1996; but see also Wuchterl 1993, 1996). Thus, in the case of close-in giants, it is very unlikely that such planets were formed at their present locations. Then, the most natural explanation for this paradox and for planets on very short orbits is that these planets have formed further away in the protoplanetary nebula and they have migrated afterwards to the small orbital distances at which they are observed. Some authors have also proposed scenarios in which migration and formation were concurrent (Terquem, Papaloizou & Nelson 2000). We refer the reader to Del Popolo et al. (2001, hereafter DP1) and Del Popolo & Ekström (2002, hereafter DP2) for a detailed discussion of the different mechanisms that have been proposed to explain the presence of planets at small orbital distances: (i) dynamical instabilities in a system of giant planets (Rasio & Ford 1996; Weidenschilling & Marzari 1996), (ii) ‘migration instability’ (Murray et al. 1998), (iii) tidal interaction with a gaseous nebula (Goldreich & Tremaine 1979, 1980; Ward 1986; Lin, Bodenheimer & Richardson 1996; Ward 1997) and (iv) dynamical friction between the planet and a planetesimal disc (DP1).

In particular, in DP1 and DP2 we showed that dynamical friction between a planet and a planetesimal disc is an important mechanism for planet migration and we pointed out that some advantages of the model are:

(i) planet halt is naturally provided by the model;
(ii) it can explain planets found at heliocentric distances of \( >0.03–0.04 \) au, or planets having larger values of eccentricity;
(iii) it can explain metallicity enhancements observed in stars having planets in short-period orbits;
(iv) radial migration is possible with modest masses of planetesimal discs, in contrast to other models.

For the planetesimal disc used in DP1, following Opik (1976), we assumed that the surface density in planetesimals \( \Sigma_s \) varies as \( \Sigma_s(r) = \Sigma_{\odot}(1 \text{ au}) r^{-3/2} \), where \( \Sigma_{\odot} \), the surface density at 1 au, was a free parameter. In DP2 the previous assumption was substituted by a more reliable model for the disc, and in particular we used a time-dependent accretion disc, since it is widely accepted that the Solar system at early phases in its evolution is well described by this kind of structure. An important assumption of DP2 was that the surface density in planetesimals remains proportional to that of the gas: \( \Sigma_s(r, t) \propto \Sigma_s(r) \). However, it is well known that the distribution of planetesimals emerging from a turbulent disc does not necessarily reflect that of the gas (e.g. Stepiński & Valageas 1996, 1997). Indeed, in addition to gas–solid coupling, the evolution of the distribution of solids is also governed by coagulation, sedimentation and evaporation/condensation. In order to take into account these effects we use the method developed in Stepiński & Valageas (1997), which is able to simultaneously follow the evolution of gas and solid particles for up to 10^7 yr. The main approximation used in this model is to associate one grain size to a given radius and time. Then, we use the radial distribution of planetesimals predicted by this model to estimate the planet migration, which is computed as in Del Popolo et al. (2001).

This paper is organized as follows. In Section 2.1 we describe the disc model we use to obtain the radial distribution of the planetesimal disc reached after 10^7 yr. Then, in Section 2.2 we briefly review the migration model introduced in Del Popolo et al. (2001). Finally, we describe our results in Section 3 and the conclusions in Section 4.

**2 DISC MODEL AND PLANET MIGRATION**

**2.1 Gas and solids distribution and evolution**

It is well known that protostellar discs around young stellar objects that have properties similar to that expected for the solar nebula are common: between 25 and 75 per cent of young stellar objects in the Orion nebula seem to have discs with mass \( 10^{-3} M_\odot < M_d < 10^{-1} M_\odot \) and size \( 40 \pm 20 \) au (Beckwith & Sargent 1996). Moreover, observations of circumstellar discs surrounding T Tauri stars support the view of discs having a limited life-span and being characterized by continuous changes during their lifetime. This evidence has led to a large consensus concerning the nebular origin of the Solar system. Moreover, it clearly appears necessary to model both the spatial and temporal changes of the disc (which cannot be handled by the minimum-mass model nor by steady-state models). Besides, one also needs to describe the global evolution of the solid material that constitutes, together with the gas, the protoplanetary disc. Of particular interest is the spatial distribution of material making up a planetary system, as this is about the only information the present observations, based on the Doppler technique, can provide. The knowledge of this distribution and its time evolution is important in understanding how planets form and in this paper it is a key issue since we wish to study the planet migration arising from the interaction between planets and the local distribution of solid matter.

As usual, the time evolution of the surface density of the gas \( \Sigma \) is given by the familiar equation (e.g. Stepiński & Valageas 1997):

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ \frac{r^{1/2}}{\Sigma} \frac{\partial}{\partial r} \left( r^{1/2} v_s \Sigma \right) \right] = 0,
\]

where \( v_s = \frac{v_{\text{turb}}}{\nu} \) is the turbulent viscosity. Since \( v_s \) is not an explicit function of time, but instead depends only on the local disc quantities, it can be expressed as \( v_s = v_s(\Sigma, r) \) and equation (1) can be solved subject to boundary conditions on the inner and outer edges of the disc. The opacity law needed to compute \( v_s \) is obtained from Ruden & Pollack (1991). Then, equation (1) is solved by means of an implicit scheme. Note that the evolution of the gas is computed independently from the evolution of particles (which only make \( \sim 1 \) per cent of the gas mass). Next, from \( \Sigma(r, t) \) we can find all other gas disc variables algebraically.

Next, as described in Stepiński & Valageas (1997) the evolution of the surface density of solid particles \( \Sigma_s \) is given by

\[
\frac{\partial \Sigma_s}{\partial t} = \frac{3}{r} \frac{\partial}{\partial r} \left[ \frac{r^{1/2}}{\Sigma_s} \frac{\partial}{\partial r} \left( v_s \Sigma_s^{1/2} \rho \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ 2 \Sigma_s (\Omega_s t_s) \right] \frac{\partial}{\partial r} \left[ \frac{\Sigma_s (\Omega_s t_s)}{\Omega_s t_s} \right].
\]

The first diffusive term is similar to equation (1), where the effective viscosity \( v_s = v_s(\Sigma_s, r) \) is given by

\[
v_s = \frac{v_{\text{turb}}}{\nu} \quad \text{with} \quad \nu = \left[ 1 + \left( \frac{\Omega_s t_s}{\Omega_s t_s} \right) \right]^{1/2} \frac{\nu}{V_{\text{esc}}}.
\]

Here we have introduced the Schmidt number \( \nu_s \), which measures the coupling of the dust to the gas turbulence. We also used the relative velocity \( \nu_s \) between a particle and the gas, the turbulent velocity \( V_{\text{turb}} \), the Keplerian angular velocity \( \Omega_s \) and the so-called stopping time \( t_s \). The dimensionless quantity \( (\Omega_s t_s)^{-1} \) measures the coupling of the solid particles to the gas. Thus, small particles (size \( s < 1 \) mm) have \( \Omega_s t_s \ll 1 \) and they are strongly coupled to gas. Therefore, they exhibit the same radial velocity. On the other hand, large particles (size \( s > 10^2 \) cm) with \( \Omega_s t_s > 1 \) show a much smaller radial drift. Finally, particles in the intermediate regime (size \( s \sim 10^2 \) cm) with \( \Omega_s t_s > 1 \) exhibit a large inward radial velocity. Therefore, the
evolution of the dust radial distribution can be significantly different from the behaviour of the gas, depending on the particle size (see Stepińska & Valageas 1996 for a detailed study).

The second advective term in equation (2) arises from the lack of pressure support for the dust disc as compared with the gas disc. Thus, it is proportional to the azimuthal velocity difference $\tau_\phi$ between the dust and the gas. The average $\langle \cdots \rangle$ refers to the vertical averaging over the disc height weighted by the solid density. We refer the reader to Stepiński & Valageas (1997) for a more detailed presentation (see also Kornet, Stepiński & Ryczka 2001).

In addition to the radial motion described by equation (2), the dust surface density also evolves through evaporation/condensation and coagulation. In this article, following Stepiński & Valageas (1997) we assume that the size distribution of solid particles at a given orbital radius and time is narrowly peaked around a mean value $s(r, t)$. This is supported by numerical simulations (Mizuno, Markiewicz & Völk 1988), which show that although a broad size distribution is maintained, most of the mass is always concentrated in the largest particles so that one can define a meaningful typical size $s(r, t)$. Then, within this approximation coagulation does not influence the dust surface density $\Sigma$, since it conserves the total mass of solids. Thus, the coagulation of solid particles only appears through the evolution of the radial distribution $s(r, t)$ of the typical size of the dust grains. We model this process as in Stepiński & Valageas (1997). We must note that we only consider collisional coagulation and we disregard gravitational interactions that would come into play at late times when large planetesimals have formed.

On the other hand, we take into account the evaporation of solid particles that takes place at the radius $r_{\text{evap}}$ where the temperature reaches $T_{\text{evap}}$. We also include the condensation of the vapour below $T_{\text{evap}}$ on to the solid grains. The velocity of the vapour is equal to the gas velocity but this component evolves in a specific way because of its own diffusion process and condensation. In this article we are mainly interested in the distribution of solids at small radii, hence we consider only one species of solid particles: high-temperature silicates with $T_{\text{evap}} = 1350$ K and a bulk density $\rho_{\text{bulk}} = 3.3$ g cm$^{-3}$. Thus, in our simplified model we follow the evolution of three distinct fluids: the gas, the vapour of silicates and the solid particles.

In this fashion, we obtain the radial distribution of the planetesimal swarm after $10^7$ yr. This yields the surface density of solids $\Sigma_{s}(r, t)$ and the mid-plane solid density $\rho_{s}(r, t)$. We also obtain the size distribution $s(r, t)$ reached by hierarchical coagulation. Of course, at these late times where planetesimals have typically reached a size of a few kilometre or larger, gravitational interactions should play a dominant role with respect to coagulation. However, if these interactions do not significantly affect the radial distribution of solids (note that the radial velocity of such large particles owing to the interaction with the gas is negligible) we can still use the outcome of the fluid approach described above to study the migration of giant planets, as detailed below.

### 2.2 Migration model

In order to study the migration of giant planets we use the model developed in DP1 (see also DP2). Since this model has already been described in these two papers, we only recall the main points here. We consider a planet revolving around a star of mass $M_\ast = 1 \, M_\odot$. As described in the introduction, in this paper we suppose that the formation mechanism is core accretion and we are not then interested in the scenarios that gravitational instability models could introduce in our model.

The equation of motion of the planet can be written as

$$\mathbf{r} = \mathbf{F}_\odot + \mathbf{R}, \tag{4}$$

where the term $\mathbf{F}_\odot$ represents the force per unit mass from the star, while $\mathbf{R}$ is the dissipative force (i.e. the dynamical friction term; see Melita & Woolfson 1996). If we assume a disc-shaped matter distribution with constant velocity dispersions $\sigma_\parallel$ (parallel to the plane) and $\sigma_\perp$ (perpendicular to the plane) and with a ratio simply taken to be 2 : 1 (i.e. $\sigma_1 = 2 \, \sigma_\perp$), then according to Chandrasekhar (1968) and Binney (1977) we may write the force components as

$$F_{\parallel} = k_1 v_{\parallel} = B_{\parallel} v_{\parallel} \left[ 2 \sqrt{2 \pi G} \log \Lambda m_1 m_2 (m_1 + m_2) \frac{1 - \epsilon^2}{\sigma_\perp^2 \sigma_\parallel} \right], \tag{5}$$

$$F_{\perp} = k_2 v_{\perp} = B_{\perp} v_{\perp} \left[ 2 \sqrt{2 \pi G} \log \Lambda m_1 m_2 (m_1 + m_2) \frac{1 - \epsilon^2}{\sigma_\perp^2 \sigma_\parallel} \right], \tag{6}$$

where

$$B_{\parallel} = \int_0^\infty \frac{dq}{(1 + q)^2(1 - \epsilon^2 + q)^{1/2}} \times \exp \left( \frac{v_{\parallel}^2}{2 \sigma_\perp^2} \left( 1 + q - \frac{v_{\parallel}^2}{2 \sigma_\parallel^2} \right) \right), \tag{7}$$

$$B_{\perp} = \int_0^\infty \frac{dq}{(1 + q)(1 - \epsilon^2 + q)^{1/2}} \times \exp \left( -\frac{v_{\perp}^2}{2 \sigma_\perp^2} \left( 1 + q - \frac{v_{\perp}^2}{2 \sigma_\parallel^2} \right) \right), \tag{8}$$

and

$$e = (1 - \sigma_\parallel^2/\sigma_\perp^2)^{1/2}, \tag{9}$$

where $\pi$ is the average spatial density of field particles, $m_1$ is the mass of the test particle, $m_2$ is the mass of a field particle, and $\log \Lambda$ is the Coulomb logarithm. Then, the frictional drag on the test particles may be written as

$$\mathbf{F} = -k_1 v_{\parallel} \mathbf{e}_\parallel - k_2 v_{\perp} \mathbf{e}_\perp, \tag{10}$$

where $\mathbf{e}_\parallel$ and $\mathbf{e}_\perp$ are two unit vectors parallel and perpendicular to the disc plane.

Since the damping of eccentricity and inclination is more rapid than radial migration (Ida 1990; Ida & Makino 1992; Del Popolo et al. 2001), we only deal with radial migration and we assume that the planet has negligible inclination and eccentricity ($i_p \sim e_p \sim 0$) and that the initial distance to the star of the planet is 5.2 au.\(^1\) For the objects lying in the plane, the dynamical drag is directed in the direction opposite to the motion of the particle and is given by

$$\mathbf{F} \simeq -k_1 v_{\parallel} \mathbf{e}_\parallel. \tag{11}$$

In order to calculate the effect of dynamical friction on the orbital evolution of the planet, we suppose that $\sigma_\parallel = 2 \, \sigma_\perp$ and that the dispersion velocities are constant. If the planetesimals attain dynamical equilibrium, their equilibrium velocity dispersion, $\sigma_{e_{\parallel}}$,

\(^1\) The planet was set at an initial distance of 5.2 au for similitudes with choices made in previous papers (e.g. Murray et al. 1998; Trilling et al. 1998).
would be comparable to the surface escape velocity of the dominant bodies (Safronov 1969) such that

$$\sigma_m \sim \left( \frac{Gm}{\theta r_s} \right)^{1/2},$$

(12)

where $\theta$ is the Safronov number, $m_s$ and $r_s$ are the mass and radius of the largest planetesimals (note that the planetesimals velocity dispersion, $\sigma_m$, now introduced, is the velocity dispersion to be used for calculating the $\sigma$ which is present in the dynamical friction force). If we consider a two-component system, consisting of one protoplanet and many equal-mass planetesimals, the velocity dispersion of planetesimals in the neighbourhood of the protoplanet depends on the mass of the protoplanet. When the mass of the planet, $M$, is $\leq 10^{25}$ g, the value of $(e_m^2)^{1/2}$ where $e_m$ is the eccentricity of the planetesimals) is independent of $M$, therefore

$$e_m \simeq 20(2m/3M_\odot)^{1/3},$$

(13)

(Ida & Makino 1993), where $m$ is the mass of the planetesimals. When the mass of the planet reaches values larger than $10^{25}-10^{26}$ g at 1 au, $(e_m^2)^{1/2}$ is proportional to $M^{1/3}$:

$$e_m \simeq 6(M/3M_\odot)^{1/3},$$

(14)

(Ida & Makino 1993). Consequently, the dispersion velocity in the disc is also characterized by two regimes and is connected to the eccentricity by the equation

$$\sigma_m \sim \left( e_m^2 + i_m^2 \right)^{1/2} v_\kappa,$$

(15)

where $i_m$ is the inclination of planetesimals and $v_\kappa$ is the Keplerian circular velocity. Following Stern (1996) and Del Popolo, Spalding & Gambera (1999) we assume that $(i_m^2) = (e_m^2)/4$. In the simulation we assume that the planetesimals all have equal masses, $m$, and $m \ll M$, where $M$ is the planet mass. This assumption does not affect the results, since dynamical friction does not depend on the individual masses of these particles but on their overall density. Note that for $m \ll M$ the frictional drag $F_i$ obtained in equation (5) does not depend on the mass $m$ of the planetesimals since the velocity dispersion $\sigma_m$ only depends on the mass $M$ of the giant planet while the explicit dependence on $m$ of equation (5) only involves the product $Rm = \rho_s$. Therefore, we do not need to follow the evolution of the size distribution of planetesimals. We merely use the planetesimal density $\rho_s$ reached after 10 yr, assuming that the height of the planetesimal disc does not evolve significantly.

An important point to discuss here is the backreaction of the planet on the swarm. As previously reported, in the calculation of the scattering of planetesimals by a protoplanet, Ida & Makino (1993) showed, by means of $N$-body simulations, that eccentricities, $e_m$, inclinations, $i_m$, and velocity dispersions, $\sigma_m$, of planetesimals in the vicinity of the protoplanet are strongly influenced by the mass of the protoplanet. In the early stage, random velocities of small planetesimals remain low during the growth of the protoplanet, since they are regulated by gravitational scatterings between planetesimals. When the protoplanet becomes massive enough $(10^{25}-10^{26}$ g) to influence the velocity distribution of small planetesimals, the random velocities and velocity dispersion of planetesimals are heated by the protoplanet and become larger than in the early stage. Furthermore, the protoplanet would scatter neighbouring planetesimals and give rise to a gap in the spatial distribution of planetesimals (see fig. 3 of Ida & Makino 1993 and fig. 1 of Tanaka & Ida 1997). As noticed by Rafikov (2001) the gaps seen in $N$-body simulations are never clean because random motion of planetesimals is naturally included, and this permits some of them to be present in the gap. The process of clearing a gap in a planetesimal disc around a massive body is analogous to gap formation in gaseous discs (Takeuchi, Miyama & Lin 1996; Rafikov 2001). The gap also reduces the growth rate of the protoplanet. In our calculation, this effect was taken into account, as previously reported. The width of the heated region is roughly given by $4(\sigma_m^2 + \dot{i}_m^2)_{\odot} + 126.4a^2\sigma_m^2$ (Ida & Makino 1993), where $a$ is the semimajor axis and $h_\mathcal{M} = ([M + m]/3M_\odot)^{1/3}$ is the Hill radius of the protoplanet. The effect on the drift velocities can be easily predicted by observing that the increase in velocity dispersion of planetesimals around the protoplanet decreases the dynamical friction force (see equation 10) and consequently increases the migration time-scale. In order to perform an order of magnitude estimation, we suppose that the planet moves on a stable circular orbit. Using for simplicity the formula from Chandrasekhar (1943), the drift velocity can be expressed as

$$\frac{dr}{dt} \simeq -\frac{4\pi \log \Lambda G^2 M \rho(r) r}{\sigma^3},$$

(Chandrasekhar 1943; Binney & Tremaine 1987; Palmer, Lin & Aarseth 1993; Hernandez & Gilmore 1998). Recalling the mass dependence of the velocity dispersion in the two regimes, we find that the drift velocity behaves as

$$\frac{dr}{dt} \propto \begin{cases} M & M \leq 10^{25} \text{g} \\ \text{constant} & M > 10^{25} \text{g} \end{cases}$$

(17)

The linear dependence of the drift velocity on the mass of the planet, corresponds to the type I drift in the density wave approach (Ward 1997), while the part of the plot independent on the mass of the planet corresponds to type II drift. The transition between the two regimes entails a velocity drop of between one to three orders of magnitude (according to the value of $a$). This is shown in Fig. 1, where we show the drift velocity, $dr/dt$, as a function of mass, $M$, of the protoplanet for $M_\odot = 0.1$ and $a = 10^{-4}$. As shown, objects having masses $< M_\odot$ have velocity drift increasing as $M$, while after a threshold mass any further mass increases begin to slow down the drift. As the threshold is exceeded the motion fairly abruptly converts to a slower mode in which the drift velocity is independent of mass. As previously explained, this behaviour is caused by the transition from a stage in which the dispersion velocity is independent of $M$ to a stage in which it increases with $M^{1/3}$ (Ida & Makino 1993). This last stage is known as the protoplanet-dominated stage. The phenomenon is equivalent to that predicted in the density wave approach (Goldreich & Tremaine 1980; Ward 1997). In this approach, the density wave torques repel material on either side of the orbit of the protoplanet and attempt to open a gap in the disc. Only very large objects are able to open and sustain the gap. After gap formation, the drift rate of the planet is set by the disc viscosity and is generally smaller than in the absence of a gap. We stress that the decaying portion of the curve corresponding to the transition from the first to the second stage does not correspond to any particular model because following Ida & Makino (1993) we do not have information on the evolution of $\sigma$ in the transition regime. Even if not necessary, in order to have an independent check of the effects of backreaction, we performed another calculation. Suppose we have the following two situations:

(i) there is an initial gap in the disc;

(ii) the planet is initially embedded in an unperturbed disc and must clear material in order to form a gap.

According to Nelson et al. (2000) the quoted situations lead to slightly different results in migration, results that tend to converge with increasing time (see fig. 3 of Nelson et al. 2000). The small initial difference in migration rate is caused by the fact that in situation
A. Del Popolo, S. Yeşilyurt and E. N. Ercan

Figure 1. Drift velocity, \( \frac{dr}{dt} \), as a function of mass. Velocities are normalized to \( V = 2(M_\oplus/M_\odot)(\Sigma \pi r^2/M_\odot)(r/\Omega_1/\sigma)^3 r/\Omega_1 \), where \( M_\oplus \) is the mass of the Earth, \( \Sigma \) is the surface density, \( \Omega_1 \) is the angular velocity and \( \sigma \) the dispersion velocity. The assumed conditions are those considered appropriate for the Jovian region and assuming that \( M_D = 0 \). The line \( \propto M_\oplus \) corresponds to type I drift described by Ward (1997) and its behaviour is valid until \( \approx 0.1 M_\oplus \) but beyond this value there is a transition to a behaviour \( \propto M_\odot^0 \) (type II motion).

(ii) the clearing of the disc material leads to a period of more rapid migration. Using this result, we compared the migration obtained assuming:

(i) an initial gap as that shown in Fig. 9 of Nelson et al. (2000) and with \( \sigma = \) constant [not given by Ida & Makino’s (1993) relation] with

(ii) the migration obtained taking account of the velocity dispersion dependence described above.

The results were in good agreement.

3 RESULTS

3.1 General framework

In this article, similarly to DP1 and DP2, we are mainly interested in studying the planet migration caused by the interaction with planetesimals. For this reason we assume that the gas is almost dissipated when the planet starts its migration.\(^2\) Moreover, we know that the behaviour of gas and dust/planetesimals is different. Usually, the decline of gas mass near stars is more rapid than the decline of the mass of orbiting solid matter (Zuckerman, Forveille & Kastner 1995). While the gas tends to be dissipated (evidence shows that the disc lifetimes range from \( 10^5 \) to \( 10^7 \) yr, see Strom, Edwards & Skrutskie 1993; Ruden & Pollack 1991), the coagulation process induces an increase of the density of solid particles with time (see Figs 5 and 6 below) and gives rise to objects of increasing dimensions, as shown in Fig. 2.

We clearly see that after times of the order of \( 10^6 \)–\( 10^7 \) yr, the coagulation process gives rise to large particles (>\( 10^5 \) cm), which have a small radial velocity, whence a negligible radial motion. This leads to a freezing of the solid surface density to the value reached at times of the order of \( 10^7 \) yr. In other terms, once solids are in the form of planetesimals, the gas coupling becomes unimportant and the radial distribution of solids no longer changes. This is why we do not need to calculate its evolution for times longer than \( 10^7 \) yr.\(^3\) Then the disc is populated by residual planetesimals for a longer period. Here it is important to stress that planetesimal formation is not independent from initial conditions. In particular, the final solid surface density depends in an intricate fashion on the initial disc mass \( M_{\text{d}} \) and on the turbulent viscosity parameter \( \alpha \), see Stepinski & Valageas (1997) and Kornet et al. (2001).

In order to investigate the dependence of the giant planet migration on the properties of the protoplanetary disc we integrated the model introduced in the previous sections for several values of the initial disc surface density (i.e. several disc masses), and different values of \( \alpha \). More precisely, as in Stepinski & Valageas (1997) we consider an initial gas surface density of the form

\(^2\)Clearly, the effect of the presence of gas should be that of accelerating the loss of angular momentum of the planet and to reduce the migration time. In our case, there is still gas after \( 10^7 \) yr, but it is in quantity inferior to that of planetesimals, especially in the case of lower-mass discs, in which it can be even two orders of magnitude less than planetesimals. Moreover, as noticed by Kominami & Ida (2002) dynamical friction and gravitational gas drag are essentially the same dynamical process and then the effect of the gravitational gas drag is incorporated in that of dynamical friction. Other forces, such as aerodynamic gas drag can be neglected when compared with gravitational gas drag (Kominami & Ida 2002), in our case.

\(^3\)Note, however, that the size distribution of planetesimals keeps changing owing to their mutual gravitational interaction.
Evolution of the mid-plane gas density $\rho$ for the two cases $M_d = 10^{-1}$ and $10^{-3} M_\odot$, and four values of $\alpha$. We can see a converged radial distribution of solids emerge at late times of the order of $10^6$ yr. Note that although the radial distribution of planetesimals depends on the value of $\alpha$, the total mass of solids in a disc is roughly independent of $\alpha$ since it remains approximately equal to the initial mass of solids in the disc. This means that solids are reshuffled within the disc but they are not lost into the star.\textsuperscript{4} The value of $\alpha$ determines the radial distribution

\textsuperscript{4}In fact, solids initially located close to the evaporation radius are lost but they constitute a small percentage of the total solid material, which is predominantly located in the outer disc.
of solids: particles in a disc with a larger value of $\alpha$ (a more turbulent disc) have larger inward radial velocities and consequently are locked into planetesimals closer to the star than particles in a less turbulent disc. Thus, the smaller the value of $\alpha$ the broader the final distribution of solids. The evaporation radius is located at $\approx 0.1$ au, while the outer limit of converged $\rho_s$ is approximately $10$ au for $\alpha = 10^{-4}$ and moves to $\approx 70$ au for $\alpha = 10^{-1}$, in the case $M_d = 0.1M_\odot$. In the case $M_d = 0.0001M_\odot$, the outer radius moves only from $\approx 10$ to $\approx 20$ au, since the initial extension of the disc is smaller.

The coagulation process gives rise to solids of $10^6$–$10^7$ cm. In discs characterized by smaller values of $\alpha$, and thus a more extended distribution of solids, the range of sizes goes from $10^6$–$10^7$ cm at the evaporation radius down to $10^3$–$10^4$ cm at the outer limit (see Fig. 2). This is because the coagulation process is less efficient at larger radii where the solid density is smaller and the velocity dispersion of the dust decreases (along with the gas temperature that governs the turbulent velocity). At later times these solids will continue to increase their sizes, but will not change their radial position, as they are already large enough to have a negligible radial motion. Note that the converged radial distribution of solids does not vary monotonically. There are bulges of matter near the evaporation radius and at the outer limit of $\rho_s$ (or $\Sigma_s$). These bulges are present in all cases, but become narrower for smaller values of $\alpha$. Their existence is a consequence of an intricate and non-linear interdependence between advection and coagulation, modulated by changing gas conditions and the character of turbulence. The formation of the inner bulge also signals the ‘freezing’ of the total mass of the solids in the disc, inasmuch as no more solids are lost to the vapour zone in subsequent disc evolution. They will either be captured by the bulge or come to rest by themselves at larger radii. Note that it is a radial squeezing caused by particle dynamics, rather than vertical squeezing caused by sedimentation, that is primarily responsible for establishing the bulge and keeping particles from falling into the vapour zone. The same mechanism is responsible for the abrupt drop in $\rho_s$, which we associate with the outer limit of solid matter distribution, and the presence of the $\rho_s$ bulge at the location of this drop.

Summarizing, the results show that the distribution of matter in the disc is sensitive to the assumed initial conditions. After times shorter than $3.2 \times 10^6$ yr the radial distribution of planetesimal mass density $\rho_s$ settles and can only change on a much longer time-scale by processes not considered in our model (such as mutual gravitational interactions between planetesimals proposed first by Safronov...
Evolution of planetesimal discs

Figure 4. (a) Evolution of the gas mid-plane density \( \rho \), for a disc with \( M_d = 0.0001 M_\odot \) and \( \alpha = 0.1 \) at \( t = 10^4 \) yr (dashed line), \( t = 10^6 \) yr (dotted line) and \( t = 10^7 \) yr (solid line). (b) Same as in (a) but with \( \alpha = 0.01 \). (c) Same as in (a) but with \( \alpha = 0.001 \). (d) Same as in (a) but with \( \alpha = 0.0001 \).

1969). We refer the reader to Stepinski & Valageas (1997) and Kornet et al. (2001) for more detailed discussions of the behaviour of protoplanetary discs.

3.4 Migration of a giant planet

Now, using the converged radial distribution of planetesimals derived in the previous section, we display in Fig. 7 the evolution of the semimajor axis \( a(t) \) of a 1 \( M_J \) planet in such a disc for \( \alpha = 0.1 \) and \( M_d = 0.1 \) (solid line), 0.01 (dotted line), 0.001 (short-dashed line) and 0.0001 \( M_\odot \) (long-dashed line), respectively. We recall here that the planet is initially located at 5.2 au with \( i_p \sim e_p \sim 0 \).

We clearly see that for a fixed value of \( \alpha \) a disc of larger mass leads to a more rapid migration of the planet. This behaviour is quite natural since a more massive disc obviously yields a stronger frictional drag. Thus, in the cases where \( M_d = 0.1, 0.01 M_\odot \) the planet migrates to \( \approx 0.08 \) au in \( \approx 1.5 \times 10^9 \) yr and to \( \approx 0.03 \) au in \( \approx 2.5 \times 10^9 \) yr, respectively. When the planet arrives at this distance the dynamical friction switches off and its migration stops. The stopping is simply caused by the inner radius of the planetesimal disc. The latter is set by the evaporation radius. Indeed, solid bodies cannot condense at such small orbital radii \( r \lesssim 0.1 \) au because the temperature is too high. Of course, this evaporation radius \( r_{\text{evap}} \) depends on the properties of the solid grains we consider. For instance, for ice particles we have \( r_{\text{evap}} \approx 1 \) au (e.g. Stepinski & Valageas 1997; Kornet et al. 2001). In this article we wish to understand the small orbital radii of observed planets, over the range 0.03–0.15 au. Therefore, we are interested in the inner regions of the disc where only high-temperature silicates with \( T_{\text{evap}} \approx 1350 \) K survive. This is why we selected this component in this study. Then, the main point of Fig. 7 is that the properties of solids known to exist in protoplanetary systems, together with reasonable density profiles for the disc, lead in a natural fashion to a characteristic radius in the range 0.03–0.2 au for the final semimajor axis of the giant planet. Note, in particular, that this process naturally explains why the migration stops at such radii.

For less massive discs, \( M_d = 10^{-3}, 10^{-4} M_\odot \), the migration is slower and the planet has not had enough time to migrate below \( \approx 4 \) au. Similarly to DP1 and DP2, note that we plotted our

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A. Del Popolo, S. Yeşilyurt and E. N. Ercan

Figure 5. (a) Evolution of the solid mid-plane density $\rho_s$ for a disc with $M_d = 0.1 M_\odot$ and $\alpha = 0.1$ at $t = 10^5$ yr (dashed line), $t = 10^6$ yr (dotted line) and $t = 10^7$ yr (solid line). (b) Same as in (a) but with $\alpha = 0.01$. (c) Same as in (a) but with $\alpha = 0.001$. (d) Same as in (a) but with $\alpha = 0.0001$.

calculation until $4.5 \times 10^9$ yr, which is the typical age of protoplanetary discs such as ours (the Sun is $\sim 4.5 \times 10^9$ yr old). Moreover, the planetesimal disc should have cleared out by this time (Ida et al. 2000; Del Popolo et al. 2001). It is interesting to note that the planet moves closer to the central star in the case where $M_d = 0.01 M_\odot$ than in the case where $M_d = 0.1 M_\odot$. This is caused by the fact that less massive discs usually have a smaller surface density that leads to a smaller temperature. This, in turn, implies a smaller evaporation radius so that the planet can move closer to the star. In order to study the effect of viscosity on migration, we performed three other calculations with $\alpha = 10^{-2}, 10^{-3}$ and $10^{-4}$, also plotted in Fig. 7. We can note that the dependence of the final radius on $\alpha$ is rather weak and non-monotonic because it competes with the dependence on the surface density, the precise value of which is an intricate function of $\alpha$. On the other hand, the migration time usually increases going from $\alpha = 10^{-4}$ up to $\alpha = 0.1$. Indeed, as seen from Figs 5 and 6 a smaller $\alpha$ can lead to a larger mid-plane solid density. This is partly caused by the fact that the dust disc height is smaller because the turbulence measured by $\alpha$ is weaker.

The results displayed in Fig. 7 show that for $\alpha \lesssim 0.01$ a Jupiter-like planet can migrate to a very small distance from the parent star, $0.03 < r < 0.1$ au, provided the disc mass is sufficiently large $M_d \gtrsim 10^{-3} M_\odot$. Only in the cases where $M_d \lesssim 10^{-4} M_\odot$ or $M_d \lesssim 10^{-3} M_\odot$ with $\alpha \gtrsim 0.1$, is the interaction with the planetesimal disc too weak to yield a significant migration. Note, however, that a giant Jupiter-like planet already makes a mass of the order of $10^{-3} M_\odot$. Hence such objects should arise from protoplanetary discs with a mass of the order of or larger than $10^{-3} M_\odot$, i.e. the mass of the initial protoplanetary disc should be at least of the same order since we do not expect all the matter to end up within this giant planet during its formation stage.7 Then, our results show that a final

6Indeed, we have the energy balance: $T_e^4 \propto \Sigma \nu \Omega_k^2$, where $T_e$ is the effective temperature, $\Sigma$ is the gas surface density, $\nu$ is the turbulent viscosity and $\Omega_k$ is the Keplerian angular velocity. Besides, the mid-plane temperature $T_s$ obeys the relation $T_s^2 \sim \tau T_e^2$, where $\tau$ is the opacity. On the other hand, the opacity $\tau$ is given by $\tau \sim \kappa \Sigma$ where $\kappa$ is the Rosseland opacity, while the turbulent viscosity scales as $\nu \sim a T_c/\Omega_k$, hence we obtain $T_s^2 \propto \kappa a \Sigma^2$.

7However, if the giant planet forms through a different process, e.g. the instability of a distinct cloud, which has no relation with the disc itself, then the latter may have any mass.
Evolution of planetesimal discs

Figure 6. (a) Evolution of the solid mid-plane density $\rho_s$ for a disc with $M_d = 0.0001 M_\odot$ and $\alpha = 0.1$ at $t = 10^5$ yr (dashed line), $t = 10^6$ yr (dotted line) and $t = 10^7$ yr (solid line). (b) Same as in (a) but with $\alpha = 0.01$. (c) Same as in (a) but with $\alpha = 0.001$. (d) Same as in (a) but with $\alpha = 0.0001$.

radius of $0.03 < r < 0.1$ au is a natural outcome, provided the planetesimal disc has not been cleared off too early on (e.g. by gravitational scattering).

Summarizing, the present model predicts that, unless the disc mass is very small $M_d \lesssim 10^{-4} M_\odot$, planets tend to move close to the parent star and to pile up to distances of the order of $0.03$–$0.04$ au. However, with some degree of fine-tuning it is also possible to find a planet at intermediate distances between its formation site and such small radii.

Before concluding this section, we have to add some hints of the way migration was calculated in the case where the disc mass, $M_d$, is smaller than that of the protoplanet. One can think that less massive discs are not able to move a more massive planet. This idea is not completely correct, as we shall see. As we know, when the planet mass is less than or comparable to the local disc mass, the planet behaves as a representative particle in the disc, as shown by Lin & Papaloizou (1986). When the protoplanet has a mass larger than that of the disc the satellite acts as a dam against the viscous evolution of the disc, and can lead to a substantial change in the disc structure in the vicinity of the planet. The coupled disc–planet evolution in this case has been studied by Syer & Clarke (1995) and by Ivanov, Papaloizou & Polnarev (1999), who showed that the inertia of the planet plays an important role in this case (see Nelson et al. 2000, equation 9). The effect of the interaction between the secondary and the disc leads to accumulation of the disc matter in the region behind the protoplanet (see equations 12 and 39 of Syer & Clarke 1995). In order to calculate the migration, in the quoted case, we used the surface density given in Ivanov et al. (1999) in the model of migration described in the previous sections.

3.5 Distribution of orbital periods

Finally, we show in Fig. 8 the predictions of our model for the distribution of planets in the inner part of the disc. More precisely, we plot the fraction of planets in the orbital period range $0$–$20$ d calculated from the model described in the previous section, assuming a uniform probability distribution in the plane $[\log(\alpha), \log(M_d)]$ in the range $10^{-4} < \alpha < 10^{-1}$ and $10^{-4} < M_d < 10^{-1} M_\odot$. This is a rather arbitrary choice but our point is simply to check whether the observed distribution can be explained by reasonable values for $\alpha$ and $M_d$ or whether it requires some fine-tuning. The right-hand panel represents the same distribution obtained with the data given at http://www.exoplanets.org (see also Kuchner & Lecar 2002). We can see that we actually obtain a reasonable agreement with the data.
Figure 7. (a) The evolution of the semimajor axis $a(t)$ of a Jupiter-mass planet, $M = 1M_J$, in a planetesimal disc for $\alpha = 0.1$ and several values of $M_d$, 0.1 (solid line), 0.01 (dotted line), 0.001 (short-dashed line) and 0.0001 $M_\odot$ (long-dashed line). (b) Same as in (a) but with $\alpha = 0.01$. (c) Same as in (a) but with $\alpha = 0.001$. (d) Same as in (a) but with $\alpha = 0.0001$.

Of course, there is some degeneracy so that different probability distributions in the plane $[\log(\alpha), \log(M_d)]$ could yield the same results. However, the main point of Fig. 8 is to show that the observed distribution of orbital periods can be easily recovered from our model, without any fine-tuning and with reasonable values for $\alpha$ and $M_d$. Moreover, as can be seen from Fig. 7, within our model the peak at 3–4 d for the orbital period comes from discs with $M_d \gtrsim 10^{-3} M_\odot$ and $\alpha \lesssim 10^{-2}$. As noted in the previous section this result actually fits nicely with the data since we can expect Jupiter-like planets to form in such protoplanetary discs with $M_d \gtrsim 10^{-3} M_\odot$ (note also that the dependence on $\alpha$ is rather weak).

Of the $\approx 20$ planets with periods less than 20 d, seven have periods in the range 3–4 d, and it seems that this trend is not an artefact of observational selection (see Kuchner & Lecar 2002). As stressed by Kuchner & Lecar (2002), phenomena such as the interactions among two planets and a star, that can leave a planet trapped by stellar tides into a circular orbit of $\approx 0.04$ au (Rasio & Ford 1996), or halting of planet migration by loss of mass to the star (Trilling et al. 1998), are rare. The migration through resonant interaction with planetesimals in the disc (Murray et al. 1998) provides a natural way of halting the planet but unfortunately changing the orbit of a Jupiter-mass planet requires approximately a Jupiter mass of planetesimals and a too
massive disc. The other possibility to explain the observed distribution of planets seen in Fig. 8 is connected with a gas disc truncated at a temperature of 1500 K by the onset of magneto-rotational instability (Kuchner & Lecar 2002). In other terms, the disc temperature determines the orbital radii of the innermost surviving planets. In our model, the distribution of internal planets is naturally connected with the evaporation of planetesimals at very short radii. As stressed in DP1, DP2, this model does not have the drawback of Murray et al. (1998), namely that of requiring a too large disc mass for migration and at the same time it has the advantage of Murray et al. (1998) of having an intrinsic natural mechanism that provides halting of migration.

4 CONCLUSIONS

In this paper, we further developed the model for the migration of planets introduced in DP1 and extended to time-dependent planetesimal accretion discs in DP2. After releasing the assumption of DP2 that the surface density of planetesimals is proportional to that of gas, we used a simplified model developed by Stepinski & Valageas (1996, 1997), which is able to simultaneously follow the evolution of gas and high-temperature silicates for up to $10^7$ yr. Then we coupled this disc model to the migration model introduced in DP1 in order to obtain the migration rate of the planets in the planetesimal disc and to study how the migration rate depends on the disc mass, on its time evolution and on the dimensionless viscosity parameter $\alpha$.

We found that in the case of discs having a total mass of $M_d > 10^{-3} M_\odot$ planets can migrate inward over a large distance, while if $M_d < 10^{-3} M_\odot$ the planets remain almost at their initial position. On the other hand, for $M_d \sim 10^{-3} M_\odot$ a significant migration requires $\alpha \leq 10^{-2}$.

If the migration is efficient the planet usually ends up at a small radius in the range 0.03–0.1 au, which is simply set by the evaporation radius of the gaseous disc that gave rise at earlier times to the radial distribution of the planetesimal swarm. Thus, our model provides a natural explanation for the small observed radii of extrasolar giant planets. In particular, the halting of the inward migration of the planet is intrinsic to this process and it does not require a second mechanism.

Finally, we noted that the observed distribution of planetary periods in the range 0–20 d can be easily obtained within this framework without fine-tuning. In particular, such small radii naturally occur for Jupiter-like planets if the initial disc has a mass of the same order or larger, which is quite likely. In order to inhibit this process (so that Jupiter-like planets such as our own remain at larger distances $\gtrsim 5$ au) the planetesimal disc must be cleared off over a time-scale of the order of or smaller than $10^6$ yr (depending on the properties of the disc) or the disc mass must be rather small (i.e. smaller than $1 M_\oplus$) which could suggest an alternative formation scenario for such a giant planet (i.e. not related to the disc itself).

ACKNOWLEDGMENTS

We are grateful to P. Valageas and T. Stepinski for stimulating discussions during the period in which this work was performed. ADP would like to thank ASI and Boğaziçi University Research Foundation and for the financial support through project code 01B304.

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A. Del Popolo, S. Yeşilyurt and E. N. Ercan

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