How many arcmin-separation lenses are expected in the 2dF QSO survey?

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ABSTRACT
Wide-separation lensing statistics offer information about the density profile and abundance of dark haloes. A possible discovery of six quasar pairs, which may be lensed multiple images, was reported by Miller et al. recently. These pairs are selected from a catalogue of the 2dF quasar (QSO) survey comprising 22 163 quasars. We calculate expected numbers of lensed quasars taking account of the redshift and magnitude distributions of the quasar catalogue. Given some of the six pairs are genuine lensed systems, we put interesting constraints on the inner slope of dark haloes, \( \Omega_0 \) and \( \sigma_8 \). We show that the detection of even one lens with separation > 30 arcsec is marginally consistent with models that have cuspy inner density profile, \( \rho \propto r^{-1.5} \) and very large \( \sigma_8 \), \( \sigma_8 \gtrsim 1.2 \) for \( \Omega_0 = 0.3 \). To reconcile with constraints from X-ray clusters or cosmic shear, much lower \( \Omega_0 \) and much higher \( \sigma_8 \) values are needed, although such high values for \( \sigma_8 \) seem too extreme. The shallower inner density profile \( \rho \propto r^{-1} \) is hardly acceptable. In particular, the expected number of lenses with separation > 200 arcsec is too small to explain the discovery of such anomalously wide-separation lens systems. These results imply that we miss some important systematic effects, there is a problem in the cold dark matter scenario, or none of these six quasar pairs is likely to be a lensed image.

Key words: gravitational lensing – galaxies: clusters: general – cosmology: theory – dark matter

1 INTRODUCTION
The cold dark matter (CDM) scenario predicts the existence of cuspy dark haloes and thus is expected to produce significant numbers of wide-separation lenses (\( \theta \gtrsim 6 \) arcsec). Statistics of such wide-separation lenses are known to be a powerful tool for probing the abundance (Narayan & White 1988; Kochanek 1995) and the density profile of dark haloes (Maoz et al. 1997; Wyithe, Turner, & Spergel 2001; Keeton & Madau 2001; Sarbu, Rusin & Ma 2001; Takahashi & Chiba 2001; Li & Ostriker 2002; Oguri 2002). Although a number of radio surveys has tried, they could not find wide-separation lensed quasars (e.g. Phillips, Browne & Wilkinson 2001a; Phillips et al. 2001b; Ofek et al. 2001, 2002). The lack of wide-separation lenses, however, does not conflict with the CDM scenario because the expected lensing rate is significantly smaller than that of small separation lensing (\( \theta \sim 1 \) arcsec).

Recently, Miller et al. (2003) reported that they found six quasar pairs which may be lensed multiple images in the 2dF quasar (QSO) catalogue comprising 22 163 quasars, although the number of lensed quasars may be even larger than this because they have carried out follow-up observations for only 11 quasar pairs among 38 quasars selected as possible lens candidates. These systems were identified to be lensed images from the detailed comparison of quasar spectra. The separations of all these quasar pairs are larger than 30 arcsec and the separations of some pairs reach even \( \sim 200 \) arcsec. If this surprising result is true, it offers a lot of information about dark haloes. In this Letter, we calculate the expected number of arcmin-separation lenses (\( \theta > 30 \) arcsec) in the 2dF QSO survey. We use the realistic density profile predicted in the CDM scenario and take account of the redshift and magnitude distribution of the quasar catalogue. Therefore our results can be directly compared with the observation. We show that the existence of such wide-separation lenses in the 2dF QSO catalogue is marginally consistent with the ‘concordance’ cosmology if the value of only \( \sigma_8 \) is very large, \( \sigma_8 \gtrsim 1.2 \) for \( \Omega_0 = 0.3 \). This means that we can strongly constrain dark halo properties if some of quasar pairs reported by Miller et al. (2003) are truly gravitational lens systems. Throughout this Letter, we assume a flat universe \( \Omega_0 + \lambda_0 = 1 \) and \( h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.7 \).

2 CALCULATION OF PROBABILITY DISTRIBUTIONS
The lensing probability distribution at wide separation reflects the properties of dark haloes, rather than galaxies (Nakamura & Suto 1997; Keeton 1998; Oguri 2002). The halo density profiles predicted by recent \( N \)-body simulations may be parametrized as a one-parameter family, the generalized NFW profile (Zhao 1996; Jing & Suto 2000):

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where \( \rho_{\text{crit}}(z) \) is a critical density at \( z \). While the correct value of \( \alpha \) is still unclear, the existence of a cusp with \( 1 \lesssim \alpha \lesssim 1.5 \) has been established in recent N-body simulations (Navarro, Frenk & White 1996; Moore et al. 1999; Jing & Suto 2000; Fukugishie & Makino 2001). For definiteness, in this Letter we consider two cases, \( \alpha = 1 \) and 1.5, which cover the range of the CDM predictions. The scale radius \( r_s \) is related to the concentration parameter \( c(M, z) \equiv r_{\text{vir}}(M, z)/r_s(M, z) \). Then the characteristic density \( \delta_c(z) \) is given in terms of the concentration parameter (see e.g. Oguri, Taruya & Suto 2001). The lens equation of the generalized NFW profile has three solutions if \( |\eta| < \eta_c \), where \( \eta \) is the position of a source in the source plane and \( \eta_c \) is a radius of the radial caustic in the source plane. The image separation is defined between the outer two solutions and is approximated as \( \theta(M, z; z_s, z_l) \approx 2 \xi/M_{\text{vir}} \), where \( \xi \) is a radius of the tangential critical curve in the lens plane, \( D_{\text{OL}} \) denotes the angular diameter distance from the observer to the lens plane and \( z_s \) and \( z_l \) indicate the redshifts of the source and lens, respectively.

The concentration parameter \( c_{\text{vir}} \) is one of the most important parameter in the generalized NFW density profile (equation 1). In numerical simulations, it has been found that the concentration parameter depends on the mass \( M \) and redshift \( z \) of haloes (e.g. Bullock et al. 2001). Moreover, it shows considerable scatter which reflects the difference in the formation epoch (Wechsler et al. 2002). The scatter in the concentration parameter is well described by a log-normal distribution. For the median of concentration parameter \( c_{\text{vir,med}} \), we adopt the value and redshift dependence reported by Bullock et al. (2001): 

\[
c_{\text{vir,med}}(M, z) = (2 - \alpha) \frac{8}{1 + z} \left( \frac{M}{10^{14} h^{-1} M_\odot} \right)^{-0.13}.
\]

Note that this fitting form is slightly different from the one Bullock et al. (2001) originally proposed and is correct at \( M \sim 10^{14} h^{-1} M_\odot \). A factor \( 2 - \alpha \) in equation (2) gives a natural way to generalize \( \alpha \neq 1 \) (Keeton & Madau 2001; Jing & Suto 2002). The scatter of the concentration parameter \( \sigma_c \) is also important element in gravitational lens statistics (Keeton & Madau 2001). We adopt the value \( \sigma_c = 0.3 \) which has been obtained from N-body simulations (Jing 2000; Bullock et al. 2001; Wechsler et al. 2002; Jing & Suto 2002).

The probability that a source at redshift \( z_s \) and having the absolute luminosity \( L \) is observed as multiply lensed system with separation larger than \( \theta \) is given by

\[
P^{b}(>\theta; z_s, L) = \int_0^{\delta_c} d\delta_c \int_{\delta_{\text{min}}}^{\delta_c} \delta_{\text{min}} d\delta_{\text{min}} M \sigma_{\text{eff}} B \left( c \frac{dt}{dM} (1 + z_l)^3 \frac{dn}{dM} \right),
\]

where \( B \) is the magnification bias (Turner 1980):

\[
B = \frac{2}{\sqrt{\pi}} \Phi(\theta, z_s, L) \int_0^{\delta_c} dy y \Phi(\theta, z_s, L/M(y)) \frac{1}{\mu(y)},
\]

with \( \Phi(\theta, z_s, L) \) being the luminosity function of sources. The lensing cross-section \( \sigma_{\text{eff}} \) is simply given by the area encompassed by the radial caustic, \( \sigma_{\text{eff}} = \pi \xi D_{\text{OS}}^2/D_{\text{OL}}^2 \), where \( D_{\text{OS}} \) indicates the angular diameter distance from the observer to the source plane. The lower limit of mass integral \( M_{\text{min}} \) is related to \( \theta \) as \( \theta = \theta(M_{\text{min}}, z_s, z_l) \). The magnification bias (equation 4) should be calculated for the faintest of the two images, because both lensed images must appear above the flux limit of the 2QZ survey (Miller et al. 2003). We use an approximation of the magnification factor \( \mu(y) \) that was derived by Oguri et al. (2002).

Since wide-separation lenses with \( \theta > 30 \) arcsec are considered to be generated by massive clusters, we should choose the mass function of dark haloes (\( dn/dM \) in equation 3) carefully. We adopt equation (B3) of Jenkins et al. (2001) which agrees well with the simulated high-mass halo abundance (Evrard et al. 2002; Komatsu & Seljak 2002; Hu & Kravtsov 2003; Pierpaoli et al. 2003). Note that \( \rho = 1800\Omega(z_0)\rho_{\text{crit}}(z) \) should be used as the mean overdensity when one adopts this mass function.

### 3 QUASAR CATALOGUE

To make a precise prediction which can be directly compared with observed lensing rate, we must properly take account of the redshift and magnitude distributions. Since the whole sample used to search lensed quasars is not publicly available, we instead use the 2dF 10k catalogue comprising \( \sim 10,000 \) quasars (Crook et al. 2001) which is a part of the whole sample. Then predicted numbers of wide-separation lenses are calculated as follows. First, from the catalogue we extract the numbers of quasars \( N(z_s, b_j) \) which are located \( z_s - \Delta z/2 < z_s < z_s + \Delta z/2 \) and have magnitude \( b_j - \Delta b/2 < b_j < b_j + \Delta b/2 \). We use \( \Delta z = 0.1 \) and \( \Delta b = 0.2 \). The average probability that quasars are lensed with separations larger than \( \theta \) is then given by

\[
P_{\text{lens}}(>\theta) = \frac{\int_0^{\delta_c} d\delta_c \int_{\delta_{\text{min}}}^{\delta_c} \delta_{\text{min}} d\delta_{\text{min}} M \sigma_{\text{eff}} B \left( c \frac{dt}{dM} (1 + z_l)^3 \frac{dn}{dM} \right) P[>\theta; z_s, L(b_j)]}{\int_0^{\delta_c} d\delta_c \int_{\delta_{\text{min}}}^{\delta_c} \delta_{\text{min}} d\delta_{\text{min}} M \sigma_{\text{eff}} B \left( c \frac{dt}{dM} (1 + z_l)^3 \frac{dn}{dM} \right)}.
\]

where \( L(b) \) is the \( B \)-band absolute luminosity corresponding to \( b \). When the quasar continuum spectrum is described by a power law, \( f_\nu \propto \nu^{-\alpha_s} \), the K-correction can be approximated as \( K(z) = -2.5(1 - \alpha_s) \log (1 + z) \). We assume \( \alpha_s = 0.5 \) to calculate the K-correction. The total number of lensed quasars expected in the 2dF QSO survey is

\[
N_{\text{lens}}(>\theta) = N_{\text{QSO}} P_{\text{lens}}(>\theta),
\]

where \( N_{\text{QSO}} = 22,163 \) is the total number of quasars in the whole sample.

The luminosity function of quasars is needed to compute magnification bias. We adopt the double power-law luminosity function:

\[
\Phi(z, L) dL = \frac{\Phi_s}{[L/L_s(z)]^{\beta_s} + [L/L_s(z)]^{\beta_i}} L_s(z) dL.
\]

We assume pure luminosity evolution models with \( L_s(z) \propto 10^{0.4(z+1)\alpha_L} \) and use a best-fitting model which was derived by Boyle et al. (2000) in the \( \Omega_m = 0.3, 0.7 \) universe: \( \beta_s = 3.41, \beta_i = 1.58, k_1 = 1.36, k_2 = -0.27 \) and \( M_s = -21.15 + 5 \log h \).

### 4 RESULTS

#### 4.1 Numbers of lenses

First we plot the predicted number distribution of image separation in the 2dF QSO survey in Fig. 1. The cases that the density profile of lens objects is described by the Singular Isothermal Sphere (SIS) are also shown for reference. The value of \( \sigma_8 \) has been constrained from X-ray clusters or cosmic shear, but resultant values show discrepancies among papers, ranging from \( \sim 0.7 \) to \( \sim 1.0 \) for \( \Omega_m \approx 0.3 \) (e.g. Pierpaoli et al. 2003). Therefore we plot both \( \sigma_8 = 0.7 \) and \( \sigma_8 = 1 \) models. Fig. 1 clearly indicates that the predicted numbers of lenses strongly depend on both density profile (\( \sigma_8 \)) and

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the abundance ($\sigma_s$) of dark haloes. In particular, numbers of arcmin-separation lensed quasars are highly sensitive to $\sigma_s$ because such very wide-separation lenses are mainly produced by massive clusters. We find that expected numbers of lensed quasars with image separation larger than 200 arcsec is quite small, $N_{\text{lens}}(>200\text{ arcsec})<10^{-6}$, while four quasar pairs which are likely to be lensed were found (Miller et al. 2003). We further find that even an ‘extreme’ model with $\alpha=1.5$ and $\sigma_s=1.4$ can produce only $\sim 3 \times 10^{-4}$ lenses with $\theta>200\text{ arcsec}$ on average. On the other hand, less wide-separation lenses ($\theta\sim30\text{ arcsec}$) may be statistically possible if both $\alpha$ and $\sigma_s$ are large.

To see what constraint can be put from the detection of wide-separation lenses, in Fig. 2 we plot contours of $N_{\text{lens}}(>30\text{ arcsec})$ in the $\Omega_{0}-\sigma_s$ plane. We also plot constraints from the observed number of lenses assuming that some of six pairs reported by Miller et al. (2003) are true lens systems. These constraints are calculated from the Poisson distribution $P(k|N)=N^ke^{-N}/k!$, where $k$ is the observed number and $N$ is the expectation. Given the numbers of true lenses range from $k=1$ to 6, the lower limits of $N$ (95 per cent confidence level) become 0.35, 0.82, 1.36, 1.97, 2.61 and 3.29, respectively. This figure indicates that the detection of even one lens with $\theta>30\text{ arcsec}$ in the 2dF QSO catalogue needs very high $\sigma_s$, $\sigma_s \gtrsim 1.2$ for $\Omega_{0}=0.3$ and $\alpha=1.5$. Our constraint and constraints from X-ray clusters or cosmic shear which are approximated as $0.4<\sigma_s\Omega_{0}^{0.5}<0.55$ (see e.g. Pierpaoli et al. 2003) shows marginal agreement if $\Omega_{0} \lesssim 0.15$, $\sigma_s \gtrsim 1.5$ and $\alpha=1.5$, although this solution seems too extreme. In particular, models with $\alpha=1.0$ are hardly acceptable because they need unusually high $\sigma_s$ or $\Omega_{0}$ in order to produce lenses with $\theta>30\text{ arcsec}$. However, this result is somewhat embarrassing because the lack of wide-separation lensing in other surveys (e.g. Phillips, Browne & Wilkinson 2001a; Phillips et al. 2001b; Oke et al. 2001, 2002) already puts the upper limit of the lensing rate. For instance, our model with $\alpha=1.5$, $\Omega_{0}=0.3$ and $\sigma_s=1.2$ predicts a lensing rate $P(>6\text{ arcsec}) \sim 3 \times 10^{-4}$ at $z \sim 1.3$ which is marginally consistent with the upper limit of the lensing rate in Phillips et al. (2001b), $P(>6\text{ arcsec}) \lesssim 3 \times 10^{-4}$ at 95 per cent confidence limit.

Figure 1. The predicted number distribution of image separations in the 2dF QSO survey (equation 6). For the density profile of lens objects, both $\alpha=1.5$ and $J$ are considered (see equation 1). The numbers assuming Singular Isothermal Sphere (SIS) density profile, $\rho \propto r^{-2}$, are also plotted for reference. The cosmological models adopted in these plots are ($\Omega_{0}$, $\sigma_s$) = (0.3, 0.7) and ($\Omega_{0}$, $\sigma_s$) = (0.3, 1.0) in the flat universe ($\Omega_{0}+\lambda_{0}=1$).

Figure 2. Contours of $N_{\text{lens}}(>30\text{ arcsec})$ in the $\Omega_{0}-\sigma_s$ plane. Constraints (95 per cent confidence level) from the detection of wide-separation lenses are shown by dashed lines. From lower to upper of dashed lines, the number of genuine lens systems is assumed to be 1, 2, 3, 4, 5, 6. Dotted lines indicate the recent constraints on $\Omega_{0}$ and $\sigma_s$ from X-ray clusters or cosmic shear, which can be approximately expressed as $0.4<\sigma_s\Omega_{0}^{0.5}<0.55$.

Note that these constraints are for the case that only one of six quasar pairs is a genuine lens system; if the number of lenses is more than one, the situation becomes worse. In this case, the discrepancy between strong lensing constraints and X-ray/shear constraints becomes more serious. Moreover, this result may conflict with other surveys which could not detect wide-separation lenses.

### 4.2 Theoretical uncertainties

We also examine possible theoretical uncertainties except for the uncertainties of the density profile and cosmological parameters. More specifically, we examine the uncertainties of the halo mass function and the quasar luminosity function. The result is summarized in Table 1. First we adopt the mass functions of Press & Schechter (1974) and Sheth & Tormen (1999) and see how the number of lensed quasars changes. We find that the uncertainty of the mass function is fairly large. In particular, the mass function of Sheth & Tormen (1999) predicts more than three times as large number of lenses with $\theta>30\text{ arcsec}$ as our fiducial model. This is because
the mass function of Sheth & Tormen (1999) seems to overestimate the number density of massive haloes (Jenkins et al. 2001; Hu & Kravtsov 2003). Moreover, we point out that the number of anomalously wide-separation lenses ($\theta > 200$ arcsec) is much more sensitive to the choice of the mass function. Therefore, in the statistics of such anomalously wide-separation lenses the uncertainty of the mass function should be carefully examined.

Since the magnification bias is sensitive to the slope of adopted quasar luminosity function, next we examine the uncertainty of the quasar luminosity function using the luminosity function from the 10k catalogue (Croom et al. 2001). This luminosity function has somewhat shallower slopes, $\beta_h = 3.28$ and $\beta_l = 1.08$, compared with our fiducial model. We find that this uncertainty is less than factor 2 and is not so large as to change our main results, because $\sigma$, $\Omega_0$, and $\sigma_8$ can change the number of wide-separation lenses by orders of magnitude.

### 4.3 Expected time delays between images

The main drawback of wide-separation lensing statistics is that it is hard to recognize quasar pairs as gravitational lens systems due to large time delays between images, any spectrum change within the time-scale of differential time delays may prevent one from selecting such systems as lens candidates. We show conditional probability distributions of time delays proposed by Oguri et al. (2002) in Fig. 3. We find that the dependence of time delays on the density profile is weak when the separation is large, although this tendency was already shown by Oguri et al. (2002). They also concluded that time delay probability distribution is insensitive to cosmological parameters. Fig. 3 suggests that lenses with even smaller separations, $\theta \sim 30$ arcsec, are likely to have time delays larger than 10 years which may be typical time-scale forming the broad absorption line (Ma 2002). Therefore, to assert quasar pairs as lensed images, one must compare spectral signature which would be unchanged within possible time delays. Needed time-scales are $\Delta t > 100$ yr for $\theta \sim 30$ arcsec and $\Delta t > 1000$ yr for $\theta \sim 200$ arcsec. We note that the information of flux ratio and the central core image may become one of the evidence of gravitational lensing (Rusin 2002).

### 5 SUMMARY

In this Letter, we have calculated the predicted numbers of arcmin-separation ($\theta > 30$ arcsec) lensed quasars in the 2dF QSO survey. We have presented realistic predictions based on the CDM scenario taking account of the redshift and magnitude distributions of the quasar catalogue. Detailed comparison between theoretical and observed numbers of lensed quasars indicates that the detection of wide-separation lenses puts interesting constraints on the density profile and abundance of dark haloes. The case that only one of six pairs is a genuine lens system is marginally consistent with the model that has a cuspy inner density profile $\rho \propto r^{-1.5}$ and the large value of $\sigma_s$, $\sigma_s \gtrsim 1.2$ for $\Omega_0 = 0.3$. To reconcile this result with X-ray or shear measurement, much smaller $\Omega_0$ ($\Omega_0 \lesssim 0.15$) and much larger $\sigma_s$ ($\sigma_s \gtrsim 1.5$) values are needed. Our result of this large $\sigma_s$ is similar to that of the Sunyaev–Zel’dovich angular power spectrum, $\sigma_s \sim 1.1$ (Komatsu & Seljak 2002). We have found also that it is quite hard to produce lenses with separation $>200$ arcsec. Thus a conservative interpretation of this observation is that none of these quasar pairs is lensed. However, if it turns out that some of these quasar pairs are genuine lens systems, we can put interesting constraints on not only the density profile of dark haloes but also the $\Omega_0$ and $\sigma_s$ that are somewhat different from X-ray/shear constraints. We note that the number of genuine lens systems may be significantly larger than the six which Miller et al. (2003) reported because only 11 of 38 candidates has been observed spectroscopically. However, such anomalously high lensing rates cannot be reproduced by even the most optimistic models. In this case, we have to examine whether we miss some important systematic effects which increase lensing rates. One possible systematic effect is the asymmetry of lensing haloes, although this effect has been considered to be small so far.

<table>
<thead>
<tr>
<th>Model</th>
<th>$N_{\text{lens}} (&gt;30$ arcsec$)$</th>
<th>$N_{\text{lens}} (&gt;200$ arcsec$)$</th>
</tr>
</thead>
<tbody>
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<td>Fiducial Model</td>
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<td>$1.8 \times 10^{-7}$</td>
</tr>
<tr>
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<td>$1.2 \times 10^{-1}$</td>
<td>$1.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>Sheth &amp; Tormen MF</td>
<td>$2.8 \times 10^{-1}$</td>
<td>$8.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Croom et al. LF</td>
<td>$4.4 \times 10^{-2}$</td>
<td>$1.1 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Figure 3. The conditional probability distributions of possible time delays are shown for each quasar pair. The definition and calculation of $P(\Delta t | \theta)$ was shown by Oguri et al. (2002). The redshift and magnitude for each image are taken into account. For the cosmological model, we adopt $(\Omega_0, \sigma_8) = (0.3, 1.0)$. 

In any case, statistics of wide-separation lensing offer a promising way to probe the abundance and density profile of dark haloes.

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