

Heat and Mass Transfer From Freely Falling Drops¹

R. Porter.² Authors employ the analogy between heat and mass transfer in order to relate the mass-transfer coefficient h_D to the sensible-heat-transfer coefficient h . They are then able to determine h in the dimensionless form of the Nusselt number by measuring the combined heat transfer due to sensible and evaporative modes. This may lead to error because the evaporative-transfer mode usually dominates and the relation between heat and mass transfer is not exactly known for the present case.

Indeed, the specific form of the analogy employed by the authors in their dimensionless groups following equation (6) and in equation (8) does not appear to be appropriate for the present case. The form used is based on

$$\frac{h_D}{h} = \frac{h_D d k_c D}{D h d k_c} = \frac{\text{Sh}}{\text{Nu}} \frac{D}{\alpha \rho c_p}$$

$$\frac{h_D}{h} = \frac{\text{Sh}}{\text{Nu}} \frac{\text{Le}}{\rho c_p}$$

being taken as

$$\frac{h_D}{h} (\text{authors}) = \frac{\text{Le}}{\rho c_p}$$

which requires

$$\frac{\text{Sh}}{\text{Nu}} (\text{authors}) = 1$$

This form of the analogy applies where Nusselt and Sherwood numbers are independent of Prandtl and Schmidt numbers, respectively, such as for pure diffusion where $\text{Nu} = \text{Sh} = 2$. Note that this is the leading term in the Ranz-Marshall correlation (5a) and (5b). However, in the present case convection dominates, and it is more appropriate to utilize the well-known empirical relation for laminar and turbulent flows over a wide range of geometries³

$$\frac{\text{Nu}}{\text{Sh}} = \left(\frac{\text{Pr}}{\text{Sc}} \right)^{1/3} = \text{Le}^{1/3}$$

which corresponds to the second terms in the Ranz-Marshall equations. In this case

$$\frac{h_D}{h} = \frac{\text{Le}^{2/3}}{\rho c_p}$$

as compared with $\text{Le}/(\rho c_p)$ used by the authors in their equation (8).

In the present case $\text{Le} > 1$, and so the heat exchange due to mass transfer is over predicted and the calculated Nusselt number is under predicted. Because of the dominance of the evaporative mode, Nusselt

number is effectively divided by $\text{Le}^{1/3}$. Fortunately, $\text{Le} = 0(1)$, and so the error is not large. Nevertheless, we estimate a reduction by about 7 percent. This may explain part of the correction factor g applied to the Ranz-Marshall relation for $x/d \geq 150$.

Actually, in most cooling-tower work the assumption of psychrometric ratio $h/(\rho c_p h_D) \equiv 1$ ($\text{Le} = 1$) is made, and combined heat and mass transfer is determined solely from the total heat (Carrier's sigma function) which is a function only of the adiabatic-saturation temperature. However, some recent work is directed toward separate computations on both modes.⁴

It may be desirable to perform experiments such as those of the present authors where the drops would be collected and weighed in addition to temperature measurement. This may allow measurement of both sensible and evaporative transport and evaluation of the appropriate analogy.

W. E. Dunn.⁵ I found the paper by Yao and Schrock quite interesting. However, my analysis suggests an alternative and equally plausible explanation of the apparent discrepancy between the authors' data and the Ranz-Marshall correlation.

I have plotted their data for 3-mm drops versus time in Fig. 1. The error bars shown are based on the authors' estimate that the data are accurate to $\pm 0.1^\circ\text{C}$. There are several key features of this plot that warrant comment.

First, to emphasize that the drop spends roughly the same time inside the thermal cavity as it does falling, I have taken the origin of the time axis as the instant at which the drop begins to form and not the instant at which it begins to fall. The time spent inside the cavity is calculated from the frequency of drop production (2 s^{-1} from Yao [21]) and the time (0.09 s) required for a 3-mm drop to fall through the 4-cm height of the thermal cavity. Unfortunately, the first data point was not taken until roughly 0.7 s after formation began (outside the thermal cavity).

Second, the dotted lines in Fig. 1 show calculations based on the Ranz-Marshall correlation but matched with the temperature measured at the first data point. The agreement here is quite good. Therefore, I conclude that the Ranz-Marshall correlation is adequate after the first 0.6 s of a drop's life. In their analysis Yao and Schrock assume that the drop loses no heat in the thermal cavity and suggest that it follows a temperature history like that shown by the dotted line of Fig. 1. This reasoning leads to the conclusion that the Nusselt number is initially very large (as indicated by the large slope of the dotted curve). Yao and Schrock further state that this enhanced cooling is due to oscillation and other dynamic effects not accounted for by the Ranz-Marshall correlation. Although I acknowledge that

¹ By S. C. Yao and V. E. Schrock, published in the Feb. 1976 issue of the JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 98, pp. 120-126.

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³ Sherwood, T. K., Pigford, R. L., and Wilke, C. R., *Mass Transfer*, McGraw-Hill, New York, Section 5.3, 1975.

⁴ Yadigaroglu, G., and Pastor, E. J., "An Investigation of the Accuracy of the Merkel Equation for Evaporative Cooling Tower Calculations," AIAA Paper No. 74-765, ASME Paper No. 74-HT-59, July, 1974.

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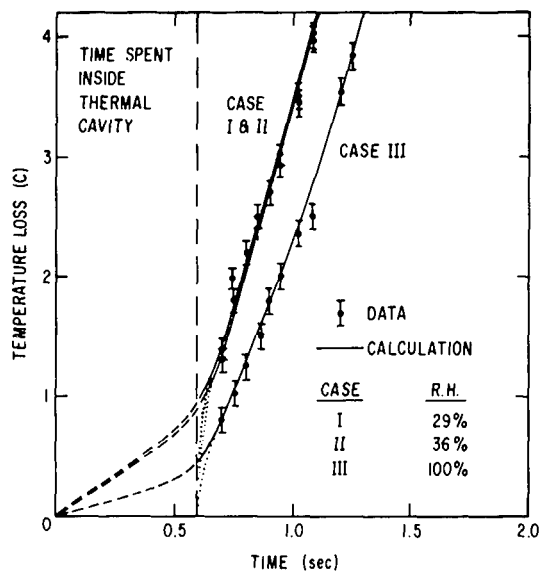


Fig. 1

such enhancement is indeed possible, I would submit that the drop could equally well follow a temperature history like that shown by the dashed line, cooling at a somewhat slower, but still nonzero, rate within the thermal cavity. I would argue further that it is impossible to determine from these data alone what factors in what proportions are responsible for the observed increase in cooling over the Ranz-Marshall correlation. Moreover, due to very complex nature of the processes involved, theoretical analysis is of little value here. The only way to satisfactorily determine the role of dynamic effects is through a very precise experimental study of the cooling that takes place in the first few centimeters of fall. This is, of course, a very difficult experiment. Finally, I would suggest that a falling distance of 3 m is probably too small to determine whether an increase in internal resistance can significantly reduce cooling during the late stages of the life of the drop.

In summary, I would conclude from these observations that the Ranz-Marshall correlation well represents the trend of the data in the range of 0.2–3 m, and that the data are insufficient to draw hard-and-fast conclusions concerning either the very early or the very late stages of a drop's life.

Authors' Closure

The authors are grateful to Porter and Dunn for their discussions which contribute to the understanding of this problem. In most cooling-tower work the analogy between heat and mass transfer is employed. Porter estimated the error as 7 percent. Yadigaroglu and Pastor⁶ conducted systematic study of the comparison of results based upon the Merkel equation and the exact formulation for a cooling tower heat exchanger problem. As to their finding, the error depends on operation conditions and is about 4–7 percent with the Merkel equation giving larger cooling range. But this error will not affect the evaluation of the correction factor g . The g factor, which is less than 1 for $x/d > 100$, fits the data adequately. This does not imply that the

transfer coefficient from the drop is less than predicted by Ranz-Marshall correlation. The g factor compensates for the overprediction by the complete mixing model while actually internal circulation is developing within the drop at $x/d > 100$.

Porter suggests to relate the mass transfer to the mass change in the drop. It will be a very hard task to measure the change of weight of 3–6-mm dia drops which undergo temperature change no larger than 3°C, even by assuming that all the cooling is due to evaporation.

In response to Dunn, in the present experiment the temperature at the drop generator needle was seen to be within 0.03°C of the thermal cavity temperature. Also the drop temperature measured in a small cup at the mount of the cavity (while it partially blocks the opening of the cavity) is in good agreement with the needle temperature. The water temperature in the calorimeter was maintained at about 0.11°C higher than the drop temperature measured in the dewar to compensate for the evaporative cooling of the water in the dewar. The differential was chosen on the basis of a simple analysis which assumed that the drops entering the calorimeter entrain about six times their own volume of air. Thus the cooling of drops in the cavity and in the dewar was prevented approximately.

The first location for measurement of the drop temperature is 17.7 cm from the needle, which is about 44 dia for 4-mm drops. In this falling range strong oscillations of large drops were observed. The augmentation of heat transfer estimated by using the formulas of Scanlan [24] gave reasonable comparisons as suggested by the proposed correction factor g in this falling range. Nevertheless, as suggested in the present paper, further investigation on the augmentation of heat transfer by large oscillations in drop shape is needed.

In the range $60 \leq x/d \leq 600$ the internal circulation of large drops is developing. The proposed correction factor gives 1.5–0.3 times the Ranz-Marshall transfer coefficient in the falling range investigated in our experiment. The procedure chosen to explain this behavior and the empirical fit of experimental data is of course not the only possible one as shown by Dunn. However, it provides a systematic approach for further understanding of the transient internal circulation upon drop heat transfer.

Nucleation Site Activation in Saturated Boiling¹

J. J. Lorenz,² B. B. Mikic,³ and W. M. Rohsenow.³ In nucleation experiments with individual cavities, the authors found that the photographically measured cavity radii were equal to those determined from the following well-known nucleation criteria:

$$\rho = \frac{2\sigma T_s V_{fg}}{h_{fg} \Delta T} \quad (1)$$

where ρ is the effective radius of nucleation. Experiments were performed with water only. The authors then suggested that a unique description of the cumulative nucleation site distribution for a given surface could be given by N/A versus equation (1).

In an earlier study of the effects of surface conditions on boiling characteristics, we performed nucleation experiments with both water and organics [4, 5, 6].⁴ In our nucleation experiments with water, we obtained results similar to those of the authors', i.e., that the photo-

¹By M. Shoukri and R. L. Judd, published in the Feb. 1975 issue of the JOURNAL OF HEAT TRANSFER, TRANS. ASME, Series C, Vol. 97, pp. 93–98.

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⁴Numbers in brackets designate Additional References at end of discussion.