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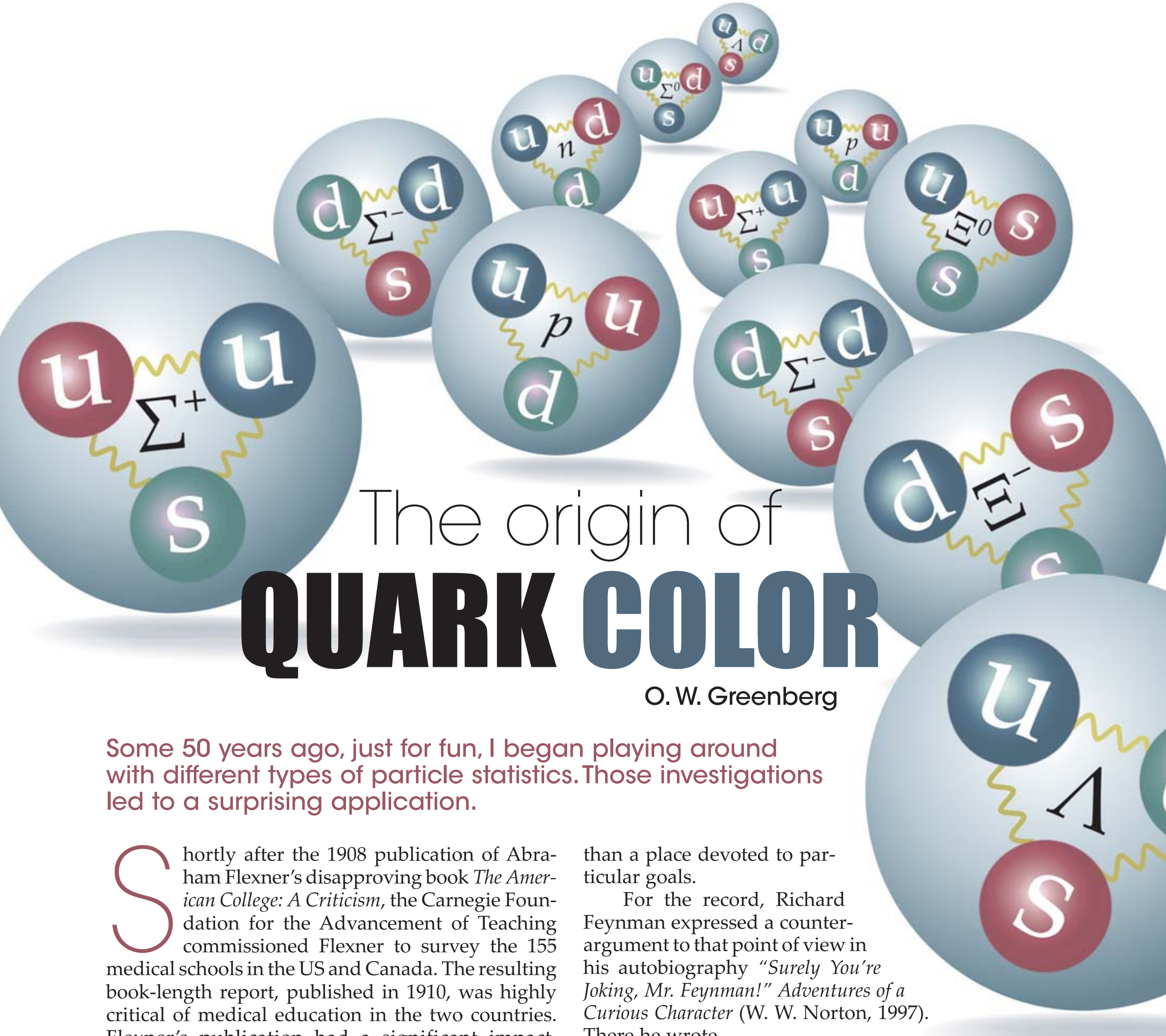

The origin of QUARK COLOR

O. W. Greenberg

Some 50 years ago, just for fun, I began playing around with different types of particle statistics. Those investigations led to a surprising application.

Shortly after the 1908 publication of Abraham Flexner's disapproving book *The American College: A Criticism*, the Carnegie Foundation for the Advancement of Teaching commissioned Flexner to survey the 155 medical schools in the US and Canada. The resulting book-length report, published in 1910, was highly critical of medical education in the two countries. Flexner's publication had a significant impact. North American medical schools completely revised their curricula, and by 1935 their number had decreased to 66.

On the strength of that report, in 1930, Louis Bamberger and his sister Caroline Bamberger Fuld, who had recently sold their large department store in Newark, New Jersey, approached Flexner and asked him to establish a medical school in Newark. Flexner was creative in his response to the siblings' request. Instead of following his previous work on medical schools, he chose a rather different course. He persuaded the Bambergers to establish the Institute for Advanced Study in Princeton, New Jersey, as a place where researchers could pursue their interests without the pressure to produce specific results.[1] Flexner became the institute's cofounder, along with the Bambergers, and its first director.

Later, Flexner wrote about the "usefulness of useless knowledge," the paradox that "the pursuit of these useless satisfactions proves unexpectedly the source from which undreamed-of utility is derived."[2] He believed that a place like the Institute for Advanced Study would be much more valuable than a place devoted to particular goals.

For the record, Richard Feynman expressed a counterargument to that point of view in his autobiography *"Surely You're Joking, Mr. Feynman!" Adventures of a Curious Character* (W. W. Norton, 1997). There he wrote,

> When I was at Princeton in the 1940s I could see what happened to those great minds at the Institute for Advanced Study, who had been specially selected for their tremendous brains and were now given this opportunity to sit in this lovely house by the woods there, with no classes to teach, with no obligations whatsoever. These poor bastards could now sit and think clearly all by themselves, OK? So they don't get any ideas for a while: They have every opportunity to do something, and they're not getting any ideas. I believe that in a situation like this a kind of guilt or depression worms inside of you, and you begin to worry about not getting any ideas. And nothing happens. Still no ideas come. Nothing happens because there's not enough *real* activity and challenge:

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> You're not in contact with the experimental guys. You don't have to think how to answer questions from the students. Nothing! (page 165)

Regardless of the state of play in the 1940s, Feynman's criticism is not valid for the current and recent faculty at the Institute for Advanced Study, who interact with physicists from all over the world.

Beyond bosons and fermions

Circulating around the physics community are several stories about scientists confronted with the "uselessness" of their work. Ernest Rutherford, when questioned in the early 1930s about the utility of his nuclear-physics experiments, replied that his work "has no use and it never will have." Michael Faraday is said to have shown some of his experiments on electricity and magnetism to a British prime minister. The prime minister said to Faraday, "That is very interesting, but what good is it?" Faraday replied, "I do not know Mr. Prime Minister, but one day you will tax it."

My own work at the Institute for Advanced Study that led to the introduction of color charge in particle physics is an example of useless knowledge that turned out to generate something of considerable use—even if it is yet to generate any tax revenue. Admittedly, the suggestion of color did not come from the pure contemplation that Flexner may have had in mind, but rather as a response to the observed masses of light baryons and the theoretical work of Feza Gürsey and Luigi Radicati.

As a graduate student I had wondered why only bosons and fermions exist in nature. That is, why does a quantum mechanical wavefunction describing many identical particles always remain unchanged when particles are permuted (in which case the particles are bosons and one speaks of Bose statistics) or simply become multiplied by ±1 (when the particles are fermions; each transposition contributes a factor of −1). I continued that interest as a faculty member at the University of Maryland, and in 1962 I went to the NATO summer school of theoretical physics, held at Robert College in Istanbul, Turkey.[3] Figure 1 shows the school participants, which included such distinguished physicists as Sidney Coleman, Sheldon Glashow, Louis Michel, Giulio Racah, and Eugene Wigner. While there, I gave a seminar on parastatistics—a generalization of Bose and Fermi statistics—a topic I had worked on with Gianfausto Dell'Antonio and George Sudarshan. Albert Messiah and I carried that work further over a period of two years and published two long papers on particle statistics other than Bose or Fermi.[4]

Messiah and I showed that such statistics are compatible with quantum mechanics. In those novel cases, exchanging the identical particles in a multiparticle wavefunction ψ can lead to wavefunctions that differ from ψ in ways more complicated than mere multiplication by ±1. Although quantum mechanics allows for more elaborate statistics than Bose or Fermi, we found no evidence for particles that take advantage of that possibility.

In our second paper, we studied field theories of particles that obey statistics other than Bose or Fermi. Parastatistics, which had been introduced by Herbert S. Green, is an example of a generalized statistics that can be incorporated into a field theory.[5]

Parastatistics comes in two families, para-Bose and para-Fermi, each labeled by a positive integer p, called the order. The usual Bose and Fermi statistics correspond to $p = 1$. A field theory with order-p parastatistics includes p particle annihilation fields ϕ^a and an equal number of creation fields $\phi^{a\dagger}$. (The index a runs from 1 to p and the dagger denotes the adjoint operation.) The parastatistics are defined by the equal-time commutation and anticommutation relations satisfied by those fields. For the para-Bose case,

$$[\phi^a(\mathbf{x},t),\phi^{a\dagger}(\mathbf{y},t)]_- = \delta(\mathbf{x}-\mathbf{y});$$
$$[\phi^a(\mathbf{x},t),\phi^{b\dagger}(\mathbf{y},t)]_+ = 0,\ (a \neq b).$$

Here, the "−" subscript denotes commutator and the "+" subscript denotes anticommutator. Para-Fermi fields satisfy similar relations with the roles of commutator and anticommutator reversed.

The above commutation rules mean that for para-Bose statistics of order p, up to p particles can be in an antisymmetric state. For para-Fermi statistics of order p, up to p particles can be in a symmetric state; in particular, as many as p particles can be in the same quantum state.

Messiah and I developed Green's ideas and found that the spin–statistics theorem could be generalized to state that particles that have integer spin

Figure 1. Participants at the 1962 NATO summer school of theoretical physics posed for this group portrait at Robert College, Istanbul, Turkey. Feza Gürsey is seated at the right end of the first row; I'm just to his right, and Louis Michel is two spots to my right. In the second row, Eugene Wigner is third from the left, and Giulio Racah is third from the right. Sheldon Glashow is second from the left in the top row, and Sidney Coleman is eighth from the left.

Box 1. Here comes the *SU*(*N*)

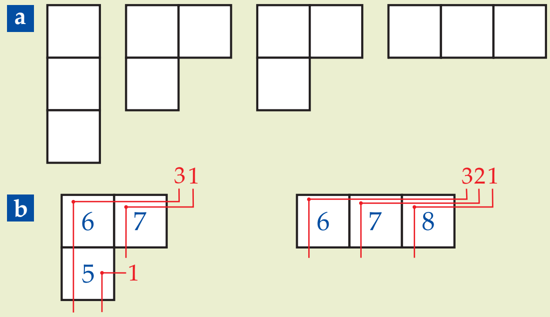
The group *SU*(*N*) is in many ways like the group of $N \times N$ rotation matrices, except that in *SU*(*N*), the matrices and the vectors on which they act have complex values. The *S* in *SU*(*N*) stands for "special," meaning that the matrices have unit determinant, just as for rotation matrices. The *U* denotes "unitary," meaning that the matrices preserve vector normalization. (Rotation matrices preserve the length of real-valued vectors; those matrices are called orthogonal.)

In light of the discussion in the main text, consider an *SU*(6) transformation acting on the three-indexed product $q_i q_j q_k$, a baryon comprising three quarks. There are $6 \times 6 \times 6 = 216$ elements of that product form, and the *SU*(6) transformations mix them up. However, group theory teaches us that if the 216 elements are organized correctly, the *SU*(6) transformations will connect only smaller collections called irreducible representations.

In general, the irreducible representations of *SU*(*N*)—and also of the *N*-object permutation group—are labeled by simple figures called Young diagrams. Panel a of the figure shows the diagrams for irreducible representations corresponding to three-quark combinations. The one with horizontally arranged boxes symbolizes the totally symmetrized combination of quarks, the one with vertically arranged boxes corresponds to the totally antisymmetric combination, and the two others denote copies of a representation with more complicated symmetry properties.

The dimension of any *SU*(*N*) representation is given by the hook rule;[18] panel b of the figure gives a couple of explicit calculations, including one for the irreducible symmetric representation of *SU*(6) proposed for baryons by Feza Gürsey and Luigi Radicati and described further in the text. According to the hook algorithm, the dimension is obtained from separate calculations of a numerator and a denominator. To get the numerator, start with *N* in the upper-left box of the diagram, add 1 to each box going to the right and subtract 1 for each box going down. Then fill in the diagram starting from the left and adding 1 to each box as you move right. The numerator is the product of all the numbers in the filled-in diagram. The denominator is the product of the hook values. The hook belonging to a specific box is a line that enters the box horizontally from the right and exits it vertically going down. The number of boxes the hook passes through is the hook's numerical value.

Gürsey and Radicati's assignment of three-quark baryons to the symmetric 56-dimensional irreducible representation was in some ways a success. But the symmetric combination was in violation of the spin–statistics theorem. That was the paradox confronting me in 1964.



$$\frac{6 \times 7 \times 5}{3 \times 1 \times 1} = 70 \qquad \frac{6 \times 7 \times 8}{3 \times 2 \times 1} = 56$$

must obey para-Bose statistics and particles that have half-odd-integer spin must obey para-Fermi statistics. We had no applications of parastatistics in mind. Our motivation was simply to stretch the formalism of quantum field theory—to see what possibilities were compatible with it. At the time, what we discovered was useless knowledge.

The quark statistics paradox

As a graduate student I was subject to disparate influences. On the one hand, very simple ideas were used to classify newly discovered particles. The only mathematics that seemed necessary was knowing how to add and subtract numbers like ⅔ and −⅓. On the other hand, I learned sophisticated mathematical techniques used in quantum field theory. Indeed, my PhD thesis on the asymptotic condition in quantum field theory, which was a formalization of the Lehmann-Symanzik-Zimmermann scattering theory, was purely theoretical—it had no numbers, except to label pages and equations. I used so-called operator-valued distributions that at the time were considered rigorous mathematics.

In 1964 Murray Gell-Mann introduced current quarks, the quark operators that appear in field theory Lagrangians and that are used to construct such things as the electric-charge operator.[6] In the same year, George Zweig introduced aces, particles that are clothed by a cloud of virtual particles and occur in bound states such as protons and neutrons.[7] I will use the word quark to refer to both those constructs. The models introduced in 1964 had three types, or flavors, of light quarks, called up (u), down (d), and strange (s), along with their corresponding antiquarks. The quarks were assumed to belong to the fundamental three-dimensional representation of $SU(3)_f$, where the subscript stands for flavor. Box 1 describes the *SU*(*N*) groups and their representation theory in some detail. For now it suffices to note that the $SU(3)_f$ group can be thought of as a collection of matrices whose action on the three flavored quarks is much like the action of a rotation matrix on the three components of a vector.

In nature, baryons such as the proton are made up of three quarks, and mesons such as the pion are made up of a quark and an antiquark. In the original models, there was no dynamical reason for just those two combinations to occur. They were chosen simply because they allowed the quantum numbers of the known baryons and mesons to be represented in terms of the quark quantum numbers.

In the original quark models, quarks were assumed to have spin ½ so that the protons and neutrons having three quarks would be spin-½ particles. Spin, however, did not play a dynamical role. In 1964 Gürsey and Radicati gave dynamics to spin by combining the two spin-½ degrees of freedom with the three flavor degrees of freedom to form a larger *SU*(6) symmetry.[8] Their model placed the three-quark baryons in a totally symmetric configuration, which, as described in box 1, corresponds to a 56D representation of *SU*(6). Such an assignment had the success that, when viewed in terms of flavor and spin, the members of the 56D representation reduce to 8 spin-½ particles (16 degrees of freedom) and 10 spin-3⁄2 particles (40 degrees of freedom), precisely as

Box 2. Colorless combinations of colored quarks

The particles made up from quarks are called hadrons, from the Greek word for thick or stout. They come in two principal varieties. Baryons, like the proton or neutron, are made of three quarks; mesons, like the pion, comprise a quark and an antiquark. Some recent experimental evidence suggests that there may exist exotic hadrons made up of four quarks (see the Quick Study by Steve Olsen, PHYSICS TODAY, September 2014, page 56).

Quarks possess a quantum number called color that comes in three varieties: red, green, and blue. The three colors are transformed into each other by the symmetry group $SU(3)_c$ (the subscript stands for "color"), which in many ways is like the familiar rotation group; see box 1 for additional details. Their colored constituents notwithstanding, all hadrons must be colorless, which means they are built from combinations of quarks that are invariant under $SU(3)_c$ transformations. Those colorless states are also called singlet states; in the language introduced in box 1, they are one-dimensional irreducible representations of the $SU(3)_c$ group.

According to the discussion in that box, a totally antisymmetric combination of three quarks $q_a q_b q_c$ remains antisymmetric under $SU(3)_c$; here the subscripts are indices giving the quark colors. Because those color indices assume one of only three values, there is only one possible totally antisymmetric combination—the unique combination of three quarks that is unchanged by $SU(3)_c$. Thus, for baryons to be colorless singlets, the quarks they are built from must be totally antisymmetric as far as color is concerned. The Pauli principle insists that the rest of the baryon wavefunction be totally symmetric under quark exchange.

The antiquarks (denoted with overbars) that live in mesons can, like the quarks, have one of three colors: antired, antigreen, or antiblue. Although quarks and antiquarks are both three-indexed objects, they are acted on differently by $SU(3)_c$. Antiquarks are analogous to the covariant vectors of special relativity, which are boosted differently than are the contravariant vectors analogous to quarks. And just as in special relativity, combinations like the "dot-product" interval $\Sigma_\mu X^\mu X_\mu$ are unchanged by boosts, in particle theory, the sum of quark–antiquark pairs $\Sigma_a \bar{q}^a q_a$ yields a colorless meson.

needed for the low-mass baryons. Further, Mirza Bég, Benjamin Lee, and Abraham Pais calculated the ratio of the proton and neutron magnetic moments in the $SU(6)$ model and found it to be $-\frac{3}{2}$, within 3% of the measured value.[9]

Those successes, however, were attended by a paradox. Because the 56D representation is totally symmetric, the wavefunction for three-quark baryons is unchanged when identical quarks are permuted. But according to the spin–statistics theorem, the spin-½ quarks must obey Fermi statistics; permutations of identical quarks should multiply the wavefunction by ±1. Something was amiss.

A resolution skeptically received

By 1964 I knew there were possibilities for identical particle statistics other than Bose and Fermi. I suggested that the way to get around the symmetry of the 56D representation was to introduce a new three-valued charge carried by quarks, now called color.[10] The idea is that if the color part of the wavefunction is antisymmetric under quark exchange, then so is the total wavefunction—a product of the symmetric space-spin-flavor wavefunction and the antisymmetric color wavefunction—as required by the exclusion principle. Moreover, if the three-quark baryon wavefunction is fully antisymmetric with respect to the color charge, then it is unchanged by $SU(3)$ matrices that mix color. Such states are known as color singlets (box 2 provides additional pedagogic detail). I called this model for baryons[11] the "symmetric quark model."

At the time, I introduced color using parastatistics of order three to construct Bose and Fermi states. Such states, as Daniel Zwanziger and I showed a little later, are in one-to-one correspondence with color singlets.[12] In that same work, we demonstrated that color provided a rationale for the phenomenon that all low-mass states are composed of three quarks (for baryons) or a quark and an antiquark (for mesons).

A new color charge was not the only proposed solution to the statistics paradox. Other physicists assumed a complicated ground state, invented various complicated models, or assumed that quarks are not real particles but only a mathematical device. None of those attempts were successful.

I was excited by the color model and predicted the spectrum of excited states of baryons based on the new color degree of freedom.[10] Over about 10 years, the patterns that I found from the color model were confirmed by the accumulating data on baryon spectroscopy, the only test for the existence of color at that time. A later theoretical paper with Marvin Resnikoff[11] and independent work by Nathan Isgur and Gabriel Karl and by Dan-Olof Riska and collaborators confirmed the symmetric quark model for baryons.[13] Fifty years later it still agrees well with the data.

In 1964 quarks with fractional electric charges that had never been seen—and that have not been seen to this day—were unacceptable to many physicists. To suggest that those quarks carry a new hidden color charge only increased the level of skepticism. The response of J. Robert Oppenheimer, who was my host at the Institute for Advanced Study, was typical. When I asked him if he had read my paper, he said, "It's beautiful." I was elated. My elation was, however, short-lived, because Oppenheimer's next statement was, "but I don't believe a word of it." At the time many physicists shared Oppenheimer's disbelief.

In 1965 Moo-Young Han and Yoichiro Nambu introduced a model in which an $SU(3)$ color group is explicit.[14] They avoided fractional electric charges by introducing nine quarks with integer charges, but they paid the price that color was not an exact symmetry. (Moreover, the notion of integer quarks ended up conflicting with later experimental evidence.) Han and Nambu also made two important suggestions. The first was that color could be gauged, which means that the theory describing color is unchanged by gauge transformations analogous to those of electrodynamics. (The historical development of gauge theories is discussed by C. N. Yang, PHYSICS TODAY, November 2014, page 45.) The second was that the color interaction could be medi-

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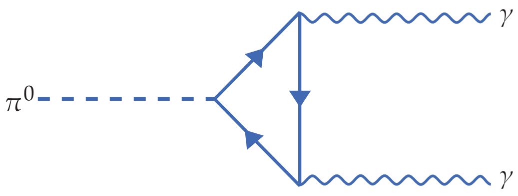

Figure 2. The decay of a neutral pion into two photons involves virtual quarks, the entities depicted with arrows in this Feynman diagram. Because quarks come in three colors, the measured decay amplitude is thrice what it would be in a theory with colorless quarks.

ated by an octet of gluons, much as how in quantum electrodynamics the interaction of charged particles is mediated by photons. The three-valued charge introduced by me in 1964 and the gauged theory introduced by Han and Nambu in 1965, taken together, contain the basis of quantum chromodynamics (QCD), today's theory of the strong interaction.

Acceptance of quarks and color

The decade following the introduction of quarks and color saw increasing evidence for the reality of quarks. Experiments at SLAC in which electrons were scattered off protons and neutrons revealed an internal structure to those "fundamental" particles, much as the Rutherford experiment revealed the internal structure of the atom. The successes of the standard model of particle physics, with its (at the time) three quarks built into it, also argued for the reality of quarks. As early as 1973, Harald Fritzsch, Gell-Mann, and Heinrich Leutwyler summarized the advantages of a "model based on colored quarks and color octet gluons."[15] But when so-called neutral currents were discovered in that same year, the standard model ran into a problem; with three quarks, it unambiguously predicted effects that were not observed.

Glashow, John Iliopoulos, and Luciano Maiani had actually floated an idea in 1970 that would suppress those later unobserved effects: a fourth quark, now called the charm quark.[16] And on 11 November 1974, independent teams at Brookhaven National Laboratory and SLAC announced the discovery of a new particle, the J/ψ, with just the right mass to be a meson built from a charm and anticharm quark. Ten days later, a second charm–anticharm meson was discovered, the ψ'. With the discovery of those two mesons in the "November revolution" of 1974 and subsequent studies quickly carried out, the case for the reality of quarks and their accompanying color was sealed.[17]

Even before the November revolution, two different phenomena had suggested the reality of color in a particularly transparent fashion. One is the decay of the neutral pion into two photons. Figure 2 shows the lowest-order Feynman diagram for the process; its central feature is a triangle representing virtual quarks. Each quark color contributes equally to the decay amplitude symbolized by the diagram, which thereby picks up a factor of three relative to the result for a theory with color-singlet quarks. The decay rate is thus multiplied by nine, in agreement with experiment. The second phenomenon is electron–positron collisions; of specific interest is the cross section (in essence, the scattering probability) for the collision to yield hadron products as compared with the cross section for the collision to yield a muon–antimuon pair. Again, the color factor of three is needed for theoretical calculations to agree with the experimental data.

To lowest order, my parastatistics model and QCD agree in their predictions for the neutral-pion decay and electron–positron annihilations. But at higher orders, the parastatistics model fails to agree with experiment: QCD is the correct theory of the strong interactions. As its name suggests, color enters in a crucial way in the theory, whose many successful predictions have further confirmed the existence of the color degree of freedom.

Early in my professional career, I spent much time pursuing "useless" knowledge—an understanding of quantum statistics that does not occur in nature. But those studies led me to propose the idea of quark color, a concept that has proved to be quite useful. I imagine that if Abraham Flexner had been able to hear the story of color's origins, he would have been pleased.

This article is an elaboration of a talk I gave at the 50 Years of Quarks and Color symposium, held at the University of Maryland on 11–12 April 2014.

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