Dynamical Correlations and the Nuclear Photoeffect\textsuperscript{(*)}

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The dipole sum rules of Levinger and Bethe are used to estimate the influence of strong dynamical correlations, generated by realistic 2-nucleon potentials, on the photoeffect in heavy nuclei. Our description of the nuclear ground state is an asymptotic one, based on a complete set of correlated functions \( \{F_{\alpha\mu}\} \), where \( F \) is a product of 2-body correlation factors \( f \) appropriate to a system with strong short-range repulsions and \( \{\Phi_{\alpha\mu}\} \) a complete set of Fermi-gas model functions satisfying periodic boundary conditions over a fundamental volume \( \Omega \) identified with the (assumed large) nuclear volume. The integral of the photonuclear cross section over all photon energies, \( \sigma_{\text{int}} = \int \sigma(W) dW \), is estimated in zeroth cluster order for Hamada-Johnston, Brueckner-Gammel-Thaler, Gammel-Christian-Thaler, and Ohmura-Morita-Yamada potentials with a ground-state wave function constructed from the \( F_{\alpha\mu} \) via a first-order perturbation theory. The bremsstrahlung-weighted cross section, \( \sigma_{b} = \int \sigma(W) W^{-1} dW \), is calculated with the "unperturbed" wave function \( F_{\Phi_{0}} \), where \( \Phi_{0} \) is the wave function of the degenerate Fermi gas; here first-order cluster corrections are included. The first energy moment of the cross section, \( \sigma_{1} = \int \sigma(W) W dW \), is also evaluated using \( F_{\Phi_{0}} \) but keeping only lowest cluster contributions. Variation of these three energy moments of \( \sigma(W) \) with an inverse range parameter \( r_{\text{inf}} \) and with the nuclear radius parameter \( r_{0} \) is studied. It is found that \( \sigma_{\text{int}} \) and \( \sigma_{1} \), which depend explicitly on the 2-nucleon potential and are sensitive to the short-range behavior of \( f \), are substantially increased, and \( \sigma_{b} \), which is sensitive to the long-range behavior of \( f \), may be appreciably diminished, compared to their respective values for a Fermi-gas model and well-behaved central potentials fitting the low energy scattering data. (In particular, for all potentials tested and for reasonable parameter choices, \( \sigma_{\text{int}} \) is enhanced by roughly a factor 2 over the TRK value.) It is anticipated that, in the future, the Levinger-Bethe sum rules may emerge as important tools for probing the validity of favored models of the nuclear ground state and of the 2-nucleon potential.

\section{1. Introduction and review}

In this paper we report rather extensive calculations of the effect of phenomenological two-nucleon potentials on energy moments

\[ \sigma_{n} = \int \sigma(W) W^{n} dW, \quad n = -1, 0, 1, \]  

of the total photonuclear cross section. Integration over all photon energies \( W \) is achieved in the dipole approximation by use of the sum rules of Levinger and Bethe\textsuperscript{(*)} (LB). The great advantage of working with these sum rules lies

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in the fact that only the ground-state wave function $\Psi_0$ of the nucleus need be known, all reference to excited states having been eliminated by closure. Here computations are performed for ground-state wave functions which include both statistical and dynamical correlations. Most simply we take

$$\Psi_0 = F \Phi,$$

(1·2)

where $F$ is a Bijl-Dingle-Jastrow correlation factor, i.e., a product of two-body correlation factors (symmetrical in the particle coordinates and hence purely dynamical), and $\Phi$ is a Slater determinant describing a completely degenerate Fermi gas (hence containing only statistical correlations) whose mean density coincides with that in the interiors of heavy nuclei. But in some cases a more elaborate $\Psi_0$ is employed, consisting of a suitable linear combination of correlated basis functions $F\Phi_m, \{\Phi_m\}$ being a complete set of Fermi-gas Slater determinants. We characterize our model as an asymptotic one, supposedly a good approximation to extended nuclear matter.

Let us first describe some general features of the sum rules and outline the relevant calculations made by other authors.

The zeroth moment $\sigma_0$ of the photonuclear cross section is also called the integrated cross section and denoted by $\sigma_{\text{int}}$. A sufficient condition for the model independence of the $\sigma_{\text{int}}$ sum rule is the vanishing of the second commutator of the proton-neutron potential $v(ij)$ with the components $z_i, z_j$ of the radius vectors $r_i, r_j$ of the proton $i$ and the neutron $j$ along the direction of polarization of the photon. We shall call any potential satisfying this condition "ordinary", although this does not coincide with common usage. One finds, as a simple generalization of the Thomas-Reiche-Kuhn (TRK) result\(^9\) for atoms,

$$\sigma_{\text{int, ord}} = \frac{2\pi^2 e^4 h}{Mc} \frac{1}{4} A \approx 15A \text{ Mev-}mb,$$

(1·3)

where $M$ is the nucleon mass, $A$ is the target mass number, and equal numbers of neutrons and protons have been assumed. (In fact, all our formulas will be written only for $N=Z$.) Realistically, however, $v(ij)$ does not satisfy the above condition and Eq. (1·3) is consequently modified by an "exchange" correction factor $1 + C^{6,9}$ Let $\vartheta(ij)$ be the irreducible component of $v(ij)$ which does not satisfy the commutation condition (meaning $\vartheta(ij)$ contains no part that does satisfy it); then the term $C$ is proportional to the matrix element, in the ground state, of the sum of two-body operators $\sum_i r_i \vartheta(ij), r_{ij} = |r_i - r_j|$. Thus, realistically, the $\sigma_{\text{int}}$ sum rule is indeed model dependent. $\sigma_{\text{int}}$ has been studied by a number of authors, for a number of assumptions on $\Psi_0$ and a number of potentials (all fitting at least the low-energy two-nucleon data) with variable Wigner-Majorana exchange mixtures.

Levinger and Bethe, in the days when purely attractive wells were popular, found
\( C = 0.8x \) \hspace{1cm} (1.4)

for such potentials, \( x \) being the fraction of Majorana exchange force \((x = 1/2 \text{ for a Serber mixture})\). A Fermi-gas model, with \( N/Z = 1 \), was assumed. This model is characterized by only one parameter, the average density, or alternatively, the Fermi wave number \( k_r \). The latter is connected to the radius \( r_0 \) of the average volume per particle by

\[ r_0 = (9\pi/8)^{1/3} k_r^{-1} \]

which in turn is identified with the nuclear radius parameter in the well-known radius formula \( R = r_0 A^{1/3} \), applied to large \( A \). The two values of nuclear radius parameter that were considered, \( r_0 = 1.37 \, \text{fm} \) and \( r_0 = 1.50 \, \text{fm} \), are both too large by present standards. In going from a square well to a Yukawa well at \( r_0 = 1.37 \, \text{fm} \), the coefficient of \( x \) decreases by 20%; the same change at \( r_0 = 1.50 \, \text{fm} \) results only in a 10% decrease. In going from \( r_0 = 1.37 \, \text{fm} \) to \( r_0 = 1.50 \, \text{fm} \), the coefficient decreases by about 10% for the square well and increases negligibly for the Yukawa well. (The quoted value 0.8 in Eq. (1.4) is for the square well at \( r_0 = 1.50 \, \text{fm} \).)

Levinger\(^6\) recalculated this coefficient, again on the basis of an independent-particle model, but employing harmonic-oscillator functions for the single-particle orbitals rather than plane waves, together with a Gaussian two-body potential. The coefficient was found to remain within 20% of 0.8 for \( A = 4, 16, 40 \). Thus surface effects are hardly overwhelming. With the idea of including dynamical correlations in the assumed ground state, Okamoto\(^7\) perturbed the degenerate Fermi-gas wave function to first order, allowing a central two-nucleon potential of Gaussian shape to mix in all Fermi-gas states with one excited neutron and one excited proton. In the computation of \( C \), terms quadratic in the perturbing potential were neglected. The resulting correction to the LB value of \( \sigma_{int} \) (called the quasideuteron correction) was found to be relatively small, producing an increase by only 10%. All the calculations just described were based on potentials without strong short-range repulsions (e.g., hard cores), or tensor or spin-orbit components, i.e., on non-singular effective central potentials. (For our purposes, an effective central potential is by definition one which fits singlet and triplet effective ranges and zero-energy scattering lengths.)

An estimate of the effect of the inclusion of hard cores of radii appropriate to the high-energy two-nucleon data was made by one of us, assuming the usual Fermi-gas-model wave function supplemented by a Bijl-Dingle-Jastrow correlation factor;\(^5\) a more detailed study employing the same method was carried out by Biem.\(^9\) In the absence of non-central forces, the results for \( \sigma_{int} \) are some tens of percent larger than for potentials without cores which fit the same low-energy two-nucleon data. A complementary step toward a more realistic treatment was made by Okamoto and Hasegawa\(^10\) (OH), who applied the perturbation procedure of reference 7) to estimate \( C \) for potentials with tensor components but without repulsive cores. Again the result for \( \sigma_{int} \) is some tens of percent larger than for well-behaved, purely central potentials fitting the same low-energy data. In the present work we join the last two methods (of Clark
and of OH) to evaluate the integrated photonuclear cross section for some of the best phenomenological potentials.

The inverse first moment $\sigma_1$ of the photonuclear cross section is called the bremsstrahlung-weighted cross section (for an obvious reason) and commonly denoted by $\sigma_b$. The dipole sum rule for $\sigma_b$ contains no reference to the two-nucleon potential save through its influence on the ground-state wave function. Further, if we suppose that $\Psi_0$ is devoid of all correlations, even statistical correlations implied by the Pauli principle, then $\sigma_b$, as shown by LB, is simply

$$\sigma_b^{LB} = \frac{e^2}{\hbar c} \frac{4\pi^2}{3} \frac{1}{4} A \langle r^2 \rangle_0 \cong 0.024 A \langle r^2 \rangle_0,$$

where $\langle r^2 \rangle_0$ is the mean-square displacement of a nucleon in the ground state (which is just $(3/5)R^2$ for uniform density over a nucleus of radius $R$). The inclusion of statistical correlations is vital, however. Levinger and Kent\(^{11}\) (LK) calculated a Pauli correction factor $1/C_1$ to convert $\sigma_b$ into the right answer for 1) the Fermi-gas model and 2) a shell model with square-well single-particle potential of radius $R = r_0 A^{1/3}$, $r_0 = 1.20$ and 1.50 fm. $C_1$ was found to be large compared to unity, 1.64–2.57 for 1) and 2.8–4.0 for 2), depending on the mass number and (in case 2) the radius parameter assumed. One way of incorporating dynamical correlations to improve the LK calculations was investigated by Okamoto: The perturbation method used in his $\sigma_{\text{int}}$ calculation led to a decrease from the corresponding LK $\sigma_b$ value of only a few percent. Another way is explored in this paper: We carry out a generalized LK calculation with $\Phi_0$ replaced by $F\Phi_0$.

Finally, the dipole sum rule for the first moment $\sigma_1$ of the photonuclear cross section—like that for $\sigma_{\text{int}}$—is devoid of explicit reference to the two-nucleon potential if this potential is “ordinary.” But as in the case of $\sigma_b$, the term $\sigma_1^{\text{ord}}$ which persists even for ordinary forces is model dependent. Discarding both dynamical and statistical correlations, LB give

$$\sigma_{1,\text{ord}}^{LB} = \frac{2\pi^2e^2\hbar}{Mc} \frac{1}{3} A \langle t \rangle_{90} = 20 A \langle t \rangle_{90} \text{ Mev-mb},$$

where $\langle t \rangle_{90}$ is the expectation value of the kinetic energy of a nucleon in the nuclear ground state (just $3\hbar^2k_f^2/10M$ for the momentum distribution appropriate to the Fermi-gas model). Again statistical correlations are essential. LK also computed a Pauli correlation factor, $C_1$, to convert the LB value for $\sigma_1$ to the correct answer for the Fermi-gas model and for their shell model. They did not consider the Fermi-gas results quantitatively reliable, however, since the corresponding $C_1$ was found to turn negative for large $A$. Using the same dynamically correlated wave function $F\Phi_0$ as for $\sigma_b$, we perform in this paper a generalized LK calculation that does not meet with such an inconsistency. For realistic potentials, there arise contributions $\sigma_{1,\text{ep}}$, $\sigma_{1,\text{exch}}$ involving the same com-
ponent of the potential as does the $\sigma_{\text{int}}$ sum rule. We also evaluate these terms, for one of the potentials considered in our $\sigma_{\text{int}}$ work and—quite naturally—find them to be of great importance.

It has been conventional to discuss the theoretical predictions emerging from sum-rule calculations, as well as the experimental data, in terms of certain mean energies. The two which may be extracted from our work are the harmonic mean energy

$$W_\text{h} \equiv W_\text{h} = \frac{\sigma_{\text{int}}}{\sigma_0},$$

and the mean energy

$$W_1 \equiv W_1 = \frac{\sigma_1}{\sigma_{\text{int}}}. \tag{1.8}$$

In regard to the comparison of dipole-sum-rule calculations with experiment, it is regrettable that $\sigma(W)$ data above 25 Mev are so meager. Usually the upper limit of integration in the experimentally constructed moments is taken at 25–50 Mev; in any case, data above the pion-production threshold are of course never included. On the other hand, the upper limit in the energy moments given by the sum rules is infinity. Thus, to the extent that contributions to present experimental cross sections from higher multipoles than $E_1$ are small (at this stage, it is, on the whole, reasonable to suppose they are), we can well expect accurate sum-rule results for $\sigma_1$, $\sigma_{\text{int}}$ and $\sigma_0$ to exceed the experimental estimates. (For pure dipole transitions, this discrepancy would obviously be more exaggerated for the higher moments; however, it must be kept in mind that in the higher moments more weight is placed on whatever higher multipole contributions are incorporated.)

In all our discussions we assume the validity of Siegert’s theorem.5)

The plan of this article is as follows: § 2 displays the dipole sum rules for $\sigma_{\text{int}}$, $\sigma_0$, and $\sigma_1$, as derived by LB; § 3, the phenomenological two-nucleon potentials to be tested in our calculations. Section 4 contains a description of the model assumed, plus comments on the approximation method (cluster expansion) which must be used to explore its consequences. In §§ 5, 6 and 7, we trace our calculations of $\sigma_{\text{int}}$, $\sigma_0$, and $\bar{W}$ respectively, and compare the results with those of other workers (sketched above) and, in tentative fashion, with experiment. Our main conclusions are stated in § 8. We find that dynamical correlations play an essential role in the nuclear photoeffect.

§ 2. The sum rules

The contributions to the desired moments of the photonuclear cross section due to a dipole transition from the nuclear ground state 0 to the excited state $m$ may be conveniently expressed in terms of the oscillator strength

$$f_{0m} = \frac{2M(E_m - E_0)}{\hbar^2} \frac{N}{A} \left[ \left\langle \sum_i z_i \right\rangle_{0m} - \frac{Z}{A} \left\langle \sum_j z_j \right\rangle_{0m} \right]^2. \tag{2.1}$$
where
\[ \langle z_b \rangle_{0m} = \langle \Psi_0, z_b \Psi_m \rangle / \langle \Psi_0, \Psi_0 \rangle^{1/2} \langle \Psi_m, \Psi_m \rangle^{1/2}, \] (2.2)
\( \Psi_0, \Psi_m \) being the wave functions and \( E_0, E_m \) the corresponding energies of the states involved. The \( z \) axis is taken along the direction of polarization of the photon. The indices \( i, i' \) will always refer to protons; \( j, j' \), always to neutrons. We let \( k, l \) be general indices running from 1 to \( A \). The inner product notation in Eq. (2.2) implies integrations over all space coordinates and sums over all spin coordinates of the \( A \) nucleons. Setting \( 2\pi e^2 \hbar/Mc = \beta \), we have, in the dipole approximation,
\[ \sigma_{int} = \beta \sum_m f_{0m}, \]
\[ \sigma_b = \beta \sum_m f_{0m} / (E_m - E_0), \]
\[ \sigma_1 = \beta \sum_m f_{0m} (E_m - E_0). \] (2.3)
The sums may be carried out by closure as in LB to yield for \( N = Z \) the rules
\[ \sigma_{int} = \beta \frac{1}{4} A (1 + C), \]
\[ C = - \frac{2}{3A} \frac{2M}{\hbar^2} \sum_{ij} \langle r_{ij} \delta (ij) \rangle_{00}, \] (2.4)
\[ \sigma_b = \frac{M}{2\hbar^2} \left[ \sum_i \langle x_i^2 \rangle_{00} + \sum_{ij} \langle x_i x_j \rangle_{00} - 2 \sum_{ij} \langle x_i x_j \rangle_{00} \right], \]
\[ \sigma_1 = \sigma_{1, ord} + \sigma_{1, exch} \]
\[ \sigma_{1, ord} = \beta \frac{1}{2M} \left[ \sum_i \langle p_{xi} \rangle_{00} + \sum_{x} \langle p_{x_i} p_{x_i} \rangle_{00} \right] \]
\[ + \sum_{ij} \langle p_{ij} \rangle_{00} + \sum_{ij} \langle p_{ij} p_{ij} \rangle_{00} - 2 \sum_{ij} \langle p_{xi} p_{xj} \rangle_{00} \], \] (2.5)
\[ \sigma_{1, exch} = - \beta \frac{2}{i \hbar} \sum_{ij} \langle (p_{xi} - p_{xj}) (z_i - z_j) \delta (ij) \rangle_{00}, \]
\[ \sigma_{1, exch} = \beta \frac{1}{3} \frac{2M}{\hbar^2} \sum_{ij} \langle r_{ij} \delta^3 (ij) \rangle_{00}. \] (2.6)
Here we have adhered to the notation of Eq. (2.2), with \( z_b \) replaced by various operators in the nuclear Hilbert space.

One might notice that the second (fourth) terms in the expressions for \( \sigma_b \) and \( \sigma_{1, ord} \) arise from both statistical and dynamical correlations between protons (neutrons) and that the fifth terms are present only by virtue of purely dynamical correlations between protons and neutrons.

Modifications of the first sum rule which are introduced by relaxing the
assumption \( N = Z \) are thought to be of negligible consequence for actual nuclei;\(^\text{13}\) modifications of the second and third may be significant. If we suppose, besides \( N = Z \), that total isospin is rigorously a good quantum number of the nuclear ground state, as is the case for our model \( \mathcal{H}_n \), numerous additional simplifications are possible. For example, the first and third terms in Eq. (2.5) are equal, and so are the second and fourth, while in the resulting 
\[
\sum_{i,j} \langle z_i z_j \rangle_0 - \sum_{i,j} \langle z_i z_j \rangle_0
\]
only exchange contributions survive. The \( \sigma_{1,3} \) part of Eqs. (2.6) is subject to similar reductions. These statements follow from the fact that \( p-p, n-n, \) and \( p-n \) correlations in the same two-particle spin-isospin state are identical.

\section{3. Two-nucleon potentials}

Five potentials, all containing hard cores, are considered in this paper. Proceeding from more to less realistic, in terms of the quantity of free two-nucleon data fitted, they are the Hamada-Johnston\(^\text{12}\) (HJ), Brueckner-Gammel-Thaler\(^\text{13,14}\) (BGT), and Gammel-Christian-Thaler\(^\text{13,14}\) (GCT) potentials and two Ohmura-Morita-Yamada\(^\text{15}\) (OMY) potentials. The first three have tensor components. In addition, the HJ and BGT potentials have linear spin-orbit components and the HJ potential has a quadratic spin-orbit component as well. All these spin-orbit terms are discarded in our calculations.

We express the interaction potential between particles \( k \) and \( l \) in the general form

\[
\nu(kl) = \sum_{\alpha} \nu^{(\alpha)}(kl) O_{\alpha}(kl),
\]

where the \( \alpha \) values 1, 2, 3, 4 identify, respectively, central, tensor, linear spin-orbit, and quadratic spin-orbit components, with associated operators \( O_{\alpha}(kl) \) given by

\[
\begin{align*}
1, & \\
S_{kl} &= 3(\sigma_k \cdot r_{kl}) (\sigma_l \cdot r_{kl}) / r_{kl}^2 - \sigma_k \cdot \sigma_l, \\
L(kl) \cdot S(kl), & \\
Q(kl) &= \delta_{LJ} + (\sigma_k \cdot \sigma_l) L^2(kl) - L(kl) \cdot S(kl).
\end{align*}
\]

The definitions necessary to clarify (3.2) are: \( r_{kl} = r_k - r_l \) (separation vector for particles \( k, l \)); \( \sigma_k, \sigma_l = \) Pauli spin operators for particles \( k, l \); \( L(kl) = r_{kl} \times p_{kl} \) (relative orbital angular momentum operator for the pair); \( p_{kl} = (p_k - p_l) \) (relative linear momentum operator for the pair); \( S(kl) = (\sigma_k + \sigma_l)/2 \) (total spin operator); \( J(kl) = L(kl) + S(kl) \) (total angular momentum operator); \( L, J = \) quantum numbers corresponding to \( L(kl), J(kl) \), respectively.

Direct display of the potential parameters quoted in the literature is facilitated by introducing isospin-space operators \( \tau_k, \tau_l, T(kl) \) with properties identical to those of the spin-space operators \( \sigma_k, \sigma_l, S(kl) \) and writing...
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\[ \psi^{(a)}(kl) = \sum_{SP} \psi_{ST}^{(a)}(r_{ul}) A_{ST}(kl), \]  
\[ \text{in which } A_{ST} \text{ is the projection operator onto two-particle states with total spin quantum number } S \text{ and total isospin quantum number } T. \]

Explicitly,

\[ A_{00}(kl) = \frac{1 - \sigma_k \cdot \sigma_l}{4}, \]
\[ A_{01}(kl) = \frac{3 + \sigma_k \cdot \sigma_l}{4}, \]
\[ A_{10}(kl) = \frac{1 - \sigma_k \cdot \tau_l}{4}, \]
\[ A_{11}(kl) = \frac{3 + \sigma_k \cdot \tau_l}{4}. \]

Note \( \psi_{00}^{(a)} = \psi_{01}^{(a)} = \psi_{10}^{(a)} = \psi_{11}^{(a)} = 0. \)

On the other hand, the representation (3·3) of \( \psi^{(a)} \) is not convenient for our work. We must isolate that (unique) component \( \psi \) of the potential which does not satisfy the commutation condition of §1 and contains no part that does. For this a representation in terms of Wigner, Bartlett, Heisenberg, and Majorana exchange operators is most natural,

\[ \psi^{(a)}(kl) = \psi_{W}^{(a)}(r_{ul}) P^W(kl) + \psi_{B}^{(a)}(r_{ul}) P^B(kl) \]
\[ + \psi_{H}^{(a)}(r_{ul}) P^H(kl) + \psi_{M}^{(a)}(r_{ul}) P^M(kl), \]
\[ P^W(kl) = 1, \]
\[ P^B(kl) = \frac{1 + \sigma_k \cdot \sigma_l}{2} \] (spin exchange operator),
\[ P^H(kl) = - \frac{1 + \tau_k \cdot \tau_l}{2} \] (isospin exchange operator),
\[ P^M(kl) = - \frac{1 + \sigma_k \cdot \tau_l}{2} + \frac{1 + \tau_k \cdot \sigma_l}{2} \] (space exchange operator).

Comparison of Eqs. (3·3) and (3·4) with Eqs. (3·5) and (3·6) yields

\[ \psi_{W}^{(a)} = \frac{1}{4} (\psi_{00}^{(a)} + \psi_{01}^{(a)} + \psi_{10}^{(a)} + \psi_{11}^{(a)}), \]
\[ \psi_{B}^{(a)} = \frac{1}{4} (-\psi_{00}^{(a)} + \psi_{01}^{(a)} + \psi_{10}^{(a)} + \psi_{11}^{(a)}), \]
\[ \psi_{H}^{(a)} = \frac{1}{4} (\psi_{00}^{(a)} - \psi_{01}^{(a)} + \psi_{10}^{(a)} - \psi_{11}^{(a)}), \]
\[ \psi_{M}^{(a)} = \frac{1}{4} (-\psi_{00}^{(a)} - \psi_{01}^{(a)} - \psi_{10}^{(a)} + \psi_{11}^{(a)}). \]
The following properties of the exchange operators are useful in later reductions (§ 5):

\[
(P^w)^2 = (P^v)^2 = (P^m)^2 = 1,
\]
\[
P^w(kl) = P^m(kl) P^v(kl),
\]
\[
S_{bd} P^B(kl) = P^B(kl) S_{bd} = S_{bd}.
\]

(3·8)

The second of these implies that, in a nuclear state with equal numbers of spin-aligned and spin-anti-aligned nucleon pairs,

\[
\left< P^v(kl) \right>_{\text{spin average}} = \frac{1}{2} \left< P^m(kl) \right>_{\text{spin average}},
\]

since in such a state

\[
\left< P^B(kl) \right>_{\text{spin average}} = \frac{1}{2}.
\]

(3·9)

(3·10)

(We are of course most vitally interested in the case that \( k \) is specifically a proton and \( l \) specifically a neutron, and the isospin formalism dropped. Then all that is required for Eq. (3·9) is that there be equal numbers of spin-aligned and spin-anti-aligned proton-neutron pairs in the nuclear state). The third line of Eq. (3·8) means that actually the tensor component of \( v \) involves only two kinds of exchange operator, \( P^w \) and \( P^m \).

It is clear that for a potential containing only central and tensor components, just the \( P^v \) and \( P^m \) terms of \( v(ij) \) written according to Eq. (3·5) survive in \( \bar{v}(ij) \); indeed, this conclusion still holds in the presence of a linear spin-orbit component.

The radial dependences of the components of the HJ potential are specified, outside a state-independent central hard core of radius \( c = 0.4853 \text{ fm} \), by

\[
\begin{align*}
\nu^g_{\tau_1}(x/\mu) &= 0.08 (\mu/3) (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2) Y(x) \left[ 1 + a_{2\tau} Y(x) + b_{2\tau} Y^2(x) \right], \\
\nu^g_{\tau_2}(x/\mu) &= 0.08 (\mu/3) (\tau_1 \cdot \tau_2) (\sigma_1 \cdot \sigma_2) Z(x) \left[ 1 + a_{2\tau} Y(x) + b_{2\tau} Y^2(x) \right], \\
\nu^g_{\tau_3}(x/\mu) &= \mu G^g_{\tau_3} Y^3(x) \left[ 1 + b_{2\tau} Y(x) \right], \\
\nu^g_{\tau_4}(x/\mu) &= \mu G^g_{\tau_4} (Z(x)/x^3) \left[ 1 + a_{2\tau} Y(x) + b_{2\tau} Y^2(x) \right],
\end{align*}
\]

(3·11)

with

\[
\begin{align*}
Y(x) &= \exp(-x)/x, \\
Z(x) &= \left[ 1 + (3/x) + (3/x^3) \right] Y(x),
\end{align*}
\]

(3·12)

and \( \mu = 139.4 \text{ Mev} \) (pion mass), \( x = \kappa r_n, \kappa = 0.7067 \text{fm}^{-1} \) (inverse pion Compton wavelength). For the remaining parameters, refer to Table I. This potential model gives good fits of all the two-nucleon data below 315 Mev (laboratory bombarding energy). It is semi-theoretical in the sense that its long-range behavior is that of the one-pion exchange potential.
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Table I. Parameters of the Hamada-Johnston (HJ) potential.

<table>
<thead>
<tr>
<th>α</th>
<th>ST</th>
<th>$a_{ST}^\alpha$</th>
<th>$b_{ST}^\alpha$</th>
<th>$G_{ST}^\alpha$</th>
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<td>01</td>
<td>8.7</td>
<td>10.6</td>
<td>----</td>
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<td>00</td>
<td>−8.0</td>
<td>120</td>
<td>----</td>
</tr>
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<td>10</td>
<td>6.0</td>
<td>−1.0</td>
<td>----</td>
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<td>3.48</td>
<td>----</td>
</tr>
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<td>10</td>
<td>−0.5</td>
<td>0.2</td>
<td>----</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>−1.29</td>
<td>0.55</td>
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<tr>
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<td>--</td>
<td>0.1</td>
<td>0.0743</td>
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<tr>
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<td>11</td>
<td>--</td>
<td>−7.12</td>
<td>0.1961</td>
</tr>
<tr>
<td>4</td>
<td>01</td>
<td>0.2</td>
<td>−0.2</td>
<td>−0.000891</td>
</tr>
<tr>
<td>4</td>
<td>00</td>
<td>2.0</td>
<td>6.0</td>
<td>−0.00257</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>1.8</td>
<td>−0.4</td>
<td>0.00287</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>−7.26</td>
<td>6.92</td>
<td>−0.000891</td>
</tr>
</tbody>
</table>

The BGT and GCT potentials have components of Yukawa radial shape, cut off by state-independent central hard cores of radii $c=0.4$ fm and $c=0.5$ fm, respectively:

$$\psi_{\alpha}(r_{st}) = A_{ST} \exp\left(-\kappa_{ST}^\alpha r_{st}\right) / \kappa_{ST}^\alpha r_{st}, \ r_{st} \geq c. \quad (3.13)$$

The parameters for these two potentials are collected in Table II. The two-nucleon data fitted extend from 0 to 310 Mev for the former potential, from 0 to 150 Mev for the latter. The fits are generally not as satisfactory as those for the HJ potential.

Table II. Parameters of the Brueckner-Gammel-Thaler (BGT) and Gammel-Christian-Thaler (GCT) potentials.

<table>
<thead>
<tr>
<th>α</th>
<th>ST</th>
<th>BGT($c=0.4$ fm)</th>
<th>GCT($c=0.5$ fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$A_{ST}^\alpha$ (Mev)</td>
<td>$\kappa_{ST}^\alpha$ (fm$^{-1}$)</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
<td>−434</td>
<td>1.45</td>
</tr>
<tr>
<td>1</td>
<td>00</td>
<td>130</td>
<td>1.0</td>
</tr>
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<td>10</td>
<td>−877.4</td>
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</tr>
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</tr>
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<td>10</td>
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<td>1.0454</td>
</tr>
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<td>11</td>
<td>22</td>
<td>0.8</td>
</tr>
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</tr>
<tr>
<td>3</td>
<td>11</td>
<td>−7315</td>
<td>3.70</td>
</tr>
</tbody>
</table>

To render more transparent the role of tensor forces in the nuclear photo-effect, we select for study two members of a class of central potentials constructed by Ohmura, Morita, and Yamada, the potentials of this class fitting, by definition, singlet and triplet zero-energy scattering lengths and effective ranges and having (state-independent) hard cores of various radii. Outside the hard
cores these potentials are of exponential form,
\[ v_{st}^{(0)}(r_{st}) = A_{st}^{(0)} \exp(-k_{st}^{(0)}(r_{st} - c)), \quad r_{st} > c, \tag{3.14} \]
with Serber exchange mixture \( A_{00}^{(0)} = A_{11}^{(0)} = 0 \). For strength and range parameters appropriate to the choices \( c = 0.4 \text{ fm} \) (OMY 4 potential) and \( c = 0.6 \text{ fm} \) (OMY 6 potential), see Table III. Mention will also be made of an OMY 0 potential, the member of this class with \( c = 0 \).

### Table III. Parameters of the Ohmura-Morita-Yamada (OMY) potentials.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( ST )</th>
<th>( A_{st}^{(0)} ) (Mev)</th>
<th>( k_{st}^{(0)} ) (fm(^{-1} ))</th>
<th>( c ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>-235.414</td>
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<td>10</td>
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<td>2.521</td>
<td>0.4</td>
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<td>1</td>
<td>01</td>
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<td>2.627</td>
<td>0.6</td>
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<td>1</td>
<td>10</td>
<td>-947.023</td>
<td>3.676</td>
<td>0.6</td>
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</tbody>
</table>

### § 4. Model for the nuclear ground state

We base our description on a complete set \( \{ F \Phi_m \} \) of correlated functions, where \( F \) is a dynamical correlation factor symmetric under particle exchange, and \( \{ \Phi_m \} \) is a complete orthonormal set of independent-particle wave functions antisymmetric under exchange of identical particles.

More specifically, \( F \) is chosen to be of Bijl-Dingle-Jastrow form,
\[ F = \prod_{i<j} f(r_{ij}), \tag{4.1} \]
the product running over all distinct pairs of nucleons. The two-body correlation factor \( F \) must have such a nature that all matrix elements of the given nuclear Hamiltonian in the representation defined by the set \( \{ F \Phi_m \} \) are finite. In particular, if the two-nucleon potential contains a hard core of radius \( c \), then we must insist
\[ f(r_{ij}) = 0, \quad r_{ij} \leq c. \tag{4.2} \]

Further restrictions implied by the requirement of finite Hamiltonian matrix elements are met if \( f(r) \) is continuous and goes to unity sufficiently rapidly as \( r \to \infty \).

A product of two normalized Slater determinants, one for the neutrons and one for the protons, is adopted for each \( \Phi_m \). We seek to give an asymptotic description of the photoeffect in heavy nuclei, hence envisage an extended uniform nuclear medium, with the Coulomb interaction between protons turned off, and with a neutron to proton ratio of unity. Then the natural single-particle functions with which to build Slater determinants are plane waves (supplemented by appropriate spin factors), normalized and satisfying periodic boundary conditions in a cubical box of volume \( \Omega \) such that \( A/\Omega \) coincides with the central
density of large nuclei. Since we take \( N=Z \), only a single Fermi wave number \( k_F \) need be considered. Throughout this paper, \( k_F=1.42 \text{fm}^{-1} \), corresponding to the asymptotic (nuclear matter) radius parameter \( r_0=1.07 \text{fm} \), will be regarded as a “standard” value, but calculations are also reported for \( k_F=1.27 \text{fm}^{-1} \), corresponding to \( r_0=1.20 \text{fm} \), the radius parameter for large but finite nuclei.\(^{16}\)

The function \( F\Phi_0 \) of our set \( \{F\Phi_m\} \) most closely approximating the ground-state wave function of nuclear matter is of course that for which \( \Phi_0 \) describes a completely degenerate Fermi gas.

Using the isospin formalism, we could write each \( \Phi_m \) as a single Slater determinant; however, this formalism is neither particularly advantageous nor conventional for photo nuclear calculations of the present sort.

All calculations of this article, save one, are carried out only to lowest cluster order.\(^{17,2} \) By this we mean that terms of order \( A\omega/\Omega \) are neglected, where the quantity

\[
\omega = \int \left( f^2(r) - 1 \right) dr \tag{4\cdot3}
\]

is a sort of effective volume of strong dynamical correlation between a given particle and a second fixed particle. For the relevant values of \( c \) and \( r_0, |A\omega/\Omega| \) is indeed small if we impose an additional restriction on \( f(r) \) for \( r>c \); its departure from unity must not be very great and must go rapidly to zero. Judging from the work of Brueckner and Gamme\(^{13}\) and Gomes, Walecka and Weisskopf\(^{18}\), however, this is not a serious restriction, since for \( f \)'s of this type we may still construct an \( F\Phi_0 \) which (forgetting about tensor effects) is fairly realistic. We feel that the inclusion of higher cluster corrections (with one exception to be noted later) is unwarranted at this stage, because the errors introduced by their neglect (typically 10-15\% of the term evaluated) will surely be masked by other idealizations, especially when comparison is made with the experimental data. It should be remarked that the cluster expansions with which we deal here should converge more sensibly than the cluster expansion for the ground-state energy.\(^{17} \) here there is no question of delicate near cancellation of individually large terms. When necessary, the order in \( \omega \) of a given quantity will be indicated within upper parentheses.

The function \( F\Phi_0 \) is spherically symmetric and hence inadequate for a description of tensor effects. The exact ground-state wave function may be expanded as

\[
\Psi_0 = \sum_m d_m F\Phi_m, \tag{4\cdot4}
\]
a convenient choice of normalization being \( d_0=1 \). Obviously, the determination of the \( d_{m=\pm} \) is equivalent to the solution of the Schrodinger equation. However, Clark and Westhaus\(^{3} \) have outlined a perturbative technique which may furnish a practical means for computing the coefficients \( d_{m=\pm} \) to a satisfactory degree of accuracy. In lowest cluster order, and to “first order” in the new perturbative
expansion, we extract the result

$$d_n^{(1)} = \frac{\langle \Phi_n, \sum_{k<l} \varpi_2(kl) \Phi_0 \rangle}{E_n^{(0)} - E_m^{(0)}}, \quad m \neq 0,$$

(4·5)

where $\varpi_2(kl)$ is a well-behaved two-body pseudopotential,

$$\varpi_2(kl) = \frac{\hbar^2}{M} (\varphi(k^*)r(k^*) + f^*(r(k^*))v(k^*)$$

(4·6)

and $E_n^{(0)}$ is the ground-state energy of the Fermi gas, $E_m^{(0)}$ its energy in the Fermi-gas state labeled $m$. When relevant, the order in $\varpi_2$ of a given quantity will be indicated within upper square brackets.

With all five of the hard-core potentials of § 3, we associate a correlation factor of the form

$$f(r_{sl}) = \begin{cases} 0, & r_{sl} \leq c, \\ 1 - \exp(-\gamma (r_{sl} - c)), & r_{sl} > c, \end{cases}$$

(4·7)

the same form as used by Iwamoto and Yamada in their cluster-method investigation of the ground-state properties of nuclear matter. These authors minimized the energy expectation value for the trial function $F\Phi_n$, neglecting the effect on the minimum of all but the lowest-cluster-order contributions. They assumed a spin-independent two-nucleon interaction, in fact just the OMY potential with $A_{10}$ set equal to $A_{01} = 397.3$ Mev, and found $\gamma = 5$ fm$^{-1}$ for the best value of the inverse range parameter in the form (4·7). We repeated the Iwamoto-Yamada calculation for the central part of the BGT potential and again found $\gamma = 5$ fm$^{-1}$ to give the lowest value for the energy. (It must be mentioned, however, that the results obtained for the energy per particle and the radius parameter should not be taken seriously because of the importance of the tensor force.) By means of a modified cluster expansion method, Temkin arrived at a similar value, $\gamma = 6$ fm$^{-1}$, for a central, spin-dependent potential fitting the low-energy data and having Serber exchange character and a hard-core radius $c = 0.5$ fm. So it seems that $\gamma = 5$ fm$^{-1}$ is a reasonable choice, and we shall take it as "standard" in this paper. An honest determination of appropriate $\gamma$ values would require elaborate calculations based on the perturbation-cluster method proposed in reference 2). At any rate, the $\gamma$'s employed in our sum-rule computations extend over a range including the standard choice; for $\sigma_{int}$ and $\sigma_h$ we take the values $\gamma = 3, 5, 7$ and $\infty$ fm$^{-1}$; for $\omega$, the values $\gamma = 5$ fm$^{-1}$ and 7 fm$^{-1}$. (The limiting case $\gamma = \infty$ (corresponding to a step-function $f$) deserves a bit more discussion. This choice is, strictly speaking, quite unrealistic, since it leads—in an Iwamoto-Yamada calculation, for example—to an infinite kinetic energy. But the results it gives for the potential energy in such a calculation are probably not too far from those to be obtained with "more realistic" $\gamma$'s. The same conjecture applies to any matrix element not involving $f$ derivatives;
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it is borne out in the expected measure by our numerical results for $\sigma_{\text{int}}$ (to zeroth order in the perturbing potential $w_2$) and for $\sigma_v$. On the other hand, the step correlation factor clearly renders $\sigma_1$ infinite.

Perhaps further studies should be made with a more flexible class of correlation factors $f$ which may exceed unity a short distance outside the core, thus allowing for the possibility of important positive correlations due to the attractive region of the two-particle potential.

§ 5. The integrated cross section

Substituting the expansion (4·4) for the ground-state wave function, with the approximation (4·5) for the $d_{nm}$, the exchange correction $C$ in the integrated cross section (2·4) becomes, to lowest cluster order,

$$C = -\frac{2}{3A} \frac{2M}{\hbar^2} \left\{ \left( \Phi_{\text{ext}} \sum \langle \xi | r_{ij}^2 f^2 (r_{ij}) \hat{\vartheta} (ij) \Phi_n \right) \right. $$

$$+ \sum_{m,n} \left[ \left( \Phi_{\text{ext}} \sum \langle \xi | r_{ij}^2 f^2 (r_{ij}) \hat{\vartheta} (ij) \Phi_n \right) \langle \Phi_m, \sum \langle \xi | w_2 (kl) \rangle \Phi_o \right] + \text{C.C.} \right\} (5·1)$$

$$+ \sum_{m,n=0} \left( \Phi_{\text{ext}} \sum \langle \xi | r_{ij}^2 f^2 (r_{ij}) \hat{\vartheta} (ij) \Phi_n \right) \langle \Phi_m, \sum \langle \xi | w_2 (kl) \rangle \Phi_o \right] \left( E_0^{(l)} - E_m^{(l)} \right),$$

$$\left. \langle l | w_{nm}^{(l')} \right) \left( E_0^{(l')} - E_m^{(l')} \right), \right\}$$

A calculation strictly to first order in the perturbing potential $w_2$ requires omission of the last term in curly brackets; a higher-order calculation is out of the question at this stage. The matrix elements that appear involve only sums of two-body operators sandwiched between independent-particle wave functions and hence may be evaluated by the usual methods. For the single-particle energies whose sums are the $E^{(l)}$'s, we shall invoke the effective-mass approximation, in order to include, in crude fashion, some dispersive effects of higher order in the perturbation expansion. It is clear, then, that the reduction of the above expression goes through just as in the paper of Okamoto and Hasegawa, with their $xV_iP^{\alpha} + x'S_{ij}P^{\alpha}$ replaced by our $f^2 (r_{ij}) \hat{\vartheta} (ij)$ and their perturbing potential $V_{ij}$ (appearing in two-body matrix elements $(V_{ij})_{\alpha \sigma \alpha' \sigma'}$) by our $w_{2} (ij)$. The potentials $w_{2} (ij), v (ij)$, like $\hat{\vartheta} (ij)$ in Eqs. (3·1) and (3·2) may be resolved into components thus:

$$w_{2} (ij) = \sum_{\alpha} w_{2}^{(\alpha)} (ij) O^{(\alpha)} (ij), \quad \vartheta (ij) = \sum_{\alpha} \theta^{(\alpha)} (ij) O^{(\alpha)} (ij). \quad (5·2)$$

We keep only $\alpha = 1$ (central) and $\alpha = 2$ (tensor) terms.

Following Okamoto and Hasegawa, certain unwieldy momentum integrals are treated in such a way as to obtain upper and lower limits on the central and tensor contributions to $C$ of first order in the perturbing potential (for the details of this approximation, the reader is asked to consult references 7) and 10). We take the liberty of bypassing the extensive intermediate manipulations. These lead ultimately to an exchange correction
\[ C = C_{0}^{(0)} + C_{1}^{(0)} + C_{2}^{(0)}, \]

consisting of a central contribution of zeroth order in \( \omega \),

\[
C_{0}^{(0)} = -\frac{2}{3A} \frac{2M}{\hbar^2} \left( \Phi_{\alpha} \sum_{r \leq l} r_{ij}^2 f^2 (r_{ij}) \bar{\Phi}_{(ij)} \Phi_{\delta} \right)
= -\frac{8M}{\hbar^2} \frac{k_{i}}{\pi} \int_{0}^{\infty} r^3 \, dr \, f^2 (r) \left( \frac{1}{2} \, \psi_{\alpha}^{(i)} (r) + \psi_{\alpha}^{(i)} (r) \right) j_{1}^2 (k_{i} r), \tag{5.4}
\]

where

\[ j_{1} (t) = \frac{(\sin t - t \cos t)}{t^2}, \tag{5.5} \]

a central contribution of first order in \( \omega \),

\[
C_{1}^{(0)} = -\frac{4}{3A} \frac{2M}{\hbar^2} \sum_{\mu \neq \delta} \left( \Phi_{\alpha} \sum_{i \leq l} r_{ij}^2 f^2 (r_{ij}) \bar{\Phi}_{(ij)} \Phi_{\mu} \right) \left( \Phi_{\alpha} \sum_{k \leq l} \omega \psi_{\alpha}^{(k)} (k \ell) \Phi_{\delta} \right)
\leq \frac{64}{45 \pi^2} \frac{M^2}{\hbar^2} k_{i}^4 (I + \bar{I}), \tag{5.6}
\]

with

\[ I = \int_{0}^{\infty} u d u P (u) V_{\omega} (2k \mu u) g_{\xi}^{(0)} (2k \mu u), \]

\[ \bar{I} = \int_{0}^{\infty} u d u P (u) \bar{V}_{\omega} (2k \mu u) \bar{g}_{\xi}^{(0)} (2k \mu u), \]

\[ V_{\omega} (2k \mu u) = \int_{0}^{u} r^4 \, dr \, f^2 (r) \psi_{\alpha}^{(i)} (r) j_{6} (2k \mu u r), \]

\[ \bar{V}_{\omega} (2k \mu u) = \int_{0}^{u} r^4 \, dr \, f^2 (r) \psi_{\alpha}^{(i)} (r) j_{6} (2k \mu u r), \]

\[ g_{\xi}^{(0)} (2k \mu u) = h_{\xi}^{(0)} (2k \mu u) + h'_{\xi}^{(0)} (2k \mu u), \]

\[ \bar{g}_{\xi}^{(0)} (2k \mu u) = \bar{h}_{\xi}^{(0)} (2k \mu u) + \bar{h}'_{\xi}^{(0)} (2k \mu u), \tag{5.7} \]

\[ h_{\xi}^{(0)} (2k \mu u) = \int_{0}^{u} r^2 \, dr \left\{ \frac{\hbar^2}{M} \left( \frac{df (r)}{dr} \right)^2 + f^2 (r) \left( \psi_{\alpha}^{(i)} (r) + \frac{1}{2} \psi_{\alpha}^{(i)} (r) \right) \right\} \]
\[ \times j_{6} (2k \mu u r), \]

\[ \bar{h}_{\xi}^{(0)} (2k \mu u) = \int_{0}^{u} r^2 \, dr \left\{ \frac{\hbar^2}{M} \left( \frac{df (r)}{dr} \right)^2 + f^2 (r) \left( \frac{1}{2} \psi_{\alpha}^{(i)} (r) + \psi_{\alpha}^{(i)} (r) \right) \right\} \]
\[ \times j_{6} (2k \mu u r), \]
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\[ h_{\psi}(2kru) = \int_0^\infty r^2 dr f^2(r) \left( \psi_\psi^2(r) + \frac{1}{2} \psi_\psi^2(r) \right) j_0(2kru), \]

\[ h_{\phi}(2kru) = \int_0^\infty r^2 dr f^2(r) \left( \frac{1}{2} \psi_\psi^2(r) + \psi_\phi^2(r) \right) j_0(2kru), \]

where

\[ j_0(t) = \sin t/t, \quad (5.8) \]

and a tensor contribution of first order in \( \omega \),

\[ C^{(1)}_{\psi\phi} = - \frac{4}{3A} \frac{2M}{\hbar^2} \sum_{\text{all}} \frac{(\phi_\psi \sum_{\text{all}} r^2 f^2(r) \psi_\psi^2 (ij) S_{ij} \phi_\psi}{E_0^{(0)} - E_0^{(m)}} \]

\[ \leq 6 \left( \frac{64}{45\pi^2} \right) \left( \frac{M}{\hbar^2} \right)^2 k^4 \int_0^\infty \frac{du}{\pi^2} P(u) V_{\psi\phi}(2kru) g_\xi^{(0)}(2kru), \quad \{ \frac{<}{\gg} \} \text{ for } \{ \frac{\xi}{\xi} = 1 \} \],

(5.9)

with

\[ V_{\psi\phi}(2kru) = \int_0^\infty r^2 dr f^2(r) \left( \psi_\psi^2(r) + \psi_\phi^2(r) \right) j_1(2kru), \]

\[ g_\xi^{(0)}(2kru) = \xi h_{\psi}(2kru) + h_{\phi}(2kru), \]

\[ h_{\psi}(2kru) = \int_0^\infty r^2 dr f^2(r) \left( \psi_\psi^2(r) + \psi_\phi^2(r) \right) j_1(2kru), \]

\[ h_{\phi}(2kru) = \int_0^\infty r^2 dr f^2(r) \left( \psi_\psi^2(r) + \psi_\phi^2(r) \right) j_1(2kru), \]

where

\[ j_1(t) = \left[ (3-t^2) \sin t - 3t \cos t \right]/t^3. \quad (5.11) \]

In the above, \( P(u) \) is an integral first evaluated by Euler,\(^{33}\) who provided the formulas

\[ 0 \leq u \leq 1 : \quad P(u) = [4 + (15/2)u - 5u^3 + (3/2)u^4] \ln (1 + u) + 29u^3 - 3u^4 \]

\[ + [4 - (15/2)u + 5u^3 - (3/2)u^4] \ln (1 - u) - 40u^3 \ln 2 \quad (5.12a) \]

\[ \approx 40(1 - \ln 2)u^2 - 10u^4 + (4/3)u^6 + (1/7)u^8, \]

\[ 1 < u < \infty : \quad P(u) = [4 - 20u^2 - 20u^3 + 4u^4] \ln(u + 1) + 4u^3 + 22u \]

\[ + [4 - 20u^2 - 20u^3 + 4u^4] \ln(u - 1) + (40u^3 - 8u^4) \ln u \quad (5.12b) \]

\[ \approx 10/3u + 1/3u^3 + 4/25u^5. \]
The power series approximations to $P(u)$, used in our calculations, are in error by less than 0.1% in the inner interval and 0.5% in the outer interval.

Note that all contributions to Eq. (5.1) linear in $S_{ij}$ or $S_{ii}$ integrate to zero.

In the energy denominators of Eqs. (5.6) and (5.9), an effective mass value $M^* = (2/3)M$, the same as that assumed by OH, has been inserted. This is a reasonable choice judging from nuclear matter calculations of the Brueckner type.

Besides introducing the pseudopotential $w_2$, with attendant inclusion of strong dynamical effects, we have generalized the OH procedure to potentials with arbitrary exchange mixture. (OH derived formulas for potentials involving only two of the exchange operators (3.6), $P^w$ and $P^m$.) This generalization has nontrivial effect only for the central contributions to $C$.

Formulas (5.3)–(5.12b) were applied to the numerical evaluation of the exchange correction for the five potentials of §3. Several values of the parameter $\gamma$ in the correlation factor ($\gamma = 3, 5$ (standard), 7, and $\infty$ fm$^{-1}$) and two values of the radius parameter ($r_0 = 1.07$ fm (standard) and $r_0 = 1.20$ fm) were considered for each potential. Complete results for $C^{(0)}$, $C^{(11)}$ and $C^{(11)}$ are assembled in Table IV. In Table V we present a comparison of our total $C$ results for standard $\gamma$ with the results of workers whose treatments we unite and with empirical estimates of $C$ for large nuclei which include, in crude fashion, high-energy contributions to the dipole cross section. The first number quoted for a given first-order contribution to $C$, or for a given total $C$ containing first-order contributions, is for $\xi = 0$ (thus a lower limit); the second, for $\xi = 1$ (thus an upper limit). The other authors whose $C$ values are incorporated into Table V specialized to potentials having only Wigner and Majorana exchange components, with constant $\xi = v_{w} / v_{w}^{CE}$. In cases for which this ratio was not specified, we have obtained numerical values by assuming a Serber mixture; in fact all earlier values given Table V refer to a Serber mixture.

We now discuss the most notable general features of the information contained in these tables:

1) Influence on $\sigma_{21}$ of a hard core, a tensor component, and odd-state interactions

Clearly, realistic two-nucleon potentials greatly enhance the integrated photonuclear cross section over corresponding LB Fermi-gas values (see the sixth and ninth entries in Table V) appropriate to well-behaved attractive wells with parameters determined from effective-range theory. Our zeroth-order contributions $C^{(0)}$ alone are about the same as typical LB results, even somewhat larger for the (effective central) OMY potentials. We know from the work of Clark and Biem and of OH, respectively, that an enhancement comes about in the presence of a) a hard core but no tensor force and b) a
Table IV. Calculated results for the exchange correction to the integrated cross section.
(A: Including extra-core odd state interactions.)
(B: Excluding extra-core odd state interactions.)

<table>
<thead>
<tr>
<th>Potential</th>
<th>$r_0$ (fm)</th>
<th>$\gamma$ (fm$^{-1}$)</th>
<th>$\frac{C^{(\bar{m})}<em>{\Omega}}{C^{(m)}</em>{\Omega}}$</th>
<th>$\frac{C^{(\bar{m})}<em>{\bar{\Omega}}}{C^{(m)}</em>{\bar{\Omega}}}$</th>
<th>$\frac{C^{(\bar{m})}<em>{\bar{\bar{\Omega}}}}{C^{(m)}</em>{\bar{\bar{\Omega}}}}$</th>
</tr>
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<td>HJ</td>
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<td>0.27 0.28</td>
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<td>0.60—1.20</td>
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<td></td>
<td>7</td>
<td>0.38 0.37</td>
<td>0.62—1.24</td>
<td>0.64—0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\infty$</td>
<td>0.47 0.43</td>
<td>0.54—1.08</td>
<td>0.59—1.18</td>
</tr>
<tr>
<td></td>
<td>1.20</td>
<td>3</td>
<td>0.26 0.26</td>
<td>0.61—1.22</td>
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Table V. Comparison of theoretical and empirical $C$ values.

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<th>$C$</th>
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tensor force but no hard core. Our results for the effective central potentials lend independent support to the conclusion of Clark and Biem. Beyond this, we can only make the weaker statement that the complete results of the present article are consistent with the obvious direct generalization of the conclusion of Clark and Biem and of OH to the presence of a (fixed) tensor force in the former case and a (fixed) central hard core in the latter. In connection with this generalization, note for example that the GCT potential has a strong, short-range extra-core triplet-even central component and a weak, but long-range even-
state tensor component, while the BGT potential has an extra-core triplet-even central component of moderate strength and range and an even-state tensor component of moderate strength and range.

It was not the intention of this paper to make a study of the relative importance of the hard core and the tensor force in the enhancements of $\sigma_{\text{int}}$ that we observe, nor is it possible to do so on the basis of Tables IV and V. Hard core and tensor effects cannot be clearly separated in our results (hence the weak statement above) because, in the first place, calculations have not been carried out for $c=0$ potentials "corresponding to" the HJ, BGT and GCT singular choices, and, in the second place, uncertainties in the tensor contributions to $C$ are large. Nevertheless, we feel that there is already adequate evidence for the assertion that a serious quantitative analysis of the integrated cross section requires simultaneous treatment of hard core and tensor effects.\textsuperscript{21}

The odd-state interactions $(v_{00}, v_{10})$ drop out of the tensor contributions $C_{5J}^{(1)}$ completely, since only the combinations $v_{00}^2 + v_{02}^2$ and $v_{10} + v_{12}$ enter. On the other hand, these interactions may produce appreciable modifications ("A values" compared to "B values" in the tables) of $C_{6J}^{(1)}$, depending in detail on the potential assumed. But for realistic potentials the very large tensor contributions $C_{6J}^{(1)}$ suppress considerably the net effect of $v_{00}$ and $v_{10}$ on $\sigma_{\text{int}}$.

It should perhaps be reiterated that our treatment neglects spin-orbit components present in HJ and BGT potentials.

2) Dependence of $\sigma_{\text{int}}$ on parameters appearing in the tables

The dependence of $C_{6J}^{(1)}$ on $r_0$ and on the core radius $c$ has been adequately investigated by Biem and will not be dwelt upon further here, except to say that our results reinforce his findings that—other things being equal—$C_{6J}^{(1)}$ decreases with $r_0$ and increases with $c$ over the physically relevant ranges of these parameters. From Table IV we see that for the OMY potentials the total $C$ shows the same behavior. In view of the uncertainties involved in the evaluation of the first-order tensor contributions, we are not justified in making this statement for the more realistic potentials, although the general trends of the results are in line with such an extension of Biem’s conclusions. (Perhaps we should remark in passing that Biem also looked into the question of the dependence of $C_{5J}^{(1)}$ on the shape of the well outside the hard core and found that a longer tail favors a larger $C_{6J}^{(1)}$. In Table V we cite only his results for exponential well shape and, of course, for the BGT and GCT potentials (extra-core odd-state interactions omitted).) We note finally that each of $C_{6J}^{(0)}$, $C_{6J}^{(0)}(\xi=0)$, and $C_{6J}^{(1)}(\xi=1)$, for given potential and $r_0$, displays a monotonic increase with $\gamma$.

3) Rapidity of convergence of the perturbation expansion for $\sigma_{\text{int}}$, as reflected in the size of the first-order contributions relative to the zeroth
For the potentials containing tensor components, $C_{ij}^{(3)}$ is always less than one-third of $C_{ij}^{(1)}$ (in most cases, considerably less than this), so that with such potentials the convergence of the purely central contributions is probably quite satisfactory (i.e. rapid enough that sufficient accuracy is practically attainable). For the effective central potentials, of course, this convergence will not be so swift, but should be essentially the same as in the perturbation treatment of Okamoto based on a Gaussian potential without hard core, since the ratio of $C_{ij}^{(3)}$ to $C_{ij}^{(1)}$ is close to one-third both in our work and his. The size of the purely tensor first-order contributions is at first disturbing: In each case the even-state tensor component is so influential that $C_{ij}^{(3)}$ is as large as, or larger than, $C_{ij}^{(1)}$. But since tensor effects do not enter in zeroth order, expeditious overall convergence is certainly not excluded—evaluation of second-order contributions is required before anything can be said. We can only point to the nuclear matter calculations of Dahlblom, Fogel, Qvist and Törn\(^29\) as indirect evidence that, even including tensor effects, convergence of the perturbation expansion for $\sigma_{\text{int}}$ is satisfactory and our results are indeed meaningful.

These observations encourage application of the method of correlated basis functions to the underlying nuclear matter problem; even for a simple correlation factor—our standard choice—the lowest-cluster-order perturbation expansion for the energy per particle may be expected to converge about as well for realistic potentials as does the conventional perturbation expansion for a well-behaved potential.

4) **Comparison with previous theoretical work**

Note that Clark’s results for the OMY 6 potential are smaller than our relevant $C_{ij}^{(3)}$ values for this potential because, in the earlier calculation, the singlet potential was used in triplet states (i.e. the difference between singlet and triplet interactions was ascribed to a tensor component). Clark’s GCT results are inordinately large because of the small radius parameter assumed, $C_{ij}^{(3)}$ becoming very sensitive to $c$ for $r_0 \lesssim 1.1$ fm. All of Biem’s calculations were based on a step correlation factor and the omission of extra-core odd-state interactions, so it is only with our $\gamma = \infty$ even-state $C_{ij}^{(3)}$ values that his results may be compared. His “exponential” results are quite in accord with our $C_{ij}^{(3)}$s for the OMY 4 and OMY 6 potentials. There is satisfactory agreement also for the BGT potential, definite (though not major) disagreement for the GCT potential. No direct comparison with the results of OH is possible.

5) **Comparison with experiment**

In the absence of adequate high-energy data, current experimental values for $\sigma_{\text{int}}$ certainly cannot provide an enlightening test of our theoretical integrated cross sections, even if, as we believe from work of Levinger and others,\(^30\) the
asymptotic nature of our results does not seriously vitiate their applicability to
large but finite nuclei. In Table V we list the results for $C$ of a crude empirical
estimate made by Okamoto and Hasegawa and of a semi-phenomenological
formula due to Carver and Peaslee, both for $A=100$ to 200. Our values for
the three "realistic" potentials lie mostly between the two (non-overlapping)
empirical ranges swept out, which we take at the present state of the art as
implying reasonable agreement with experiment. From our work it seems likely
that, with the accurate inclusion of higher-energy processes (up to the threshold
for meson production), the experimental dipole $\sigma_{\text{int}}$ for heavy nuclei will exceed
the TRK value by close to a factor two. Clearly, extensive and definitive meas-
urements of $\sigma(W)$ at high photon energies could be most helpful in distin-
guishing between proposed free two-nucleon potentials. From the theoretical
side, it is necessary to make more precise the association of approximate
ground-state wave function with given potential (for our asymptotic model in
particular, more accurate calculations of the first-order perturbative corrections
are essential) and to estimate the contributions to $\sigma_{\text{int}}$ from higher multipoles
than $E_1$.

§ 6. The bremsstrahlung-weighted cross section

The bremsstrahlung-weighted cross section is experimentally the best known
of the three moments we consider, and it has the additional theoretical advantage
that its sum rule does not involve the potential explicitly and thus provides a
direct test of nuclear wave functions.

Unfortunately, the evaluation of $\sigma_b$ in the presence of dynamical correlations
is considerably more difficult (in given approximation) than that of $\sigma_{\text{int}}$. How-
ever, there is evidence that the "unperturbed" wave function $\Phi_b$ should be
adequate: Okamoto's perturbation calculation of $\sigma_b$ assuming a central mo-
notonically attractive two-nucleon potential led to a negative dynamical correction
to the LK Fermi-gas result of only 5 %. A comparable perturbation correction
is expected in our approach, the (central) pseudopotentials generated by plausible
correlation factors and realistic potentials being, at worst, of about the same
effective strength (and the same range) as Okamoto's Gaussian potential. (It
should be noted here that to first order in the perturbing potential—using either
the conventional perturbation expansion or our modification of it—the tensor
component of the potential does not contribute to $\sigma_b$.) At any rate, such a
simplification of our model is necessary, to convert an otherwise formidable
calculation into one that can be executed in a reasonable amount of machine
time.

We can argue qualitatively that the effect of dynamical correlations on the
$\sigma_b$ sum rule should be relatively minor. Since the lower photon energies are
emphasized in the $\sigma_b$ integral, high-momentum components of the wave function
(corresponding to short-range correlations) should be correspondingly less im-
important than for $\sigma_{1\text{st}}$. In short, one would expect the hard core to have little effect on $\sigma_b$; so perhaps the choice made for $f(r)$ will not be critical. To see in what measure these anticipated features are realized, we perform calculations for finite $c$ on the basis of our model, allowing a wide range of $r$ values, and compare with Fermi-gas results like those of LK.

Inserting $F\Phi_0$ into Eq. (2.5), and, as usual, keeping only contributions of lowest cluster order, the following formulas emerge:

\begin{align}
\sigma_b &= \sigma_b^{p-p} + \sigma_b^{p-p'} + \sigma_b^{p-n} \\
\sigma_b^{p-p} &= \beta \frac{M}{h^2} \sum_r \langle z_1^r \rangle_0 = \beta \frac{M}{h^2} Z R^2 = \sigma_b^{\text{LB}}, \\
\sigma_b^{p-p'} &= \beta \frac{M}{h^2} \sum_{r \neq r'} \langle z_1^r z_{r'} \rangle_0 = \beta \frac{M}{h^2} \sum_{\text{pair}} (0, 0_l | f^2(r_{12}) z_1 z_2 | 0_l 0_{l'}, 0_l 0_{l'}), \\
\sigma_b^{p-n} &= \beta \frac{M}{h^2} \sum_{I \neq j} \langle z_1^I z_j \rangle_0 = \beta \frac{M}{h^2} \sum_{\text{pair}} (0, 0_l | f^2(r_{12}) z_1 z_2 | 0_l 0_j).
\end{align}

Here $\sigma_b^{p-p'}$ is a correction to the LB result arising from both statistical and dynamical correlations of like particles—it is our dynamically modified version of the Pauli correction calculated by LK. $\sigma_b^{p-n}$ is a correction which is present only by virtue of the dynamical correlations among unlike particles. We have adopted the usual notation

\begin{align}
(ab|\mathcal{O}(12)|cd) &= \int d(1) \int d(2) \varphi_a^*(1) \varphi_b^*(2) \mathcal{O}(12) \varphi_c(1) \varphi_d(2) \tag{6.3}
\end{align}

for two-particle matrix elements of the operator $\mathcal{O}(12)$, in which the $\varphi$'s are single-particle wave functions with the indicated sets of quantum numbers $a, b, c, d$, the coordinates of particle $l$ are symbolized by $l$ itself, and $\int d(l)$ signifies an integration over the space coordinates and a summation over the spin coordinates of the $l$th particle. Our single-particle spatial functions are plane waves, so $0_l = (k_0, s_0)$, where $k_0, s_0$ are, respectively, the wave vector and spin quantum number of the corresponding single-particle state, and similarly for $0_{l'}$. The abbreviations $k_{0l} = k_l, s_{0l} = s_l,$ etc., should lead to no confusion. We shall give more steps of the development than for $\sigma_{1\text{st}}$, since the formulas we seek cannot be so easily extracted from earlier work.

Our model ignores the difference between neutron and proton single-particle states and between $N$ and $Z$, so the direct contribution of $\sigma_b^{p-p'}$ is identical to $\sigma_b^{p-n}$. Only the exchange part of $\sigma_b^{p-p'}$ persists as a correction to $\sigma_b^{\text{LB}}$. The general summand of the latter is

\begin{align}
\mathcal{E}_{0_l 0_{l'}} &= (0_l 0_{l'} | f^2(r_{12}) z_1 z_2 | 0_l 0_{l'}) \\
&= \delta_{0_l 0_{l'}} \frac{1}{Q^2} \int dq \int dr_1 \int dr_2 f^2(r_{12}) z_1 z_2 \exp (iq \cdot r_{12}), \tag{6.4}
\end{align}

with $q = k_l - k_{l'}$. 
Moving on to the details of integration, the product $z_1z_2$ in Eq. (6·4) is replaced according to

$$z_1z_2 \rightarrow (r_1 \cdot r_2) / 3 = (r_1^3 + r_2^3 - r_{12}^3) / 6 \rightarrow (2r_1^3 - r_{12}^3) / 6,$$

leaving us with two basic integrals to evaluate:

$$I_1 = \int dr_1 \int dr_2 f^3(r_{12}) r_{12}^2 \exp(iq \cdot r_{12}),$$

$$I_2 = \int dr_1 \int dr_2 f^3(r_{12}) r_{12}^2 \exp(iq \cdot r_{12}).$$

These two-body coordinate space integrals are a bit more troublesome than those which arise in the case of $\sigma_{1n}$. The integrands of the latter are always short-range functions only of the separation $r_{12}$, thus may be converted to independent integrals over $R_{12} = (r_1 + r_2) / 2$ and $r_{12} = r_1 - r_2$ in which the upper limit of $r_{12}$ may be taken infinite. Essential for this reduction is the assumption that the normalization volume $Q$ of our system is so large that surface effects are negligible. Appealing to the same assumption, we are justified in carrying out the $r_1$, $r_2$ integrations of Eqs. (6·6) over spheres of radius $R$, $4\pi R^3/3 = Q$, instead of cubes of side length $L$. (The first term of $\sigma_h$ was of course given in terms of $R$—rather than $L$—by Levinger and Bethe.) Then $I_1$ and $I_2$ may be transformed into one-dimensional integrals suited to numerical computation by application of the relations

$$1 / 4\pi \int_{r_2 \leq R} dQ \int dr_2 \delta(r - r_{12}) = \begin{cases} 1, & 0 \leq r \leq R - r_1, \\ R - (r_1 - r)^3, & R - r_1 \leq r \leq R + r_1, \\ 0, & R + r_1 \leq r, \end{cases}$$

and

$$\int_{r_1 \leq R} dr_1 \int_{r_2 \leq R} dr_2 \delta(r - r_{12}) = \frac{(2\pi/3)(2R^3 - (3/2)R^3 + r^3/8)}{r \leq 2R},$$

respectively. In the first of these, $dQ$ is the element of solid angle for the variable $r_1$. The second relation is just an expression for the volume common to two spheres, both of radius $R$, their centers a distance $r$ apart.

It might be helpful to record the manipulations involved in reducing $I_1$. We write, using Eq. (6·7),

$$I_1 = \int_{r_1 \leq R} r_1^4 dr_1 \int dr \int dQ \int dr_2 \delta(r - r_{12}) f^3(r_{12}) \exp(iq \cdot r_{12})$$

$$= 4\pi \int_{0}^{R} r_1^4 dr_1 \int_{0}^{R - r_1} dr f^3(r) \exp(iq \cdot r)$$

$$+ 4\pi \int_{0}^{R} r_1^4 dr_1 \int_{R - r_1}^{R + r_1} dr f^3(r) \frac{R^2 - (r_1 - r)^2}{4r_1 r} \exp(iq \cdot r).$$
The limits on the \( r_1, r \) integrations may be rearranged as follows (consider the surfaces of integration in the \( r_1, r \) plane):

\[
\int_0^R \int_0^{r-r_1} \int_0^{r-r} \int_0^r \, dr_1 \, dr \rightarrow \int_0^R \int_0^{r-r} \int_0^{r-r_1} \int_0^r \, dr_1 \, dr \rightarrow \int_0^R \int_0^{r-r} \int_0^{r-r} \int_0^R \, dr_1 \, dr \.
\]

Upon performing the resulting \( r_1 \) quadratures, we find

\[
I_1 = \frac{16\pi^2}{q} \int_0^{2\pi} r \, dr \, f^2(r) \sin qr \left( \frac{1}{5} R^5 - \frac{1}{4} R^4 r + \frac{1}{16} R^2 r^2 - \frac{1}{240} r^4 \right).
\]

After similar treatment of \( I_2 \), we arrive at

\[
E_{\theta \theta'} = \frac{1}{\rho^2} \frac{16\pi^2}{q} \int_0^{2\pi} r \, dr \, f^2(r) \sin qr \left( \frac{2}{5} R^5 - \frac{1}{2} R^4 r + \frac{3}{16} R^2 r^2 - \frac{1}{80} r^4 \right) .
\]

Asymptotically, the required sums of \( E_{\theta \theta'} \) over single-particle states are to be carried out according to

\[
\sum_{\theta \theta'} \frac{Q}{(2\pi)^3} \int dk_1 \frac{Q}{(2\pi)^3} \int dk_2 \sum_{\theta \theta'}.
\]

To reduce the momentum integrations we again invoke Eq. (6.8), but with the coordinate-space vectors replaced by the appropriate wave vectors, and in particular \( R \rightarrow k_r \). The final formula for \( \sigma_b^{\nu-\nu'} + \sigma_b^{\nu-\nu} \), used for numerical calculation, is

\[
\sigma_b^{\nu-\nu'} + \sigma_b^{\nu-\nu} = -\frac{M}{\hbar^2} \frac{128k_1^2 R^4}{45\pi^2} \int \int f^2(2Rt) X_2(t) \int \sin(4k_1 Rtu) X_1(u),
\]

where

\[
X_1(u) = u - (3/2)u^3 + (1/2)u^4, \quad X_2(t) = t - (5/2)t^3 + (5/2)t^4 - t^6.
\]

As a check on this expression, we have used it to perform Fermi-gas-model calculations for \( A=63 \) and \( A=181 \), setting \( c=0 \) and \( f^2=1 \); the results at \( r_0=1.20 \text{ fm} \) coincide with values obtained by LK using their simpler but specialized formula. Also, it is readily seen that, for \( f^2=1 \), the direct contribution to \( \sigma_b^{\nu-\nu'} \) vanishes, as in LK. Generally, we denote the Fermi-gas \( \sigma_b \) by \( \sigma_b^{\text{FK}} \).

The above procedure leads to a correction which decreases \( \sigma_b \) from \( \sigma_b^{\text{FK}} \) by about 30–60% (the smaller values for the smaller \( \gamma \)'s, \( A \)'s, the larger values...
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for the larger $r$'s, $A$'s). But recall that $\sigma_b^{LB}$ is only the zeroth-cluster-order result for the first term of Eq. (6.1). For the smallest $r$ we consider, it turns out that the first-cluster-order contribution $(\sigma_b^{p-p})^{(1)}$ to this term provides a correction to $\sigma_b^{LB}$ of about the same size as does Eq. (6.11), so a consistent calculation demands its inclusion. Straightforward application of the formalism of Iwamoto and Yamada\textsuperscript{15} or Clark and Westhaus\textsuperscript{9} to the required (diagonal) matrix element of the sum $\sum_{\epsilon} z_\epsilon^2$ of one-body operators yields

$$\sigma_b^{p-p} = \sigma_b^{LB} + \sum_{\epsilon<\epsilon'} \{ (0,0,\epsilon | (f^2(r_{12})-1) z_\epsilon^2 | 0,0,\epsilon) - (0,0,\epsilon | f^2(r_{12})-1 | 0,0,\epsilon) \}$$

a result accurate to first cluster order. Here the third addend just doubles the direct part of the second. The same methods as used for $\sigma_b^{p-p'} + \sigma_b^{p-n}$, specifically, application of relation (6.7), serve to prepare this formula for numerical computation. We obtain

$$\begin{align*}
(\sigma_b^{p-p})^{(1)} &= \frac{512}{\hbar^2} \left\{ \frac{2k_t R^3}{27\pi^2} \int_0^1 \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \! \![6.13]
\end{align*}$$

Numerical calculations were carried through for $c=0.4$ fm, $r_0=1.07$ fm and $r_8=1.20$ fm, and $r$ values including 3, 5, 7 and $\infty$ fm$^{-1}$. The $A$ values selected range from $A=36$ to $A=238$; with the exception of $A=36$ they all correspond to nuclei for which an experimental $\sigma_b$ has been constructed by Khokhlov\textsuperscript{26} (and possibly Levinger\textsuperscript{9}) or Rustgi and Levinger\textsuperscript{27} as well.\textsuperscript{28}

Information on the relative importance of the various contributions to the calculated $\sigma_b$ follows. Let an arrow $\rightarrow$ stand for the words "and increases (decreases), in magnitude, monotonically with $A$ up to." Then we have, first of all,

$$\frac{\sigma_b^{p-p'} + \sigma_b^{p-n}}{(\sigma_b^{p-p})^{(0)}} \times 100 \%$$

$$= -34\% (A=36) \rightarrow -52\% (A=238) \text{ for } r=5 \text{ fm}^{-1}, r_0=1.07 \text{ fm}, \quad (6.16)$$

$$= -34\% (A=36) \rightarrow -54\% (A=238) \text{ for } r=5 \text{ fm}^{-1}, r_0=1.20 \text{ fm}, \quad (6.16)$$
with roughly the same results for other \( r \)'s, an overall increase in magnitude of the ratio of about 20% being observed over the whole \( r \) range. This ratio appears almost independent of \( r \).

Next,

\[
\frac{(\sigma_{b}^{p-p})^{(1)}}{(\sigma_{b}^{p-p})^{(0)}} \times 100\%
\]

\[
= -35\% \rightarrow -42\% \text{ for } r = 3 \text{ fm}^{-1}, \quad r_0 = 1.07 \text{ fm},
\]

\[
= -25\% \rightarrow -30\% \text{ for } r = 3 \text{ fm}^{-1}, \quad r_0 = 1.20 \text{ fm},
\]

\[
= -15\% \rightarrow -17\% \text{ for } r = 5 \text{ fm}^{-1}, \quad r_0 = 1.07 \text{ fm},
\]

\[
= -10\% \rightarrow -12\% \text{ for } r = 5 \text{ fm}^{-1}, \quad r_0 = 1.20 \text{ fm},
\]

\[
= -9\% \rightarrow -10\% \text{ for } r = 7 \text{ fm}^{-1}, \quad r_0 = 1.07 \text{ fm},
\]

\[
= -6\% \rightarrow -7\% \text{ for } r = 7 \text{ fm}^{-1}, \quad r_0 = 1.20 \text{ fm},
\]

\[
= -7\% \text{ for } r = \infty \text{ fm}^{-1}, \quad r_0 = 1.07 \text{ fm},
\]

\[
= -5\% \text{ for } r = \infty \text{ fm}^{-1}, \quad r_0 = 1.20 \text{ fm}.
\]

(Beginning and ending \( A \) values are the same as before.) These results are interesting in themselves: They furnish an indication of the usefulness of the cluster expansion for the first term of Eq. (6.1) as a function of the parameter \( r \). The present ratio is much more sensitive to \( r \) than the preceding one, and manifests a substantial \( r_0 \) dependence. Last, the integral from 0 to \( c/2R \) in the expression (6.14) for \((\sigma_{b}^{p-p})^{(1)}\) produces a negative contribution to \(\sigma_{b}^{p-p}\) amounting, in magnitude, to only about 2% of the whole, thus pointing to the relative insignificance for \(\sigma_{b}\) of the parts of configuration space excluded by the hard cores. Notice that all corrections to \(\sigma_{b}\) that we have considered separately are negative.

Following LK, we may define a correction factor

\[
C_{b} = \left(1 + \frac{(\sigma_{b}^{p-p})^{(1)} + \sigma_{b}^{p-p'} + \sigma_{b}^{p-p'}}{\sigma_{b}^{LB}}\right)^{-1},
\]

so that our \(\sigma_{b}\) is given by \(\sigma_{b} = \sigma_{b}^{LB}/C_{b}\). For the LK calculation \(C_{b}\) was simply a Pauli correction factor, and its use was, for them, more natural in that \(W_{NI}^{LK} = C_{b}^{LK}W_{NI}^{LB}\). We might say here that \(C_{b}^{LK}\) is independent of \(r_0\); our \(C_{b}\), in general, is not.

In Table VI are compiled the results for \(\sigma_{b}/A^{1/3}\) of numerical calculations based on the Fermi-gas model and on our model including dynamical correlations. Table VII presents, for tentative comparison, the corresponding results of Khokhlov and of LK based on the model of nucleons moving independently in a finite square well of radius \(R = r_0 A^{1/3}\), \(r_0 = 1.20 \text{ fm}\), along with the experimental values emerging from the aforementioned data analyses of Khokhlov, Levinger, and Rustgi and Levinger.
Dynamical Correlations and the Nuclear Photoeffect

We first discuss the effect of dynamical correlations as treated in this paper. There is a quenching of \( \sigma_b \) (relative to the Fermi-gas or LK values) for all \( r, r_0, A \), but this quenching is a strong function of these parameters over their physically relevant ranges (increasing monotonically with \( r \) and always being larger for the smaller \( r_0 \)). The sensitivity to \( r \) is especially striking: At \( r = 3 \text{ fm}^{-1} \) the dynamical effect is overwhelming, amounting to a decrease of 42\% \((A=36)\rightarrow 76\% \((A=238)\) for \( r_0 = 1.07 \text{ fm} \) and 29\% \((A=36)\rightarrow 42\% \((A=238)\) for \( r_0 = 1.20 \text{ fm} \), but by \( r = \infty \text{ fm}^{-1} \) it has practically died out, the decrease being only 9\% \((A=36)\rightarrow 5\% \((A=238)\) for \( r_0 = 1.07 \text{ fm} \) and 3\% \((A=36)\rightarrow 2\% \((A=238)\) for \( r_0 = 1.20 \text{ fm} \). For \( r \) values which we consider reasonable on the basis of energy calculations, the quenching is around 20\%. Both the possibility

<table>
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<th>( A )</th>
<th>( r_0 (\text{fm}) )</th>
<th>( c = 0.4 \text{ fm} ) ( r = 3 \text{ fm}^{-1} )</th>
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<th>( r = \infty ) LK ((c = 0) )</th>
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Table VII. Calculated results for \( \sigma_b/A^{1/3} \) (mb).
Table VII. Results for $\sigma_b/A^{4/3}$ from a finite independent-particle model and from experiment.

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</table>

$^a)$ See reference 26).

$^b)$ See reference 11).

$^c)$ See references 26) and 28).

of sizeable dynamical effects (even as large as those for $\sigma_{in}$) and their sensitivity to $\gamma$ might be regarded with surprise, in view of the preconceptions expressed at the beginning of this section. But these preconceptions are in part supported by our results—for certainly, from the fact that the $\gamma=\infty$ results deviate so little from the LK values, it is apparent that the hard core is indeed of little importance for $\sigma_b$. On the other hand, the bremsstrahlung-weighted cross section does depend critically on $f$, but only on its long-range behavior, in contrast to the energy, which depends mainly on its short-range behavior. Thus the $\sigma_b$ sum rule may provide a far more valuable test of dynamically correlated nuclear models than anticipated. We shall have more to say on this point in a moment.

Note that $\sigma_b/A^{4/3}$ varies with $A$ in all six columns of Table VI (increases, except for $\gamma=3$ fm$^{-1}$), the variation being least (almost nil) for the standard parameter values $\gamma=5$ fm$^{-1}$ and $r_0=1.07$ fm.

Direct comparison of the results of this paper with those of the LK-Khokhlov finite square-well model or with experiment would be misleading, but we can make some interesting and pertinent remarks of qualitative nature. Levinger$^{28}$ observed that both the theoretical predictions and the (predigested) experimental data of Table VII may be usefully represented by the formula

$$\sigma_b=0.30 A^{4/3} \text{ mb.}$$ (6·19)

(The fluctuations of the experimental values away from this formula are, however, significant; the LK-Khokhlov results show somewhat less structure.) Oddly
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enough, our standard results are very well summarized by precisely the same formula (there being, of course, no local fluctuations from it, since such structure effects are inaccessible to us). For a harmonic-oscillator model with the same mean-square radius of the nucleon distribution as in the square-well model, Levinger\(^6\) found

\[ \sigma_b = 0.36A^{1/3}\text{ mb}. \]  

(6.20)

(The reader should take cognizance of the fact that all of these models rely on \(N=Z\)—all "symmetry" effects are disregarded.) A semi-phenomenological analysis carried out by Carver and Peaslee\(^{25}\) yielded

\[ \sigma_b \approx (0.31 \pm 0.05)A^{1/3}\text{ mb}, \]  

(6.21)

which allows both Eqs. (6.19) and (6.20).

The remarkable agreement of our standard results with Eq. (6.19) is surely fortuitous because, quite obviously, surface and shell effects are large: For example, the quenching from the corresponding Fermi-gas values of \(\sigma_b\) for the LK-Khokhlov finite model, is 34% at \(A=63\), 33% at \(A=209\), and 46% at \(A=238\). Piling on an additional negative correction due to (residual) dynamical correlations—as predicted by our treatment with \(\gamma = 5\text{ fm}^{-1}\) and \(r_0 = 1.20\text{ fm}\)—destroys all agreement with experiment. Still, this is no real evidence for an inconsistency; there is plenty of latitude to alleviate the difficulty, by

1) going over to a harmonic-oscillator well, in which case, on the basis of Eq. (6.20), the quenching due to finite nuclear effects is, roughly, only 18% at \(A=63\), 29% at \(A=209\), and 31% at \(A=238\), and/or

2) going over to a larger \(\gamma\) than those which seem favored by energy calculations for the extended system.

Indeed, an elaborate calculation is required to settle this problem of consistency. Our method may be adapted in a straightforward way to a finite nuclear system by taking for \(\Phi_0\) in \(\Psi_\alpha = F\Phi_0\) a suitable (e.g. self-consistent) shell-model wave function. Then, corresponding to 1) and 2),

1') Neither the square well nor the harmonic-oscillator well would be the best choice, the resulting surface being too sharp in the former case, too diffuse in the latter; one would be driven to something in between.

2') As is well known, such a \(\Phi_0\) can account, in average fashion, for some long-range dynamical correlations, leaving \(F\) with lighter burden in the attempt to realize in \(\Psi_\alpha\) a fairly realistic ground-state wave function. So it is understandable how an \(f^2 - 1\) with shorter range (larger \(\gamma\)) than we consider standard for nuclear matter might be appropriate when we go to the finite system.

Nevertheless, we would not be surprised if detailed study revealed the need for a (non-monotonic) correlation factor which approaches unity from above, the short-range behavior being such as to get a good energy (\(f\) not too steep) and the long-range behavior such as to get a good \(\sigma_b\) (\(f^2 - 1\) not too long-
ranged), both in the infinite medium and in finite nuclei. (In this connection, attention should be called to the prediction made near the end of § 1 that $\sigma_b$ as calculated via the sum rule should exceed present experimental estimates, and also to the view that the first-order perturbation correction to our results will be negative, each implying a small accentuation of the seeming inconsistency remarked above.)

§ 7. The mean energy

Our calculation of the first moment $\sigma_1$ of the photonuclear cross section proceeds in the same spirit as that of $\sigma_b$. The "unperturbed" wave function $F\Phi_0$ is employed throughout this section. Since high energies are emphasized in the $\sigma_1$ integral, and since the potential enters explicitly into the corresponding sum rule, this crude model is not expected to be as satisfactory as for $\sigma_b$. Nevertheless, judging from our $\sigma_b$ results and our $\sigma_{\text{int}}$ results for the effective central potentials, we should be able to obtain, for such potentials, a useful preliminary estimate of the mean energy $\bar{W}=\sigma_1/\sigma_{\text{int}}$. Indeed, a more detailed evaluation of $\sigma_1$ would be premature at the present stage of theoretical and experimental knowledge.

It might be mentioned that the problem of calculating

$$\sigma_1=\sigma_{1,\text{ord}}+\sigma_{1,\text{ep}}+\sigma_{1,\text{xch}} \quad (7.1)$$

(see Eq. (2.6)) is even more complicated than that of calculating the ground-state energy of the system. For a given trial function, $\sigma_{1,\text{ord}}$ is, apart from a constant, just the expectation value of a quadratic function of momentum operators; $\sigma_{1,\text{xch}}$ is, apart from a constant, just the expectation value of a sum of potential-like operators, $\sum_{i<j} r_{ij}^2 \partial^2 \langle ij \rangle$; however, $\sigma_{1,\text{ep}}$ involves, linearly, both momentum and potential operators. Furthermore, the above $\sigma_1$ lacks the extremum property of the energy expectation value. On the other hand, since the three terms in Eq. (7.1) are all positive, their evaluation need not be so accurate as that of the kinetic and potential contributions to the energy, which are of opposite sign and nearly compensate one another.

All computations will be carried out only to lowest cluster order.

As we saw earlier, $\sigma_{1,\text{ord}}$ for our model collapses to

$$\sigma_{1,\text{ord}} = \frac{\beta}{M} \left\{ \langle p_i^2 \rangle_{\text{00}} + \sum_{n} \langle p_{i_n} p_{i_n} \rangle_{\text{00}} - \sum_{ij} \langle p_{i_n} p_{j_n} \rangle_{\text{00}} \right\}. \quad (7.2)$$

The first term is proportional to the expectation value of the kinetic energy, so we may take over a (lowest-cluster-order) formula of Iwamoto and Yamada$^{29}$ or (more conveniently) Bell and Squires$^{30}$ to write

$$\frac{\beta}{M} \sum_i \langle p_i^2 \rangle_{\text{00}} = \frac{\beta \hbar^2 k^2}{2M} + \frac{2\pi\beta}{3} \int_0^\infty r^2 dr \ d_\ell(kr) \left( \frac{df(r)}{dr} \right)^2, \quad (7.3)$$
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The second and third terms of Eq. (7·2) may be reduced via the machinery of the general cluster expansion formalism, to

\[
\frac{\beta}{M} \left\{ \sum_{ij} \langle p_i p_j \rangle \right\} + \frac{1}{M} \left\{ \sum_{ij} \langle p_i p_j \rangle \right\} = \sigma_{1,01} + \sigma_{1,02} + \sigma_{1,03}, \tag{7·5}
\]

with

\[
\sigma_{1,01} = \beta \frac{\hbar^2}{M} \sum_{q \neq q'} \langle 0| \psi_{0c} | f^2(r_{13}) \frac{\partial^2}{\partial z_1 \partial z_2} | 0, 0 \rangle,
\]

\[
\sigma_{1,02} = \beta \frac{\hbar^2}{M} \sum_{q \neq q'} \langle 0| \psi_{0c} | f(r_{13}) \left( \frac{\partial f(r_{13})}{\partial z_1} + \frac{\partial f(r_{13})}{\partial z_2} \right) | 0, 0 \rangle,
\]

\[
\sigma_{1,03} = \beta \frac{\hbar^2}{M} \sum_{q \neq q'} \langle 0| \psi_{0c} | f(r_{13}) \frac{\partial^2 f(r_{13})}{\partial z_1 \partial z_2} | 0, 0 \rangle,
\]

where advantage has been taken of the fact that, as in the case of the \( \sigma_v \) sum rule, only exchange contributions survive. The further reduction of this (lowest-cluster-order) result to a form suitable for numerical integration requires only straightforward application of the techniques outlined in the last section. In working on \( \sigma_{1,01} \) and \( \sigma_{1,02} \), repeated use is made of the relations (6·7), (6·8) and their momentum space analogs. To remodel the \( \sigma_{1,02} \) formula, we notice that, for any radial function, and for \( f \) in particular,

\[
\frac{\partial f(r_{13})}{\partial z_1} = \frac{\partial f(r_{13})}{\partial z_2} = \frac{\partial f(r_{13})}{\partial r_{13}} \frac{r_{13} - z_{12}}{r_{13}}, \tag{7·7}
\]

while

\[
\exp(iq \cdot r_{13}) = \frac{1}{i(k_{13} - k_{13}')} \frac{\partial}{\partial z_{12}} \exp(iq \cdot r_{13}), \quad q = k_{13} - k_{13'}, \tag{7·8}
\]

so that an integration by parts leads to a momentum integral which can be immediately evaluated analytically, leaving only a radial integral to be done numerically. The formulas used for computations are

\[
\sigma_{1,01} = -\frac{128 \beta}{45} \frac{R^4 k^2}{\pi^2} \frac{\hbar^2}{M} \int_0^1 \left\{ f^2(2Rt) X_1(t) \right\} \int_0^1 du \sin(4k_t R u) X_1(u), \tag{7·9}
\]

\[
\sigma_{1,02} = -\frac{4 \beta}{9} \frac{A k^2}{\pi} \frac{\hbar^2}{M} \int_0^\infty r^2 dr j^4(k_t r) \rho(c(r), \tag{7·10}
\]

\[
\sigma_{1,03} = -\frac{64 \beta}{27} \frac{R^4 k^2}{\pi^2} \frac{\hbar^2}{M} \int_0^\infty dr \rho(r) X_1(r/2R) \int_0^1 du \sin(2k_t R u) X_1(u). \tag{7·11}
\]
where $X_1$, $X_2$ are given by Eq. (6.12), and

$$
\rho_1(r) = r^2 e^{-\gamma(r-\alpha)} \left( 1 - \frac{2}{\gamma r} - 2 e^{-\gamma(r-\alpha)} + \frac{2}{\gamma r} e^{-\gamma(r-\alpha)} \right), \\
\rho_2(r) = r^2 e^{-\gamma(r-\alpha)} \left( -1 + \frac{2}{\gamma r} + e^{-\gamma(r-\alpha)} - \frac{2}{\gamma r} e^{-\gamma(r-\alpha)} \right).
$$

(7.12)

The cross product term in Eq. (7.1) becomes, for our model, again to lowest cluster order,

$$
\sigma_{1,ex} = \sigma_{1,el} + \sigma_{1,ex},
$$

(7.13)

$$
\sigma_{1,el} = 2\beta \sum_{\delta} (0, 0) f^2(r_{12}) \varpi(12) \left( \frac{\partial}{\partial z_1} - \frac{\partial}{\partial z_2} \right) |0, 0\rangle,
$$

$$
\sigma_{1,ex} = 2\beta \sum_{\delta} (0, 0) f^2(r_{12}) \left( \frac{\partial f(r_{12})}{\partial z_1} - \frac{\partial f(r_{12})}{\partial z_2} \right) (z_1 - z_2) \varpi(12) |0, 0\rangle.
$$

(7.14)

Integration by parts facilitates the simplification of the second term. In both cases Eq. (7.8) is useful, and we arrive finally at

$$
\sigma_{1,el} = -\frac{4\beta}{3\pi} A k^3 \int_0^\infty r^3 dr f^2(r) j_1^3(kr) \left\{ \frac{r}{3} \frac{\partial}{\partial r} \left( \frac{1}{2} v^{(\alpha)}_l(r) + v^{(\beta)}_l(r) \right) \\
+ \left( \frac{1}{2} v^{(\alpha)}_l(r) + v^{(\beta)}_l(r) \right) \right\},
$$

(7.15)

$$
\sigma_{1,ex} = \frac{8\beta}{9\pi} A k^3 \int_0^\infty r^3 dr f(r) j_0^3(kr) l(r) \left( \frac{1}{2} v^{(\alpha)}_l(r) + v^{(\beta)}_l(r) \right),
$$

(7.16)

with

$$
l(r) = r e^{-\gamma(r-\alpha)}.
$$

(7.17)

The last term on the right in Eq. (7.1), the exchange term $\sigma_{1,ex}$, assumes the form

$$
\sigma_{1,ex} = \frac{\beta}{3} \frac{2M}{h^2} \sum_{\delta} (0, 0) f^3(r_{12}) r (\varpi_{12}^3(12))^3 |0, 0\rangle,
$$

(7.18)

which may be readily converted to a one-dimensional integral,

$$
\sigma_{1,ex} = \frac{4\beta}{9\pi} A k^3 \frac{M}{h^2} \int_0^\infty r^4 dr f^3(r) \left( \frac{1}{2} v^{(\alpha)}_l(r) + v^{(\beta)}_l(r) \right)^2.
$$

(7.19)

In the above the potential $\varpi(ij)$ has been taken central.

The mean energy is now expressed as

$$
\overline{W} = \frac{\sigma_1}{\sigma_{int}} = \frac{\sigma_{1,el} + \sigma_{1,ex} + \sigma_{1,ex} + \sigma_{1,el} + \sigma_{1,ex} + \sigma_{1,ex} + \sigma_{1,ex}}{(\beta A/4) (1 + \overline{C})}.
$$
where \( \sigma'_{1,06} = \sigma_{1,06} / (\beta A/4) \), etc. The complexity induced by our correlated model function is partially revealed by comparison with the formula of LB. If we put \( c = 0 \) and \( f = 1 \), and neglect statistical correlations, only the \( \sigma'_{1,06} \), \( \sigma'_{1,01} \) and \( \sigma'_{1,e} \) terms in the numerator endure, and these are then exactly the three terms of LB.

Our generalization of the (Pauli) correction factor introduced by LK to convert the LB value of \( \sigma_1 \) to their value is

\[
C_1 = 1 + \frac{\sigma'_{1,01} + \sigma'_{1,02} + \sigma'_{1,03}}{\sigma_{1,00}},
\]

which has the property

\[
\sigma_{1,e} = C_1 \sigma_{1,00}.
\]

LK computed \( C_1 \) using a Fermi-gas model (for which \( \sigma'_{1,01} = \sigma'_{1,02} = 0 \) and found it to decrease with increasing \( A \), eventually going negative—a catastrophic result which was taken as symptomatic of the inadequacy of the Fermi-gas approximation in the photonuclear problem. On the other hand, our correlated model gives non-negative \( C_1 \)'s for all \( A \), and, in fact, this remains true when we go to the limiting case (\( c = 0, f = 1 \)) which should lead back to the LK values.

Numerical calculations were carried out only for the OMY 4 potential. (A more realistic choice would present us with a laborious perturbation-theoretic computation of tensor contributions.) The usual radius parameters, along with \( \gamma \) values of 5 fm\(^{-1}\) and 7 fm\(^{-1}\), were selected as representative. To be definite, we have taken the upper limits of the relevant \( C \)'s (see Table IV). Results for the \( \sigma' \) quantities at \( \gamma = 5 \) fm\(^{-1}\) (ranging over the same set of \( A \)'s as in \( \S \) 6) are collected in Tables VIII and IX, while Table X displays the corresponding \( W \) values. Our \( C_1 \) results for the Fermi-gas model are compared with those for the dynamically correlated model at \( \gamma = 5 \) fm\(^{-1}\) in Table XI.

It should be remarked that the quantities in Table VIII are independent of \( A \); those in Table IX, almost independent of \( A \). Hence \( W \), as seen, changes (in fact, decreases) only very slightly as \( A \) increases. The dominant contributions to the numerator of \( W \) come from \( \sigma'_{1,00} \) and \( \sigma'_{1,e} \). The first cluster cor-

<table>
<thead>
<tr>
<th>( r_0 ) (fm)</th>
<th>( \sigma'_{1,00} ) (Mev)</th>
<th>( \sigma'_{1,02} ) (Mev)</th>
<th>( \sigma'_{1,03} ) (Mev)</th>
<th>( \sigma'_{1,04} ) (Mev)</th>
<th>( \sigma'_{1,e} ) (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.07</td>
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<td>-9.5</td>
<td>13.9</td>
<td>-0.31</td>
<td>76.6</td>
</tr>
<tr>
<td>1.20</td>
<td>50.8</td>
<td>-5.8</td>
<td>7.6</td>
<td>-0.33</td>
<td>54.3</td>
</tr>
</tbody>
</table>
Table IX. Breakdown of $\sigma_1$ results at $r=5$ fm$^{-1}$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\sigma_{1,\alpha'}$ (Mev) $r_0=1.07$ fm</th>
<th>$\sigma_{1,\alpha'}$ (Mev) $r_0=1.20$ fm</th>
<th>$\sigma_{1,\alpha'}$ (Mev) $r_0=1.07$ fm</th>
<th>$\sigma_{1,\alpha'}$ (Mev) $r_0=1.20$ fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>-13.3</td>
<td>-10.7</td>
<td>-12.3</td>
<td>-9.6</td>
</tr>
<tr>
<td>93</td>
<td>-15.3</td>
<td>-12.3</td>
<td>-12.7</td>
<td>-9.9</td>
</tr>
<tr>
<td>64</td>
<td>-15.7</td>
<td>-12.6</td>
<td>-12.8</td>
<td>-9.9</td>
</tr>
<tr>
<td>75</td>
<td>-16.0</td>
<td>-12.8</td>
<td>-12.9</td>
<td>-9.9</td>
</tr>
<tr>
<td>98</td>
<td>-17.0</td>
<td>-13.7</td>
<td>-13.1</td>
<td>-10.1</td>
</tr>
<tr>
<td>109</td>
<td>-17.3</td>
<td>-14.0</td>
<td>-13.2</td>
<td>-10.2</td>
</tr>
<tr>
<td>119</td>
<td>-17.6</td>
<td>-14.2</td>
<td>-13.2</td>
<td>-10.2</td>
</tr>
<tr>
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<td>-13.2</td>
<td>-10.2</td>
</tr>
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<td>-19.0</td>
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</tr>
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<td>-15.8</td>
<td>-13.5</td>
<td>-10.4</td>
</tr>
<tr>
<td>238</td>
<td>-20.0</td>
<td>-16.1</td>
<td>-13.6</td>
<td>-10.5</td>
</tr>
</tbody>
</table>

Table X. Mean energy $W$ (Mev) at $r=5$ fm$^{-1}$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$W$ $r_0=1.07$ fm</th>
<th>$W$ $r_0=1.20$ fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>62</td>
<td>46</td>
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<tr>
<td>98</td>
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<td>43</td>
</tr>
<tr>
<td>238</td>
<td>58</td>
<td>42</td>
</tr>
</tbody>
</table>

Corrections to these terms would probably be (individually) about the same size as $\sigma_{1,\alpha'}$, hence substantially smaller than the sum of the remaining terms of $\sigma_{1,\alpha'}$. For our purposes, we do not regard their effect on the results as serious, nor their calculation worthwhile.

At $r=7$ fm$^{-1}$, all these features of the $r=5$ fm$^{-1}$ results persist, but $W$ has increased (remember, for $r\to\infty$, $W\to\infty$) by about 13%, to $\approx 67$ Mev at $r_0$. 


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Table XI. Correction factor $C_1$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$r_0=1.07$ fm</th>
<th>$r_0=1.20$ fm</th>
<th>$c=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c=0.4$ fm</td>
<td>$c=0.4$ fm</td>
<td>$c=0$</td>
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<tr>
<td>36</td>
<td>0.48</td>
<td>0.49</td>
<td>0.60</td>
</tr>
<tr>
<td>63</td>
<td>0.48</td>
<td>0.45</td>
<td>0.60</td>
</tr>
<tr>
<td>64</td>
<td>0.47</td>
<td>0.44</td>
<td>0.53</td>
</tr>
<tr>
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</table>

$=1.07$ fm and $\approx49$ Mev at $r_0=1.20$ fm. Thus $\tilde{W}$ is highly sensitive to $\gamma$. (In these estimates, the $C_{0}^{[s]}$'s appropriate to $\gamma=7$ fm$^{-1}$ were taken from Table IV, but the $C_{0}^{[p]}$'s were left at their $\gamma=5$ fm$^{-1}$ values. This could hardly give rise to an error of more than 2%.)

We cannot fully assess the dynamical effects of our model without calculating $\sigma_{1,e}$ and $\sigma_{1,c}$ for the OMY 0 potential and $f=1$. But it is clear from the LB results $\sigma_{1,e}^{[s]}=18$ Mev and $\sigma_{1,e}^{[p]}=12$ Mev (Yukawa well, Serber mixture, $r_0 =1.37$ fm) that, even allowing for the difference in potential shapes and $r_0$ values, the effect of hard-core correlations on the last two terms of Eq. (7.1) is enormous. For $\sigma_{1,e}$ we can be more definite. The appropriate LB values for $\sigma_{1,e}^{[s]}$ (just the Fermi-gas kinetic energy) are 33.3 Mev for $r_0=1.07$ fm and 26.8 Mev for $r_0=1.20$ fm. The appropriate "LK" values accounting for statistical correlations, but with all dynamical correlations turned off, are obtained by multiplying by $C_1$ according to Table XI. We see that statistical correlations alone reduce $\sigma_{1,e}$ by nearly a factor two. But dynamical effects are even more important: On the basis of our $\gamma=5$ fm$^{-1}$ and $\gamma=7$ fm$^{-1}$ results, the additional inclusion of dynamical correlations so enhances $\sigma_{1,e}$ over $\sigma_{1,e}^{[s]}$ that the LB values are almost reinstated. The dominant effect here is the great increase of the "kinetic" term $\sigma_{1,0}$.

A few observations on our results for $C_1$ are in order. As calculated here, $C_1$ is always positive. It is independent of $r_0$ when the dynamical correlations are shut off and is almost independent of $r_0$ for $\gamma=5,7$ fm$^{-1}$. Further, $C_1$ is insensitive to $\gamma$, with the consequences that the $\gamma=5,7$ fm$^{-1}$ $C_1$ results are hardly
distinguishable and that the Fermi-gas results are not far different from those with dynamical correlation (the former being, however, more strongly A-dependent). Also, our $\gamma = 5.7 \text{ fm}^{-1}$ results for this correction factor are close to those of LK using their finite square-well model with $r_0 = 1.20 \text{ fm}$.

If we wished to compare our predictions with experiment, then (from remarks made near the end of §1) it would appear safer to do so in terms of $W$ rather than $\sigma_i$. For if we include dynamical correlations in the treatment of $\sigma_{\text{int}}$, the extra contribution to $\sigma_1$ from high-energy processes is partially compensated by the corresponding contribution to $\sigma_{\text{int}}$, lessening, in $W$, the conjectured discrepancy between theoretical predictions and experimental measurements. It must be admitted, though, that our information on the $\gamma$ dependence of $\sigma_1$ and $\sigma_{\text{int}}$ indicates only a nominal compensation.

At any rate, even putting aside the asymptotic nature of our model, the extreme sensitivity, on the one hand,

1) of the predicted mean energy to $r_0$ and $\gamma$ as well as to variations in the exchange, repulsive, and non-central character of the potential (not studied here) and, on the other hand,

2) of the experimental $W$ to the poorly known high-energy tail of $\sigma(W)$ precludes for now a meaningful comparison.

§ 8. Conclusions

Strong dynamical correlations implied by realistic two-nucleon potentials have substantial effect on energy moments of the photonuclear cross section calculated by means of the dipole sum rules. In our model, we find that $\sigma_{\text{int}}$, which is sensitive to the long-range behavior of the two-body correlation factor $f$, may be significantly decreased, and $\sigma_{\text{int}}$ and $\sigma_1$, which are sensitive to the two-nucleon potential and to the short-range behavior of $f_1$, are strongly enhanced (especially $\sigma_1$), relative to values of these moments based on a Fermi-gas model and monotonic central potentials. Our numerical results are not inconsistent with the most relevant experimental data. Eventually—upon refinement of the method applied here (in particular, its extension to finite systems) and upon accumulation of more extensive high-energy data—the sum rules may well provide a valuable testing ground for proposed free two-nucleon potentials and for their corresponding models of the nuclear ground state.

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