

## Optimization of water distribution for open-channel irrigation networks

Sothea Hong, Pierre-Olivier Malaterre, Gilles Belaud and Cyril Dejean

### ABSTRACT

Water distribution for open-channel irrigation networks is more and more complex due to increasing constraints on water resources and changing demand patterns, whereas the performance of such systems is expected to increase. In this regard, an optimization approach is developed in order to schedule a fair scenario of water distribution among different users, where water demand is formulated in term of start time, duration and flow rate. This study investigates how to optimize the water distribution over a finite scheduling horizon while respecting the constraints linked to the system. The optimization approach forces the scheduled start time and the volume to be closer to the demanded ones, to minimize water losses and to reduce manpower. The constraints take into account the flow routing processes, the physical infrastructure, the available water resource, and the gate keeper timetable. The numerical resolution is done by using an optimization software IBM-Ilog Cplex. The method is then illustrated with the scheduling of off-take withdrawals for a typical traditional open-channel network: a lateral canal of the Gignac canal, in southern France.

**Key words** | arranged schedule, gate operation scheduling, irrigation canal, irrigation scheduling, mixed integer linear programming, optimization, water distribution

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### INTRODUCTION

In traditional open-channel irrigation networks, the flow of water from the source to off-takes is regulated mostly by manual gates. Three main strategies to deliver water to users are found in the world and described in the literature: rotation schedule, arranged schedule and on-demand distribution (Zimbelman 1987). Rotation schedule is a method used to determine irrigation water distribution scenarios among users sequentially. The frequency, flow rate and duration of water distribution are fixed at the beginning of the irrigation season for the entire season and for all users. In this case, management is easy but water delivery rarely matches crop needs (Merriam *et al.* 2007), leading to water waste. Most of the irrigation canals in the world are still using this method, at least officially. This type of system was constructed all over the world mostly for simplicity and economical reasons.

On the other hand, on-demand distribution is the most flexible approach which allows all users, in theory (without

resource or structural constraints), to satisfy their requirements since they can take water freely. This method is used in some irrigation districts such as the Société du Canal de Provence or the Canal de la Neste in France. However, this technique cannot be applied to old systems that have been designed for upstream control (Malaterre *et al.* 1998) in particular, due to the fact that they have a small inline storage capacity. Arranged scheduling is a solution to adapt water distribution to users' requirements in systems with limited storage. In this method, the users can place orders a few days or hours in advance, and the manager tries to satisfy them as much as possible. At regular times (e.g. daily, weekly), a new water distribution scenario is arranged in order to satisfy the demands received from the users. Nowadays, this can be considered as a fair method and good compromise for water distribution. This method is used more and more in the USA, Australia and Europe. Nevertheless, the required manpower for gate operations is

considerable (Zimbelman 1987) and coping with the demand from the users is a complex task for the manager. Concomitant demands may also exceed canal conveyance capacity, and may even be impossible to satisfy in case of water shortage. This complexity is becoming very common on most irrigation canals due to the development of mixed usages (irrigation, industrial and domestic users), the diversification of crops, and the use of different irrigation techniques (gravity, sprinkler, drip). By consequence, the needs of the users become very different compared to the original ones.

Optimization is therefore a way to establish schedules taking account of the multiple constraints. Suryavanshi & Reddy (1986) introduced the 'stream tube method' to minimize water flows in lateral canals by stricter application of the duration of the irrigation turn. The objective function was further corrected by introducing a dummy variable called 'activated function' (Wang *et al.* 1995). Later, it was improved by Anwar & Clarke (2001) and De Vries & Anwar (2004, 2006) to be able to satisfy the users' request time. De Vries & Anwar (2006) included the water travel time concept in their calculation, and then could reduce iniquity between users due to flow propagation. Another technique called 'Time bloc approach' was developed by Reddy *et al.* (1999) for more flexible scheduling of water demand (time and volume) at the lateral canal level. This technique can handle the irrigation scheduling for mixed water demands where the flow, frequency and duration are different. Yet, it is a static calculation water delivery process, in the sense it does not take into account the dynamics of the flow from the upstream-end of the pools to the off-takes, or tail-end of the pools. Irrigation scheduling with flow of water through canal network and water travel time was developed by Nixon *et al.* (2001) and Alende *et al.* (2009). Their method considered the dynamic water distribution process, which was closer to real systems than static methods. These studies only considered canals without bifurcations. However, off-takes can usually be branched to every canal level (primary, lateral or distributor) as seen in Figure 1, which increases the complexity to the problem. Hong *et al.* (2012) developed an irrigation scheduling optimization method for such systems. They also considered manpower issues and possible restriction due to water shortage. Nevertheless, the duration of gate keeper travel was not explicitly taken into account, whereas this travel is essential to perform high distribution adequacy and water saving.

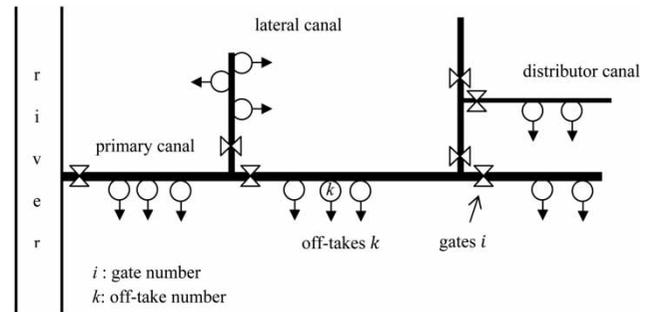


Figure 1 | Water distribution for open-channel irrigation networks scheme.

A second limitation was to consider the demanded flows as variables of the optimization problem, whereas irrigation equipment usually imposes a fixed flow rate.

The objective of this paper is to show how the water distribution optimization method of Hong *et al.* (2012) can be improved in order to take account of the gate keeper trajectory. In addition, we consider the propagation dynamics of flow by using hydraulic delay times.

The paper is structured as follows: after detailing the scheduling problem of irrigation networks, the optimization method is presented. Then, as an illustration, the method is applied to a traditional network located in Southern France.

## WATER DISTRIBUTION SCHEDULING PROBLEM

To ensure water distribution within open-channel networks, gate operation is necessary to satisfy the flow deliveries. To do so, an irrigation schedule must be defined, in which gate operations are specified. For arranged water distribution method, the off-take schedule is generally defined for 3 days, 1 week, 10 days or 2 weeks. Irrigation managers must then analyze these demands and find a way to satisfy them, as much as possible. The calculated solution must then be communicated to the users and to the gate keepers some time in advance (e.g. 1 or 2 days).

A satisfactory distribution scenario is the result of a trade-off between the user's and the irrigation manager's satisfactions. For the users, if their demands are accepted without modifications, they are then supposed to be satisfied. This requirement may imply increasing manpower for gate operations while the irrigation manager wishes to optimize this in an efficient way in order to reduce service costs. On the

other hand, limitation of manpower may imply increased water losses at the tail-end of the canal. This can occur, for example, when the water withdrawal at downstream off-takes has stopped while the upstream water delivery is still unchanged. Nevertheless, water saving is also an important issue, in particular during a restriction period.

When the supply is insufficient, there may be a need to define priorities between users. Besides, the scheduling of water withdrawal for each off-take should be arranged with a new start time and a new volume. Because of irrigation equipment constraints, the flow rate is fixed as the demanded one. The decision on volume may be between the acceptable specified ranges.

The off-takes can be linked to the canal network at any canal level (Figure 1). The distance from the upstream source to the canal tail-end can be more than several kilometers. To avoid the time mismatch between the water demand and the water supply due to the water travel time, this travel time should be taken into account. Moreover, the inflow of the system may vary during the season. The conveyance capacities are also limited by the infrastructure. So, the flow of water into the system must take into account these capacities.

In traditional systems, these operations are performed by gate keepers who need to travel between structures and open or close the gates, according to a schedule. In the past, they could work, or at least be available, almost 24 h a day, 7 days per week during the irrigation season. Gate operations at night are now considered as difficult conditions which should be avoided. Sometimes, these gate keepers have other extra-activities, such as maintenance tasks, meetings, administrative work, and personal constraints. Therefore, it is important to consider these constraints while designing irrigation schedules in order to avoid, or at least reduce, the missing gate operations and to reduce the drudgery.

## OPTIMIZATION APPROACH

The management principles presented above are formulated into an optimization problem with Boolean variables and mixed variables (integer and Boolean). Mixed integer programming (MIP) and evolutionary methods are able to handle such an optimization problem. The main advantage

of MIP is to be a deterministic algorithm, which is essential for transparency between users and manager. It was therefore largely adopted to solve the irrigation scheduling problem (Wang et al. 1995; Reddy et al. 1999; Anwar & Clarke 2001; De Vries & Anwar 2006; Alende et al. 2009; Hong et al. 2012). This algorithm requires large CPU time and hardly converges toward optimum solution when the problem size is huge. For this reason, some authors preferred using evolutionary algorithms, such as genetic algorithms (GAs) (Wardlaw & Bhaktikul 2004; Haq & Anwar 2010; Peng et al. 2012; Anwar & Haq 2013). GA is an algorithm based on techniques inspired by natural evolution, such as selection, crossover and mutation (Goldberg 1989). However, the huge size of solution space and the number of constraints makes it also a limitation for GA, and we did not find applications of GA to such large problems. We prefer to use efficient solvers for the optimization problem with linear quadratic programming and mixed integer linear programming (MILP), like Cplex and Gurobi. An MILP problem is expressed as:

$$\text{MILP: } \begin{cases} \min_X C'X \\ \text{subject to: } AX = or \sim = b \\ lb \leq b \leq ub \end{cases} \quad (1)$$

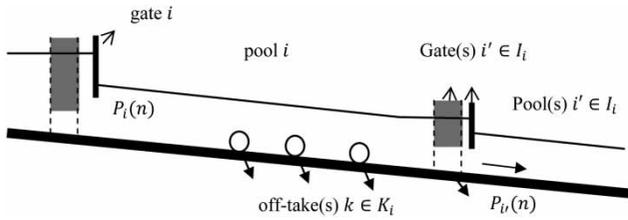
where  $X$  = set of mixed variables; Boolean, integer, and continuous variables;  $C$  = objective function vector;  $A$  = constraints square matrix;  $b$  = vector of constant terms in the constraints (right hand side);  $lb$  and  $ub$  = lower and upper bounds of variables  $X$ , respectively.

## Variables declaration and problem formulation

We use discrete-time for all state variables. Each time slot is denoted by its number  $n$ , while  $N$  is the total number of time slots ( $n \in \{1, 2, \dots, N\}$ ). The whole system is split into a series of  $N_G$  pools delimited by an upstream control gate (Figure 2). Pool  $i$  directly supplies the set of off-takes  $K_i$ , and a set of pools  $I_i$ .

The water distribution process is formulated pool by pool from upstream to downstream with the following hypotheses:

1. Inflow changes of each pool is performed only when its control upstream gate is operated. In this context, the flow perturbations of upstream and downstream pools of pool



**Figure 2** | Pool division hypothesis,  $I_i$  and  $K_i$  are the set of downstream pools and the set of off-takes linked directly to pool, respectively;  $i'$  of the set  $I_i$ .

- $i$ , due to water withdrawal or gate operation in those pools, do not affect the inflow in pool  $i$ . The inlet flow  $P_i(n)$  of pool  $i$  at a time slot  $n$  can vary only when gate  $i$  is operated at  $n$ . Conversely, if the control gate of pool  $i$  is not operated,  $P_i(n)$  is assumed to be the same as  $P_i(n - 1)$ .
- Water travel time  $\tau_i$  from upstream end (gate  $i$ ) to downstream end is fixed by the characteristics of the pool. Any off-take within the pool is assumed to be supplied with the delay  $\tau_i$  from the upstream end.
  - Poolshave no storage capacity. The inflow must be equal to outflow.

It is possible that the inflow does not match with the flow distributed at off-takes and downstream pools. The corresponding difference, which is the undistributed flow, is denoted  $L_i(n)$  for pool  $i$  and time  $n$ . This flow is lost, since there is no storage in the pool. The water losses from the pool are obtained by integrating the undistributed flow over the time, say  $\sum_{i,n} L_i(n)\delta t$ .

Gate operation is carried out by one gate keeper. We introduce the Boolean variable  $G_i(n)$  to specify the need to operate gate  $i$  at time slot  $n$ :

$$G_i(n) = \begin{cases} 1 & \text{if gate } i \text{ is operated at time slot } n \\ 0 & \text{otherwise} \end{cases}$$

The travel of the gate keeper also needs to be considered, as it must be compatible with his timetable. For operation  $m$ , the gate keeper goes from gate  $i$  to gate  $j$ . The existence of this travel is defined by the Boolean variable  $F_{i \rightarrow j}^m(n)$ :

$$F_{i \rightarrow j}^m(n) = \begin{cases} 1 & \text{if operation number } m \text{ is performed for} \\ & \text{operation of gate } j \text{ at time slot } n, \\ & \text{after operation of gate } i \\ 0 & \text{otherwise} \end{cases}$$

To take account of constraints on the labor force, for example unavailable gatekeeper at certain times, we need to consider different periods. A period  $p$  is a set of time slots when the gatekeeper is available continuously, defined as  $[t_b(p), t_e(p)]$  with  $t_b(p)$  and  $t_e(p)$  beginning and ending times of period  $p$ .

Each off-take can start only once during the schedule. We introduce the Boolean variable  $S_k$  to denote the start of functioning of off-take  $k$ :

$$S_k(n) = \begin{cases} 1 & \text{if offtake } k \text{ starts functioning at time slot } n \\ 0 & \text{otherwise} \end{cases}$$

The Boolean variable  $D_k$  is introduced to specify the state of off-take  $k$ :

$$D_k(n) = \begin{cases} 1 & \text{if offtake } k \text{ is functioning at time slot } n \\ 0 & \text{otherwise} \end{cases}$$

$D_k(n)$  must be equal to 1 during the scheduled duration of off-take  $k$ , starting at the time when  $S_k$  is equal to 1.

**Objective function**

Water distribution must satisfy as close as possible the water needs. This notion was qualified as ‘adequacy’ by Gates et al. (1991) who selected a series of performance indicators to analyze the quality of distribution in irrigation networks. They defined adequacy as the ratio between the supplied and demanded water volume. De Vries & Anwar (2004) considered adequacy in terms of irrigation start time, so that earliness and tardiness were avoided. In order to minimize the gap between scheduled and demanded time and duration, we use a combination of both concepts in the adequacy objective function, denoted  $J_1$ , which is then written as follows:

$$J_1 = \frac{1}{2} \sum_k \left\{ \frac{\alpha_k |s_k - \sum_n n S_k(n)|}{\sum_k \Delta t_k} + \frac{\beta_k q_k [d_k - \sum_n D_k(n)]}{\sum_k \Delta v_k} \delta t \right\} \tag{2}$$

where  $\alpha_k$  and  $\beta_k$  = priority coefficients linked to start time and volume, respectively;  $q_k, s_k$  and  $d_k$  = demanded flow, start time and duration of off-take  $k$ , respectively;

$\Delta t_k$  and  $\Delta v_k$  = maximum gap between scheduled and demanded start time and volume, respectively.

Any scheduled volume exceeding the demand is seen as wasteful. Therefore, the maximum scheduled volume should be equal to the demanded volume ( $q_k d_k \delta t$ ). A minimum volume should also be supplied. This minimum is defined as a proportion  $\varepsilon_k$  of the demanded volume. Then, the maximum gap between scheduled and demanded volume is:

$$\Delta v_k = (1 - \varepsilon_k) q_k d_k \delta t \tag{3}$$

Considering that the arranged schedule period ranged from 1 to  $N$ , and that  $\varepsilon_k d_k$  is the minimum scheduled duration, the start time of off-take  $k$  could be at  $n = 1$  at the earliest, and at  $N - \varepsilon_k d_k$  at the latest (Figure 3). Therefore,  $\Delta t_k$  is obtained as follows:

$$\Delta t_k = \max (s_k - 1, N - s_k - \varepsilon_k d_k) \tag{4}$$

A second objective is to minimize water losses. Efficiency is commonly used as a performance indicator to quantify the waste of water within irrigation systems (Gates et al. 1991). Network efficiency can be defined as the ratio of supplied volume to diverted volume into the system. Maximizing the efficiency is therefore equivalent to minimize water losses. Then, the second objective function  $J_2$  is:

$$J_2 = \frac{\sum_{i,n} L_i(n)}{\sum_n r(n)} \tag{5}$$

where  $r(n)$  is the total available flow at time slot  $n$ .

The last objective concerns the manpower. As mentioned earlier, the duration dedicated to gate keeper work must be minimized. We need to determine this duration through the trajectory between successive positions (gate  $i$  to  $j$ ). This issue is referred to as the ‘Travelling Salesman’ problem (Verfaillie 2008; Fouilhoux 2011). In that problem, the trajectory from town to town was selected in order to find the shortest

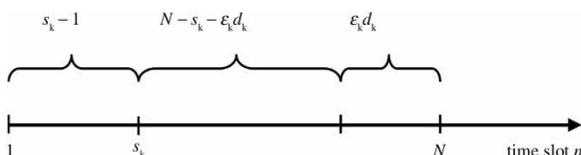


Figure 3 | The worst scheduled start time of off-take calculation schematic.

total distance of the trip. For the water distribution problem, we consider the total time of gate operation and travel from gate to gate ( $i$  to  $j$ ) instead of distance. Using the Boolean variable  $F_{i \rightarrow j}^m(n)$  denoting gate keeper trajectory, the criterion on gate operation is written as:

$$J_3 = \sum_{i,j,m,n} \frac{\varphi_{i \rightarrow j} F_{i \rightarrow j}^m(n)}{\psi} \tag{6}$$

where  $\psi$  denotes the predicted duration of gate keeper work before optimization for the whole arranged schedule, and  $\varphi_{i \rightarrow j}$  is the matrix duration of gate keeper travel from gate  $i$  to gate  $j$ , plus operation of gate  $j$ .

We finally obtain the objective function  $J$  by summing the three objective functions  $J_1, J_2$ , and  $J_3$  with penalty costs  $w_f$ :

$$J = \sum_f w_f J_f \tag{7}$$

Penalty costs  $w_f (f = 1, 2, 3)$  are chosen so that  $\sum w_f = 1$ . Since  $0 \leq J_f \leq 1$ , we also find that  $J$  is bounded by 0 and 1 ( $J \in ]0, 1[$ ) (1 corresponds to the worst distribution scenario). This is called the weighting optimization technique which is commonly used (Anwar & Clarke 2001; De Vries & Anwar 2004, 2006; Haq & Anwar 2010; Anwar & Haq 2013). Nevertheless, the multi-objective optimization approach used by Babel et al. (2005), Alfonso et al. (2010), and Peng et al. (2012) is a helpful optimization technique to make a better decision. Therefore, we will carry out this approach in a future work.

### Constraints

The constraints concern the water delivery process, gate operation, gate keeper trajectory, and water distribution policies. These constraints are formulated below.

### Water delivery process

Based on hypothesis #1, we can write the constraint to determine the gate operation needs as below:

$$G_i(n) = 0 \rightarrow P_i(n) - P_i(n - 1) = 0, \forall i \text{ and } \forall n \geq 2 \tag{8}$$

This flow must be lower than the pool conveyance capacity. It is also limited by the available resource, so

$$P_i(n) \leq \min [c_i, r(n)], \forall i \text{ and } n \tag{9}$$

The other hypotheses imply a relationship between inflow, distributed and undistributed flows, by taking account of the hydraulic delay  $\tau_i$  (which is rounded up as a number of time slots) within the pool. Since pool  $i$  supplies the set of pools  $I_i$  and the set of off-takes  $K_i$  the water balance writes as:

$$P_i(n - \tau_i) = \sum_{k \in K_i} q_k D_k(n) + \sum_{i' \in I_i} P_{i'}(n) + L_i(n), \forall i \text{ and } n, n - \max(\tau_i) \geq 1 \tag{10}$$

**Gate operation and gate keeper trajectory**

A flow change in pool  $i$  is possible only if gate  $i$  has been operated (Equation (8)). To respect the gate keeper timetable, all Boolean variables describing gate movements are set to zero outside of the gate keeper work period. This writes as:

$$\sum_{i,n} G_i(n) = 0, n \notin [t_b(p), t_e(p)] \tag{11}$$

The duration of gate keeper travel and gate operation are related to departure gate and destination gate, which

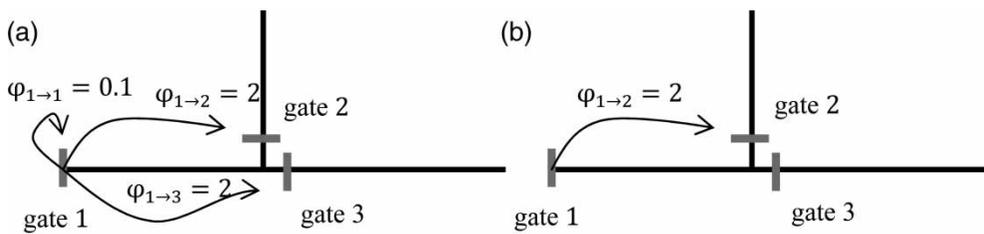
we call trajectory. Setting gate keeper trajectory from gate ( $i$ ) to gate ( $j$ ) is more complex than for the ‘Travelling Salesman’ issue. The salesman must go across all towns only once, whereas a gate can be operated several times. Indeed, operation of gate  $j$  can be done at any time slot ( $n' = \{n, n + 1, \dots, N\}$ ) after operation of gate  $i$  at time slot  $n$ . Thus the trajectory from gate  $i$  at time slot  $n$  has  $N_G(N - n + 1)$  possibilities ( $N_G$  is number of gates).

To illustrate the notation, we take an example with three gates ( $N_G = 3$ ) as shown in Figure 4, four time slots ( $N = 4$ ) and two gate operations. The first operation ( $m = 1$ ) is accomplished for gate 1 at time slot 1, so  $G_1(1) = 1$ . For the next operation ( $m = 2$ ), there will be  $3(4 - 1 + 1)$  possible trajectories. If gate 2 is chosen as the destination for the next operation, then the next step is to determine time slot  $n'$  when gate 2 could be accomplished. As  $\varphi_{1 \rightarrow 2} = 2$ , the gate keeper is still travelling at time slot 2, so  $F_{1 \rightarrow 2}^2(1)$  and  $F_{1 \rightarrow 2}^2(2)$  must be equal to 0. Only  $F_{1 \rightarrow 2}^2(3)$  and  $F_{1 \rightarrow 2}^2(4)$  could be equal to 1.

As  $G_i(n)$  does not naturally include the operation number, we introduce an auxiliary variable  $E_i^m(n)$  to formulate the link between the operation number and operated gate.

$$E_i^m(n) = \begin{cases} 1 & \text{if operation number } m \text{ concerns gate } i \text{ at time slot } n \\ 0 & \text{otherwise} \end{cases}$$

$E_i^m(n)$  plays the role as mediator between  $G_i(n)$  and  $F_{i \rightarrow j}^m(n)$ .  $E_i^m(n)$  is related to  $G_i(n)$  after operation of gate  $i$



Departure situation: gate ( $i = 1$ )

Travel and arrival situation: gate ( $j = 2$ )

operated at time slot ( $n = 1$ ) for operation

is selected. The arrival time is at least equal

number ( $m = 1$ ). The destination for next

to  $1 + \varphi_{1 \rightarrow 2}$ .

operation ( $m = 2$ ) is searched (gate 1, 2

or 3).

Figure 4 | Example of gate keeper trajectory for successive operations.

through the equation below:

$$G_i(n) = \sum_m E_i^m(n), \forall i \text{ and } n \quad (12)$$

To define the trajectory from  $i$  at time slot  $n$  to  $j$  at time slot  $n'$  for the operation number  $m$ , we write:

$$E_i^{m-1}(n) - \sum_{j,n'} F_{i \rightarrow j}^m(n') \leq 0, \forall i, m \geq 2 \text{ and } n; \\ n' = \{n, n+1, \dots, N\} \quad (13)$$

where  $n'$  is the time slot numbers starting from  $n$  when gate ( $i$ ) is operated. Then, when the gate keeper arrives at the destination gate  $j$  and this gate is decided to be operated,

$$E_j^m(n) = \sum_i F_{i \rightarrow j}^m(n), \forall j, m \text{ and } n \quad (14)$$

The duration between two operations ( $m-1$  and  $m$ ) must be greater or equal to the duration of the travel  $\varphi_{i \rightarrow j}$  between the operated gates  $i$  and  $j$ . For all  $\varphi_{i \rightarrow j} > 1$ , this condition writes as:

$$- \sum_{i,n} n E_i^{m-1}(n) + \sum_{j,n} n E_j^m(n) \\ \geq \sum_{i,j,n} \varphi_{i \rightarrow j} F_{i \rightarrow j}^m(n), \forall m \geq 2 \text{ and } \varphi_{i \rightarrow j} > 1 \quad (15)$$

If  $\varphi_{i \rightarrow j} \leq 1$ , several gate operations could be performed during one time slot. To respect this condition, we write:

$$\sum_{i,j,m} \varphi_{i \rightarrow j} F_{i \rightarrow j}^m(n) \leq 1, \forall n \text{ and } \varphi_{i \rightarrow j} \geq 1 \quad (16)$$

### Water distribution policies

Each off-take starts functioning only once during the arranged schedule. Therefore, when  $S_k(n)$  gets 1,  $D_k(n)$  also gets 1 and must continue until the end of functioning:

$$\begin{cases} \sum_n S_k(n) = 1, \forall k \in K \\ S_k(n) = D_k(n), \forall k \in K \text{ and } n = 1 \\ S_k(n) = 0 \rightarrow D_k(n-1) - D_k(n) \geq 0, \forall k \in K \text{ and } n \geq 2 \end{cases} \quad (17)$$

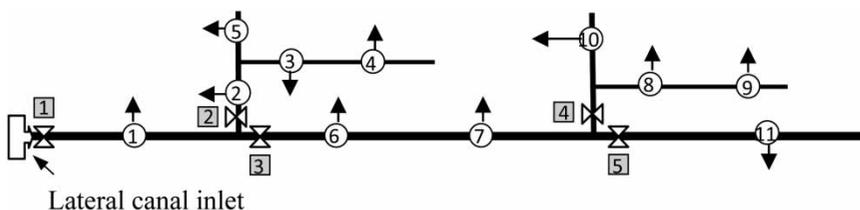


Figure 5 | Schematic representation of the studied irrigation network.

The water withdrawal from off-take  $k$  must be completed before the end of the schedule period, say

$$\sum_n n S_k(n) + \sum_n D_k(n) \leq N, \forall k \in K \quad (18)$$

In addition, the scheduled volume must be between the lower bound and demanded volume. Since  $q_k$  is constant, the off-take volume constraint is written only with duration as below:

$$\varepsilon_k d_k \leq \sum_n D_k(n) \leq d_k, \forall k \in K \quad (19)$$

## APPLICATIONS

The method developed and presented in the previous section is applied to a command area of a secondary irrigation canal of the Gignac Canal (South of France) for scheduling water users' demands. This sector, located close to the downstream end of the right bank main canal, has about 130 ha of irrigated land, mainly vineyards. Nowadays, this command area is in the complex situation of diversified crop patterns, mixed uses (farming and urban), different irrigation techniques (furrow and drip irrigation) and mixed distribution scheduling (rigid rotation and arranged schedules). In addition, it faces water scarcity in July and August. This sector has 201 off-takes including nine urban off-takes.

### Problem declaration

We illustrate our method (Figure 5) on a simplified sub-system within this command area. The selected network has 11 off-takes to be supplied during a 12 h period. The irrigation starts at 8:00 and finishes at 20:00. In this example, five gates are operated manually by a gate keeper during

**Table 1** | Off-take numbers linked to the corresponding gate, priority coefficients, irrigation start times, durations and flows for each off-take

<i>k</i>	<i>i</i>	$\alpha$	$\beta$	$\varepsilon$	<i>S</i>			<i>d</i> (min)	<i>q</i> (l/s)
					Day	Hour	Minute		
1	1	1	1	1	1	8	0	60	30
2	2	1	1	0.5	1	8	0	180	40
3	2	1	1	0.75	1	12	0	120	40
4	2	1	1	0.75	1	10	0	60	20
5	2	1	1	0.5	1	11	0	60	30
6	3	1	1	0.75	1	10	0	60	30
7	3	1	1	1	1	18	0	50	50
8	4	1	1	0.5	1	15	0	60	30
9	4	1	1	0.5	1	16	0	60	30
10	4	1	1	0.5	1	17	0	60	30
11	5	1	1	0.75	1	13	0	300	35

two possible intervention periods: (1) from 8:00 to 12:00, and (2) from 14:00 to 20:00.

The requested water demand is presented in Table 1 (using the numbering of off-takes and gates defined in Figure 5).

We consider in this example that all off-takes have the same priorities. The priority coefficients we introduce in the formulation are therefore set to be equal ( $\alpha_k = \beta_k = 1, \forall k$ ). The available inflow discharge at the upstream-end of the lateral canal is  $r = 70 \text{ L/s}$  from 8:00 to 20:00. Water travel time from upstream to downstream pools and pool conveyance capacities are given in Table 2.

The durations of the gate keeper travels from gate to gate and gate operations are reported in Table 3 based on time slots of 30 min.

This example illustrates a case where the total demanded volume is smaller than the available one and the total

**Table 2** | Water travel time (from upstream to downstream end of the pool) and conveyance capacity

Pool <i>i</i>	$\tau$ (min)	<i>c</i> (L/s)
1	15	100
2	30	70
3	20	70
4	20	35
5	10	35

**Table 3** | Durations of gate keeper travels and gate operations ( $\varphi_{i-j}$ ) in time slots of 30 min

Gate ( <i>i</i> / <i>j</i> )	1	2	3	4	5
1	0.16	0.36	0.36	0.66	0.66
2	0.36	0.16	0.16	0.46	0.46
3	0.36	0.16	0.16	0.46	0.46
4	0.66	0.46	0.46	0.16	0.16
5	0.66	0.46	0.46	0.16	0.16

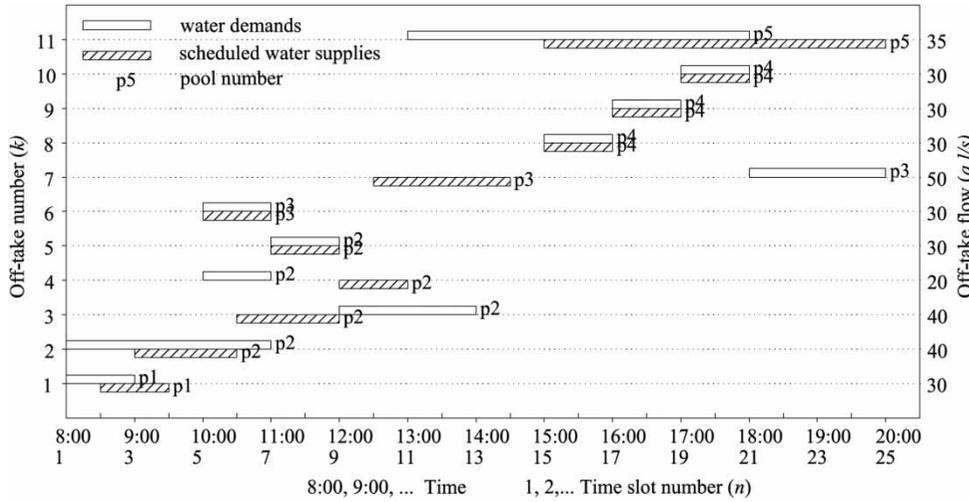
For example,  $\varphi_{1-2} = 0.36$  time slot, signifies that the gate keeper need  $0.36 \times 30 \text{ min} = 11 \text{ min}$  to travel from gate 1 to gate 2 and to operate the gate 2 ( $\delta t = 30 \text{ min}$ ).

demanded flow is larger than the available in flow ( $r = 70 \text{ L/s}$ ). The problem then is how to shift these demands in an optimal way. Our method can be applied to such a problem to find a new optimal schedule for the water distribution to the off-takes and for the gate operations.

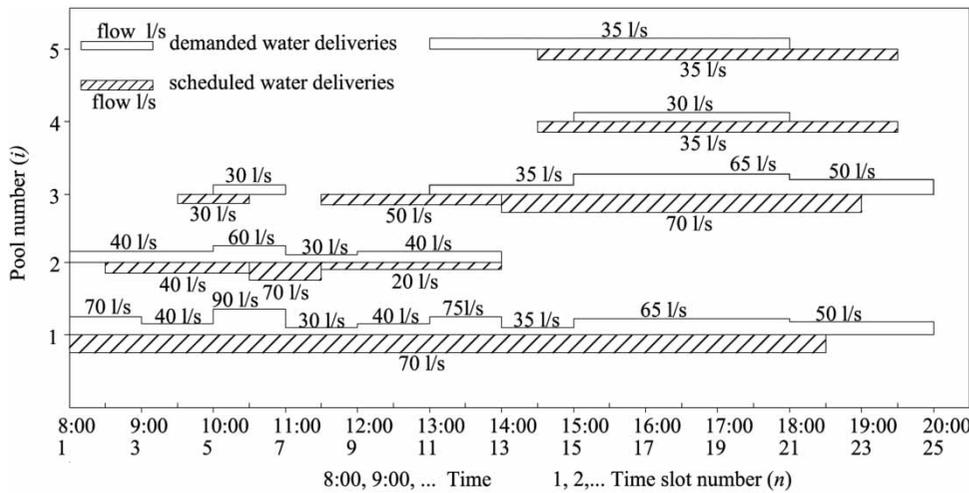
### Computation process and solution

The computation was carried with a computer based on an Intel(R) Core (TM) i7-2600 CPU@3.4 GHz RAM 8Go. Equations (7)–(19) were transformed into a matrix formulation as Equation (1) through the Matlab software. The IBM-Ilog Cplex software was called to find an optimum solution using MILP algorithm. For 11 off-takes ( $k = 1, \dots, 11$ ) during a time horizon of 12 h, giving 24 time slots of 30 min ( $n = 1, \dots, 24$ ), and 5 gates ( $i = 1, \dots, 5$ ) with 20 available operations ( $m = 1, \dots, 20$ ), the problem has 15,288 variables (15,048 Boolean and 240 integers) and 5,402 constraints to be solved. For such a problem, the solution converges after 85,754 s of CPU time with 28.95% of gap. The gap is calculated as (best integer – best node) / (abs (best integer) +  $1e-10$ ). The objective function value (*J*) is found to be 0.29 (by considering  $w_f = \frac{1}{3}, \forall f$ ).

The obtained optimized irrigation schedule (Figure 6) shows that the off-take # 5, 6, 8, 9 and 10 perform their operation at the right time and with the right quantities. Off-takes # 2 and 3 obtain less volume than their requests. The smaller available inflow rate compared to the demanded one, as shown in Figure 7 (between 10:00 to 11:00 and 13:00 to 14:00), implies that some off-takes are scheduled to be supplied later than demanded, such as off-takes # 1, 2, 4 and 11, and some others earlier (off-takes # 3 and 7).



**Figure 6** | Demanded and scheduled water for the different off-takes (time, duration and flow). There is no scheduled flow in this example. The time slot starting at 1 corresponds to 8:00 in real time, 2 corresponds to 8:30 and so on.

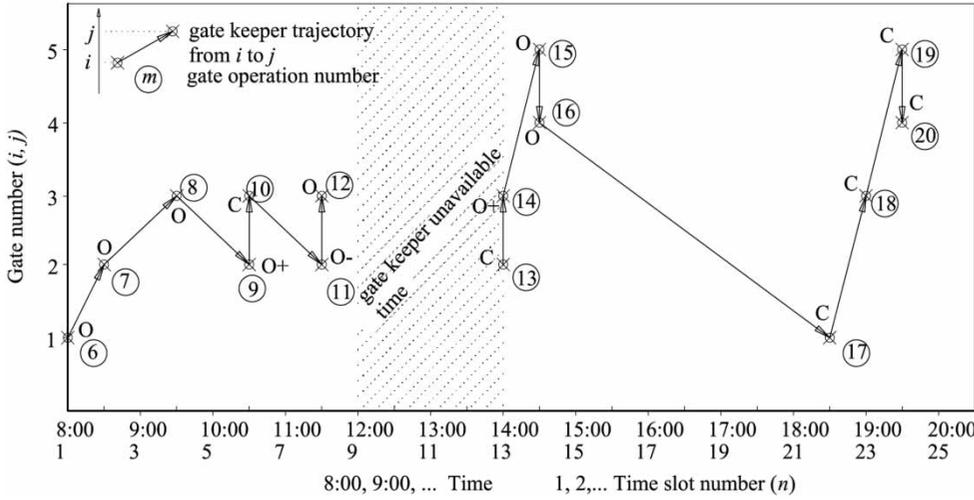


**Figure 7** | Demanded and scheduled water distribution for the pools. Pool #1 is at the head of the system. The total water distribution must be equal to the inflow of this pool. The flow passing through this pool supplies two pools (#2 and 3) and one off-take (#1). The pool #3 supplies water to the pools #4 and 5, and to off-takes #6 and 7.

Figure 7 shows the demanded and the scheduled inflows for each pool. The demanded inflow has many perturbations while the scheduled one has less. These are due to the objective of minimizing the duration of the gate keeper work and the limitation of the gate keeper available time. Moreover, as the water travel time is taken into account in the method, the off-takes start their functioning a time-lag ( $\tau_i$ ) later (Figure 6) compared to the inflow of the corresponding pool (Figure 7). For instance, the inflow of pool #2 begins at time slot 2 while the off-take #2 takes

water from pool #2 at time slot 3. This is caused by the water travel time  $\tau_2 = 1$  time slot.

The gate keeper trajectory is shown in Figure 8. The maximum number of gate operations is set to 20 operations. The scheduled number is found to be 15, starting at operation #6. The first five other gate operation numbers ( $m = 1 - 5$ ) are not performed. During 12:00 to 14:00, the gate keeper has no time available to operate the gates. For this reason, there is no scheduled operation during this time.



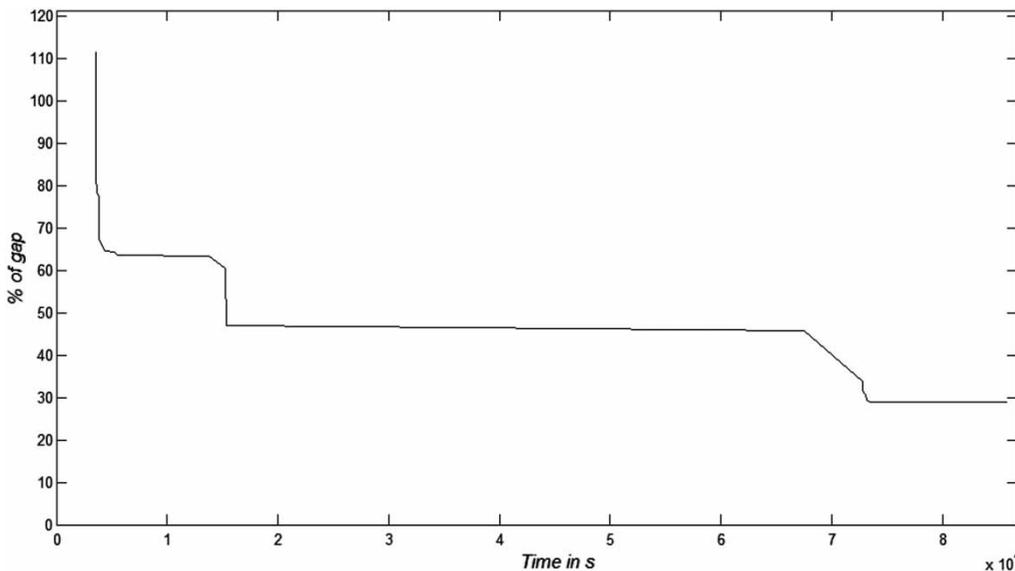
**Figure 8** | The gate keeper trajectory and gate operations resulting from the optimization. O refers to open gate, O+ moves up (increases the opening compared the previous state), O- moves down and C closes the gate completely.

**DISCUSSION**

In this section, we want to further analyze the computation time, the satisfaction of the users and of the manager. We also want to study the sensitivity of the objective function with reference to the penalties cost.

**Computation time**

The solver uses Branch and Cut method for discrete optimization problem and the simplex method for the real variables relaxation. After 3,602 s of CPU time, the first integer solution is found (111.22% of gap). Figure 9 shows the improvement of the optimal solution as a function of the CPU time. The



**Figure 9** | Improvement of the solution as a function of the CPU time.

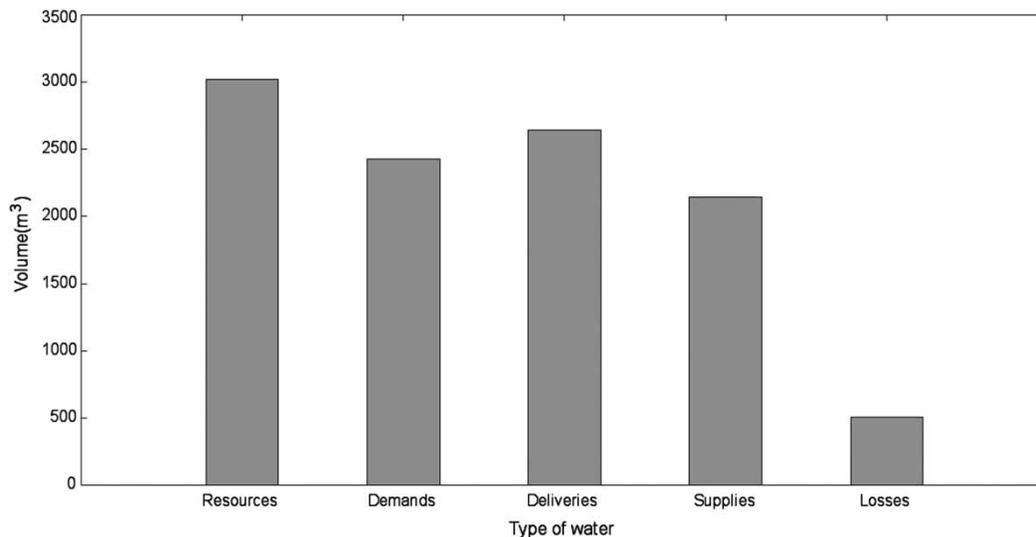
intermediate solution corresponding to 46.92% of gap is then hardly improved for a long time. The difficulty comes from the constraints imposed by the gate keeper trajectory issue. The huge numbers of constraints are due to time discretization and gate operation numbers. The gate operation numbers are needed to allow the taking into account of the gate operation constraints. These constraints must allow several operations in the same time slot (time step). It may not be necessary if the duration of time slot ( $\delta t$ ) is set to the smallest duration of gate keeper travel and gate operation from one gate to another one. In contrast, a small value of the time slot ( $\delta t$ ) increases the problem size, due to the larger size of the time horizon  $N$ .

### Performance analysis

To analyze the results of the optimization, we use two indicators from Gates *et al.* (1991): adequacy and efficiency. We introduce two other indicators linked to the start time adequacy and to the gate operations.

The volume adequacy ( $I_{VA}$ ) is calculated by dividing the total scheduled volume (result of optimization) by the total demanded volume.

$$I_{VA} = \frac{\sum_{k,n} q_k D_k(n) \delta t}{\sum_k q_k d_k \delta t} \quad (20)$$



**Figure 10** | Comparison of total water distribution with equal penalty cost for the objective function: 'Resources' refers to available volume resource, 'Demands' refers to off-take demanded volume, 'Deliveries' refers to volume diverted to system, 'Supplies' refers to off-take scheduled volume, 'Losses' refers to water losses.

The start time adequacy ( $I_{SA}$ ) is also used to check how the solution obtained from the optimization satisfies the demand. It is estimated as the complement to 1 of the second objective function  $J_2$  (Equation (3)):

$$I_{SA} = \left( 1 - \sum_k \frac{|S_k - \sum_n n S_k(n)|}{\sum_k \Delta t_k} \right) \quad (21)$$

The efficiency criterion of Gates *et al.* (1991) is used in our analysis as the water losses indicator ( $I_{WL}$ ). Therefore, the water losses indicator is computed by:

$$I_{WL} = \frac{\sum_{i,n} L_i(n)}{\sum_{i,n} P_i(n)} \quad (22)$$

The gate operation is analyzed by comparing the maximum number of operations that we allow in the optimization problem, with the scheduled number of gate operations (result of optimization):

$$I_{GO} = \frac{\text{scheduled number of gate operation}}{\text{maximum number of gate operation}} \quad (23)$$

Figure 10 shows the total volumes of: available water resources at the upstream end of the canal, the water demands, deliveries, supplies at off-takes, and losses. We

note that the available water resource is larger than the demanded one, but the scheduled volume is still less than the demanded one. The adequacy is 88 and 87% for volume and start time, respectively (Table 4). The low adequacy is caused by the available flow constraint (limited to 70 L/s), while the demanded flow was requested for 90 L/s from 10:00 to 11:00 and 75 L/s from 13:00 to 14:00. The water losses indicator is 19% of the total distributed volume, while 75% of gate operations are needed to perform the required flow changes (Table 4). The water losses could be reduced if the number of gate operations was increased. To achieve that, the penalty cost of water losses objective function must be increased and the one of the gate operations must be decreased. For this scenario, we considered equal penalty costs for each terms of the objective function.

### Sensitivity analysis to the penalty costs

Penalty costs of the objective function were introduced to set priorities between sometimes antagonistic criteria: water saving, adequacy and manpower. It is interesting to study the sensitivity to these penalty costs, and to analyze how they are able to change the optimized schedule. We consider the same optimization problem as above, but with different values of each penalty cost ( $w_f$ ) as shown in Table 4.

For optimizations #1, 4 and 7, we give the priority to the adequacy objective function ( $w_1$ ). The adequacies in terms of volume and start time are found to be better than with other optimizations. In the same way, when we set the priority to one of the three objective functions, the indicator

value linked to that criterion is improved. This is not a surprising conclusion, but our optimization program allows quantifying the possible improvements for each option.

We note that the minimum water loss is 12%. It could be reduced even more if the gate operation constraint was relaxed. The maximum water loss is limited to 37%. This limitation comes from the available water resource and the minimum volume constraint of each off-take expressed by a proportion coefficient  $\varepsilon$  (Table 1). Also, the gate operation constraints (gate keeper available time and operations number) implies a limited maximum adequacy of 88 and 89% in terms of volume and start time respectively, even when we set the penalty cost of the adequacy objective function to one and the others to zero. To sum up, it is clear that the changes of the penalty cost can drastically influence the solution. By choosing appropriate penalty coefficients, the canal manager has the possibility to select a schedule that fulfills his priorities.

### CONCLUSIONS

This paper introduces a new method for the scheduling of water distribution scenarios for heterogeneous open-channel networks. The proposed technique used MILP algorithm with objective functions based on user adequacy, water distribution cost linked to manpower and water losses. The constraints include water distribution policies scheduling and hydraulic characteristics such as water travel time, canal conveyance capacity, and available water resource. This optimization procedure allows a flexible decision making on irrigation start time and volume, as close as possible to the demands. It enables taking into account a priority concept between optimization objective and between users. Moreover, it can explicitly take into account the work of the gate keeper by optimizing his work duration and selecting the best trajectory between the gates that need to be operated.

This initial formulation is limited to strong hydraulic hypothesis, and small scale systems due to computation time and one gate keeper working in the system. The problem size relies largely on the number of time slots and on the number of gate operations. This increases the problem size because of the  $N$  times repeating for each operation.

**Table 4** | Result of the optimization using different values of the penalty costs

Opt. #	$w_1$	$w_2$	$w_3$	Obj. value	$I_{VA}(\%)$	$I_{SA}(\%)$	$I_{GO}(\%)$	$I_{WL}(\%)$
1	1	0	0	0.51	88	89	100	28
2	0	0	1	0.14	74	72	55	37
3	0	1	0	0.08	85	74	100	12
4	20/30	0	10/30	0.39	88	89	60	32
5	10/30	20/30	0	0.24	88	87	100	15
6	0	10/30	20/30	0.19	85	74	85	16
7	15/30	10/30	5/30	0.35	88	89	90	22
8	10/30	10/30	10/30	0.29	88	87	75	19

Therefore, future works will address this issue by reducing the number of unused variables and constraints through considering gate operation time as a variable. This solution may help to clean up  $n$  from  $F_{i,j}^m(n)$  that significantly increases the size of the problem. Another perspective is to reduce the total number of variables by decomposing the whole problem into smaller size problems. On the other hand, we will improve the hydraulic hypothesis by considering flow perturbation of each pool associated not only with water withdrawal, and gate operation at upstream and downstream ends, but also to flow perturbation of the upstream pool. Besides, the method is based on one gate keeper, and used the single optimization technique to solve the problem. Thus, we will also address the case when there are several gate keepers, and a multi-criteria optimization technique.

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