Clusters of galaxies with modified Newtonian dynamics

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ABSTRACT
I consider X-ray emitting clusters of galaxies in the context of modified Newtonian dynamics (MOND). Self-gravitating isothermal gas spheres are not good representations of rich clusters; the X-ray luminosity at a given temperature is typically an order of magnitude larger than observed, and the predicted X-ray surface brightness distribution is not well-matched by the standard ‘β-model’ fits to the observations. Pure gas spheres with a density distribution described by a β-model also fail because, with MOND, these objects are far from isothermal and again overluminous. These problems may be resolved by adding an additional dark mass component in the central regions, here modelled by a constant density sphere contained within two core radii and having a mass typically of one to two times the total cluster mass in the gas. With this additional component, the observed luminosity–temperature relation for clusters of galaxies is reproduced, and the typical mass discrepancy in actual clusters is three to four times smaller than implied by Newtonian dynamics. Thus, while MOND significantly reduces the mass of the dark component in clusters it does not remove it completely. I speculate on the nature of the dark component and argue that neutrinos, with mass near the experimental upper limit are a possible candidate.

Key words: gravitation – galaxies: clusters: general – dark matter – X-rays: galaxies: clusters.

1 INTRODUCTION
The modified Newtonian dynamics (MOND) is an empirically-based modification of Newtonian gravity or inertia in the limit of low accelerations (\(<a_0 \approx cH_o\)) suggested by Milgrom (1983a,b,c) as an alternative to cosmic dark matter. In addition to explaining galaxy scaling relations (Tully–Fisher, Faber–Jackson, fundamental plane), this simple algorithm allows us to accurately predict the shapes of spiral galaxy rotation curves from the observed distribution of gaseous and stellar matter. MOND also accounts for the kinematics of small groups of galaxies (Milgrom 1998) and of superclusters, as exemplified by the Perseus–Pisces filament (Milgrom 1997) without the need for unseen mass. These well-documented phenomenological successes (Sanders & McGaugh 2002, and references therein) challenge the cold dark matter (CDM) paradigm and provide some support for the suggestion that the current theory of gravity and inertia (general relativity) may need revision in the limit of low accelerations or field gradients.

However, problems do arise when we attempt to apply MOND to the large clusters of galaxies. The & White (1988) first noted that, to successfully account for the discrepancy between the observed mass and the traditional virial mass in the Coma cluster, the MOND acceleration parameter, supposedly a universal constant, should be about a factor of 4 larger than the value implied by galaxy rotation curves. With MOND, the dynamical mass of a pressure-supported system at temperature \(T\) is \(M \propto T^2/a_0\); therefore, the result of The & White (1988) could also be interpreted as an indication that the MOND dynamical mass is still larger than the detectable mass in stars and gas.

In astronomical tests involving an individual extragalactic object, such as the Coma cluster, a contradiction is not necessarily a falsification. We can always argue that the peculiar aspects of an object, such as deviations from spherical symmetry or incomplete dynamical relaxation, exempt that particular case. However, Gerbal et al. (1992), looking at a sample of eight X-ray emitting clusters, noted that the problem is more general; although MOND reduces the Newtonian discrepancy by a factor of 10, there is still a need for dark matter, particularly in the central regions. Later, considering a large sample of X-ray emitting clusters, I found that the mass predicted by MOND remains, typically, a factor of 2 or 3 times larger than the total mass observed in the hot gas and in the stellar content of the galaxies (Sanders 1999). More recently, Aguirre, Schaye & Quataert (2001) pointed out that MOND is inconsistent with the observed temperature gradient in inner regions of three clusters for which such data are available. Again, the problem can be remedied by additional non-luminous mass, primarily in the inner regions, of the order of two or three times the observed gas mass. This discrepancy is also
evident from strong gravitational lensing in the central regions of clusters – the formation of multiple images of background galaxies. Here, MOND does not apply because accelerations are Newtonian, and the implied surface density greatly exceeds that of visible matter and hot gas (Sanders 1999). So, although MOND clearly reduces the classical Newtonian mass discrepancy in clusters of galaxies, there still remains a missing mass problem, particularly in the cores.

Here I reconsider the issue of rich clusters and whether or not these systems represent a fundamental problem for MOND. First of all, it is evident that MOND self-gravitating isothermal gas spheres are not good representations of clusters of galaxies. The predicted radial dependence of X-ray surface brightness does not reproduce that observed in X-ray emitting clusters – observations which are well fit by the traditional ‘β-model’ (Sarazin 1988). Moreover, the predicted X-ray luminosity at a given temperature is typically an order of magnitude higher than observed. On the other hand, pure gas spheres with a density distribution described by a β-model also fail. These objects are far from isothermal in the context of MOND, but the most nearly isothermal models are larger, in terms of core radius, and overluminous with respect to observed clusters of the same mean temperature.

I find that these problems may be solved by postulating the existence of a second rigid component in the mass distribution, here modelled as a constant density sphere having a radius about twice that of the β-model core and a central surface density comparable to \( a_o / G \). By rigid, I mean a component with a fixed density distribution which does not respond to the gravitational field of the hot gas or galaxies. The presence of such a constant density dark component in the Coma cluster is implied by the MOND hydrostatic gas equation for cumulative mass, and is consistent with a near isothermal β-model. If this component is generally present in clusters, it contributes to the gravitational force in the inner regions and reduces the core radius at a given temperature. In this way the observed luminosity–temperature relation for clusters is reproduced.

About 40 individual clusters have been considered in terms of such a two-component model; i.e. the observed surface-brightness distributions and mean temperatures are fit by specifying the density of the non-luminous component which, in all cases, is assumed to extend to two gas core radii. The total mass of this additional component varies between a few times \( 10^{15} \) and \( 10^{14} M_\odot \) and the implied mass-to-light ratio is typically 50 in solar units. Therefore, the required rigid component is not a standard stellar population. Although the total discrepancy between dark and detectable mass is reduced by, on average, a factor of 4 over that implied by Newtonian dynamics, it is clear that there remains a major dark matter problem for MOND in the central regions of most rich clusters. This is the point stressed by Aguirre et al. (2001). Here, I discuss the issue of whether or not this is a falsification. I speculate on the possible nature of the non-luminous component and argue that neutrinos of finite mass are a possible candidate.

2 THE STRUCTURE AND PROPERTIES OF MOND GAS SPHERES

2.1 MOND isothermal spheres

The structure of isotropic gas spheres may be determined by solving the equation of hydrostatic equilibrium

\[
\frac{dp}{dr} = -\rho g
\]  
\((1a)\)

with the pressure \( p \) given by

\[
p = \rho \sigma^2
\]  
\((1b)\)

where \( \sigma \) is the one-dimensional velocity dispersion (constant for an isothermal sphere), and \( \rho \) is the density. The gravitational acceleration, \( g \), in the context of MOND, is given by

\[
g_M(g/a_o) = g_n
\]  
\((2a)\)

where \( g_n \) is the usual Newtonian gravitational acceleration,

\[
g_n = \frac{GM(r)}{r^2}
\]  
\((2b)\)

\( a_o \) is the presumably universal constant of acceleration below which gravity deviates from Newtonian (found to be about \( 10^{-8} \) cm s\(^{-2}\) from fits to galaxy rotation curves), and \( \mu(x) \) is a function which interpolates between the Newtonian regime (\( \mu(x) = 1 \) when \( x \gg 1 \)) and the MOND regime (\( \mu(x) = x \) when \( x \ll 1 \)). A function having this asymptotic form,

\[
\mu(x) = x(1 + x^2)^{-1/2}
\]  
\((2c)\)

works well for galaxy rotation curves and is also used here.

Milgrom (1984) demonstrated that MOND isothermal spheres have a finite mass and a density which falls off as \( r^{-\alpha} \) where \( \alpha \approx 4 \) at large radii. In the outer regions, where \( M(r) = M = \) constant, it follows immediately from equations (1) and (2) that

\[
\sigma^4 = \alpha^{-2}GMa_o
\]  
\((3)\)

which means that the mass is uniquely determined by the specified velocity dispersion or temperature. For an isothermal gas sphere this relation becomes

\[
M \approx 16 \frac{GM a_o}{G v_0} \left( \frac{kT}{f m_p} \right)^2 = (2.9 \times 10^{15}) T_{a_o}^{-2} M_\odot
\]  
\((4)\)

where \( f \) is the mean atomic weight (\( \approx 0.62 \) for a fully ionized gas with solar abundances) and \( m_p \) is the proton mass.

This would be, in effect, the extension of the Faber–Jackson relation to clusters of galaxies (Sanders 1994). However, the observational definition of such a relation is ambiguous because the total gas mass defined by the β-model is typically divergent. If we consider the mass inside a fixed radius (Sanders 1994), or within a radius where the density falls to some fixed value (Mohr, Mathiesen & Evrard 1999), a relationship of this form (equation 4) is observed for clusters, but the mass at a given temperature is typically a factor of 5–10 below that implied by equation (4).

Although the mass of an isotropic, isothermal sphere is effectively determined by the temperature, the detailed structure depends upon the central density; for a single temperature there is a family of solutions bounded by a limiting solution with a \( 1/r \) density cusp at the centre (Milgrom 1984). The limiting MOND solution resembles the Hernquist model (Hernquist 1990) with the \( 1/r \) cusp steepening to \( 1/r^4 \) beyond a radius \( r_m \approx \sigma^2 / a_o \). Lower density spheres are characterized by a constant density core with a \( 1/r^4 \) gradient at large \( r \); the central gas densities observed in clusters of galaxies would place these objects in this category of the sublimiting solutions. The X-ray surface brightness distribution resulting from such a MOND isothermal sphere is shown in Fig. 1 compared to that observed for the Coma cluster (Briel, Henry & Böhringer 1992). These observations are well fit by the traditional β-model (Cavaliere & Fusco-Femiano 1976; Sarazin 1988)

\[
I(r) = I_o \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right]^{-3\beta+1/2}
\]  
\((5)\)

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dependence shallower than observed, but the predicted luminosities are at least an order of magnitude larger than actual clusters at the same temperature.

2.2 MOND $\beta$-models

The radial dependence of electron number density which produces the X-ray intensity distribution described by equation (5) is

$$n_e = n_o \left[ 1 + \left( \frac{r}{r_c} \right)^{2\beta - 1} \right]$$

(Cavaliere & Fusco-Femiano 1976). Rather than assuming a constant temperature and solving equation (1) for the density distribution, we may alternatively take equation (6) as the density distribution and solve equations (1) and (2) for the radial dependence of the temperature. It is necessary to specify $\beta$ (typically 0.6–0.7), a central electron density, $n_o$ (of the order of a few times $10^{-3} \text{ cm}^{-3}$), a core radius, $r_c$ (ranging from 50 to 300 kpc), and a central gas temperature, $T_o$. Because the $\beta$-model does not have an outer boundary, we usually solve the structure equations out to a radius where the gas density falls to some fixed multiple of the mean cosmological density, in this case 250 times the mean cosmic baryonic density.

For a given central temperature, the run of temperature is completely determined by the core radius. There is one specific value of $r_c$ which minimizes the temperature gradients. For smaller core radii, the temperature rapidly increases toward the boundary (the models are very far from isothermal); for larger core radii, the temperature decreases to zero before the boundary is reached. Because clusters of galaxies are observed to be near isothermal, these MOND $\beta$-models with the smallest temperature variations are taken to be appropriate for clusters.

The emission-weighted temperature of this most nearly-isothermal model as a function of projected radius is shown by the dashed curve in Fig. 3 for $n_o = 0.006 \text{ cm}^{-3}$, $\beta = 0.62$, and $T_o = 6.5 \text{ keV}$. The required core radius is 436 kpc. It is evident from Fig. 3 that the model still deviates significantly from purely isothermal, with roughly a factor of 2 variation around the central temperature. Moreover, compared to actual clusters, the implied core radius for this temperature is too large by a factor of 2.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The radial distribution of X-ray surface brightness for a MOND isothermal sphere ($T = 2.5 \text{ keV}$) compared to observations of the Coma cluster ($T = 8.6 \text{ keV}$). The dashed curve is the $\beta$-model fit (Reiprich 2001) to the observations.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The X-ray luminosity–temperature relation for clusters of galaxies. The points are the clusters tabulated by Ikebe et al. (2002) scaled to $h = 0.7$. The dashed curve is the relation for MOND isothermal spheres and the dotted curve is the relation for the near-isothermal MOND $\beta$-models. The solid curve is the relation for the MOND two-component models discussed in Section 3.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** The projected emission-weighted temperature of a MOND $\beta$-model as a function of projected radius (dashed curve). The radius is given in terms of $r_{\text{max}}$ where the gas density has fallen to $10^{-28} \text{ g cm}^{-3}$. This is the most nearly isothermal $\beta$-model. The solid curve shows the projected emission-weighted temperature for the most nearly isothermal two-component model where the dark central component is a constant density sphere. The dotted curve is the same for a two-component model where the dark central component is represented by a Hernquist sphere.
For these near isothermal $\beta$-models, I determined the mean emission-weighted temperature and the X-ray luminosity within the 0.1–2.4 keV band. The resulting luminosity–temperature relation for these MOND $\beta$-models is shown in Fig. 2 by the dotted curve. Not surprisingly, this nearly coincides with the calculated relation for MOND isothermal spheres and is clearly an equally poor description of reality. The basic problem is that, for both MOND isothermal spheres and MOND $\beta$-models, the core radii are too large. It is evident that some ingredient is missing from these models; that an additional mass component must be added to decrease the gas core radius at a given temperature.

3 RESOLUTION OF THE PROBLEM: TWO-COMPONENT MODELS

The essential problem for MOND self-gravitating gas spheres can be simply summarized. If they are considered to be isothermal (as observed), the gas density distribution is not at all similar to the fitted $\beta$-models which work well as a phenomenological description of the X-ray intensity distribution in clusters. However, if the density distribution is taken to be that of a $\beta$-model, the MOND gas spheres are far from isothermal. In both cases, the objects are overluminous for a given temperature. A different approach would be to take a cluster with an observed density and temperature distribution, and apply equations (1) and (2) directly to yield $M(r)$, the interior dynamical mass as a function of radius. In the Newtonian regime this is given simply by

$$M_N(r) = \frac{r kT}{G m_p} \left[ \frac{d \ln(\rho)}{d \ln(r)} + \frac{d \ln(T)}{d \ln(r)} \right].$$

(7)

With MOND, taking $\mu(x)$ to be given by equation (2c), the dynamical mass is

$$M_\text{d} = \frac{M_N}{\sqrt{1 + (a_\text{c}/a)^2}}$$

(8)

where $a$ is the “observed” acceleration

$$a = \frac{1}{\rho} \frac{d \rho}{dr} = \frac{kT}{m_p r} \left[ \frac{d \ln(\rho)}{d \ln(r)} + \frac{d \ln(T)}{d \ln(r)} \right].$$

(9)

Obviously, from equation (8) in the limit of large accelerations ($a \gg a_\text{c}$) the MOND dynamical mass is equivalent to the Newtonian dynamical mass.

We may apply these relations to the Coma cluster which has a density distribution well fit by a $\beta$-model with $\beta = 0.71$, $r_c = 276$ kpc, $n_0 = 0.0036$ (Reiprich 2001), an observed radial temperature profile (Arnaud et al. 2001) and an average emission-weighted temperature of 8.6 keV. The results are shown in Fig. 4 which is the cumulative Newtonian dynamical mass (dotted curve), the MOND dynamical mass (solid curve), and the gas mass (dashed curve). Here it is evident that while the Newtonian dynamical mass continues to increase at a radius of 1 Mpc, the MOND mass has essentially converged. However, the total MOND mass is still a factor of 4 times larger than the detected mass in the X-ray emitting gas alone. This discrepancy cannot be accounted for by the stellar content of the galaxies which, assuming a mass-to-light ratio of 7, amounts only to about $10^{10} M_\odot$ within 1 Mpc (The & White 1988). This is, in fact, the discrepancy pointed out by The and White – a discrepancy which can be resolved by increasing $a_\text{c}$ by a factor of 3 or 4 over the value required for galaxy rotation curves, or by admitting the presence of non-luminous mass which MOND does not remove.

The density of this non-luminous component is roughly constant and contained within two gas core radii. In other words, the missing mass is essentially present in the inner regions of the cluster as implied in the work of Aguirre et al. (2001). This suggests that, with MOND, clusters might be described by a two-component model: a gas component with a density distribution given by the $\beta$-model and a dark central component of roughly constant density extending to twice the core radius of the gas distribution.

In Coma, the central surface density of the dark component is about $\Sigma_\text{d} = 240 M_\odot$ pc$^{-2}$ (the characteristic MOND surface density of $a_\text{c}/G \approx 700 M_\odot$ pc$^2$). Therefore, to determine scaling relations, I assume that the non-luminous component is a rigid constant density sphere having a central surface surface density equal to that in the Coma cluster and a radius twice that of the gas core radius, as in Coma. Then the density of the dark component is

$$\rho_\text{d} = \frac{\Sigma_\text{d}}{2 r_c}$$

and the total dark mass is

$$M_\text{d} = \frac{16\pi}{3} \Sigma_\text{d} r_c^3.$$  

(11)

Evidently, with this assumption, larger clusters have a more massive dark component, but because the gas mass scales as $r_c^2$, the dark-to-gas mass ratio decreases with increasing core radius or temperature.

The procedure followed is identical to that of the pure MOND $\beta$-models described above; I assume a gas distribution described by equation (6) with $\beta = 0.62$, $n_0 = 0.006$. Then for a given central gas temperature, I determine the core radius of the model for which the temperature gradient is minimized. The new aspect is the second component which makes its presence felt by contributing to the total cumulative mass ($M(r)$ in equation 2) and hence the total gravitational force. This has the effect of decreasing the core radius at a given temperature compared to the single-component MOND $\beta$-models. The emission-weighted temperature as a function of projected radius is shown by the solid curve in Fig. 3 again for a model with a central temperature of 6.5 keV. This is the most-nearly isothermal model, and we see that the temperature gradients are much smaller than in the single-component $\beta$-model. The total variation about the central temperature is less than 40 per cent. In other words, isothermal $\beta$-models require, in the context of MOND, this second central component with roughly constant density.

The luminosity–temperature relation for these two-component cluster models is shown by the solid curve in Fig. 2 where it is evident that these models provide a reasonable description of the observed relation. This is due to the fact that, in the low-temperature systems,
a relatively larger fraction of the mass is not in gaseous form. It is also not in the form of luminous material in galaxies as the implied mass-to-light ratios would be too large. Agreement of MOND with the cluster scaling relations is achieved at the expense of adding unseen matter.

Must the central rigid component be represented by a constant density sphere? While that is consistent with the derived mass distribution in the Coma cluster, we might reasonably ask if this is generally true. To address this question, I have also modelled the central component by a Hernquist sphere (Hernquist 1990) with a density law given by

$$\rho(r) = \frac{\Sigma_0}{r(1 + r/r_d)^3}. \quad (12)$$

This form is similar to the universal CDM dark halo (NFW) implied by cosmic N-body simulations (Navarro, White & Frenk 1996). If \( \Sigma = 240 \, M_\odot \, \text{pc}^{-2} \) and \( r_d = 2 r_c \), the observed luminosity–temperature relation (Fig. 2) is reproduced, and the corresponding dark mass is 50 per cent larger than that of the constant density models. However, modelling the dark component by a Hernquist sphere leads to larger temperature gradients than does the constant density model, as shown by the dotted curve in Fig. 3. The actual (unprojected) gas temperature falls by 40 per cent within one core radius (not reflected in Fig. 3 due to projection effects). Highly isothermal clusters are more consistent with a constant density central component than with a Hernquist or NFW sphere, but the present observational uncertainty in the temperature profile permits a range of models.

4 MODELLING INDIVIDUAL CLUSTERS

X-ray observations of individual clusters may be reproduced by such two-component models. Given the parameters of a \( \beta \)-model fit to the X-ray emission from a specific cluster (i.e. \( \beta, n_c \), and \( r_c \)), and a characteristic temperature for the entire cluster, I determine, via equations (1) and (2), the central temperature, \( T_0 \), and the density (or surface density, related by equation (10)) of the dark component which yields the observed emission-weighted temperature and the observed core radius, \( r_c \), of the \( \beta \)-model. In all cases, the dark component is assumed to extend to \( 2 r_c \). Uniqueness is ensured by requiring that the temperature gradients be minimized, i.e. these are again the most nearly isothermal models. The fitting parameter is the density of the central dark component which yields the total dark mass via equation (11) above.

Table 1 lists the clusters which have been modelled in this way along with the parameters of the \( \beta \)-model fit and the mean temperature, all from the compilation of Reiprich (2001) with cluster properties scaled to \( \hbar = 0.7 \). The required central surface density of the dark component, \( \Sigma_0 \), is given along with the total gas mass (out to the cut-off radius) and the total mass of the dark component. The enclosed Newtonian dynamical mass is also given.

This sample was chosen to include a number of objects with significant inferred cooling flows (such as Abel 1689 and 2029). These clusters are characterized by relatively small core radii (\( r_c < 100 \, \text{kpc} \)) and large central electron densities (\( n_e > 0.01 \, \text{cm}^{-3} \)). The sample also includes clusters at the other extreme – large core radii (\( r_c > 250 \, \text{kpc} \)) and low central electron densities (\( n_e < 0.005 \, \text{cm}^{-3} \)) with no inferred cooling flow (such as Abel 119 and 2256).

In general, MOND reduces the classical Newtonian discrepancy for clusters of galaxies. For this sample, with MOND the ratio of the total dark mass to the gas mass is \( 1.60 \pm 1.7 \). The Newtonian dark-to-gas mass ratio is \( 7.14 \pm 2.7 \). While this is a significant reduction, it is also clear that MOND does not fully resolve the mass discrepancy in clusters. Moreover, it should be noted that the ratio of masses of the dark-to-hot gas components in the central two core radii (where the additional component is required) can be as large as 10, as emphasized by Aguirre et al. (2001). This rigid component cannot be stars of a normal population because the mass-to-light ratio within two core radii would still be, on average, in excess of about 50. The principal question is whether or not this dark component is a fundamental problem or can be accommodated in the context of MOND.

It could be that a high mass-to-light population of low-mass stars or substellar objects is deposited in the central regions of clusters as a result of cooling flows. Fig. 5 is a plot of the total mass in the dark component versus the cooling rate as estimated by White, Jones & Forman (1997). The solid points and open points are those clusters with discrepancies respectively larger than or smaller than \( M_\delta/M_g = 1.5 \). The errors on the dark mass result primarily from uncertainty in the cluster temperature determinations. Given that the mean error in the dark mass is roughly 25 per cent, those objects requiring 1.5 times more dark mass than gas mass have at least as much dark as detectable mass. We see that there is no obvious correlation between the total mass of the dark component and the cooling rate – especially for those clusters with the largest discrepancy. On the other hand, in Fig. 6 we see a plot of the surface density of the dark component versus the mass deposition rate. Here, there does appear to be a correlation. This is because those clusters with the highest central dark matter densities are not the clusters with the largest mass discrepancies.

It is unclear if this apparent correlation between surface density and mass deposition rate is significant. The mass deposition is not actually observed but calculated from the central gas density. Those clusters with large inferred cooling flows are clusters with high central gas densities and small core radii, but it is precisely these clusters which require a large surface density of dark matter to produce the small core radius. While it may be the case that cooling flows contribute to the dark component of those clusters with the largest central density of dark matter, it is also evident from Fig. 5 that this cannot be the explanation for the discrepancy in clusters with the largest dark mass problem. These tend to be the clusters with low central gas densities and low inferred dark matter densities – but large core radii. In the next section, I consider another possibility: particle dark matter in the form of massive neutrinos.

5 NEUTRINOS AS CLUSTER DARK MATTER

The most well-motivated form of particle dark matter consists of neutrinos with finite mass. Aspects of the observed fluxes of atmospheric and solar neutrinos provide strong evidence for neutrino oscillations and hence non-zero neutrino masses (Gonzalez-Garcia & Nir 2002). The fact that the number density of neutrinos produced in the early Universe is comparable to that of photons then implies that there is a universal dark matter sea of neutrinos. The contribution to cosmological mass density would be \( \Omega_h^2 = \sum m_\nu/94 \, \text{eV} \) where the sum is over neutrino types.

The neutrino oscillation experiments do not provide information on the actual masses of neutrino species but on the square of the mass differences. These are small, such that, the largest mass difference, suggested by the atmospheric oscillations, is \( \Delta m \approx 0.05 \, \text{eV} \). If \( m_\nu \approx \Delta m \) then the three active neutrino types would have no significant cosmological mass density (\( \approx 10^{-5} \)) and could not contribute to the mass budget of any bound astronomical object. However, another possibility is that \( m_\nu \gg \Delta m \) and that the masses of all three
active types are nearly equal. In this case, an upper limit to the masses is provided by an experimental limit on the mass of the electron neutrino, i.e. $2.2 \pm 0.3$ eV at 90 per cent confidence level (Groom et al. 2003). Constraints on neutrino mass arising from observations of the cosmic background radiation combined with the power spectrum of observed large-scale structure (Spergel et al. 2003; Elgarøy et al. 2002) rest upon the standard model for structure formation, and are not necessarily relevant in a MOND universe (Sanders 2001). If it were the case that the electron neutrino mass were near 2 eV, then neutrinos would constitute a significant fraction of the cosmic density ($\Omega_\nu \approx 0.13$ for $h = 0.7$).

However, neutrinos of this mass could not contribute to the mass budget of galaxies. This follows from a classic argument by Tremaine & Gunn (1979) based upon conservation of the phase-space density of the neutrino fluid. Relativistic neutrinos are created with a maximum phase-space density of $(2\pi \hbar)^{-3}$ per type including antineutrinos (this is a factor of 2 less than the absolute limit implied by quantum mechanical degeneracy). In subsequent evolution of the neutrino fluid involving gravitational instability and collapse, the final phase-space density cannot exceed this value. This provides a relation between the final density of neutrino dark matter and the velocity dispersion of the system; with three types

$$\rho_\nu \leq \left(4.8 \times 10^{-27}\right) \left(\frac{m_\nu}{2\text{ eV}}\right)^4 \left(\frac{T_{\text{keV}}}{2}\right)^{1/2} \text{ g cm}^{-3}. \quad (13)$$

Equivalently, for virialized systems, this may be written as a relation between the effective core radius of a dark halo and its velocity dispersion; roughly, this is
the clusters. The solid points are the objects with large discrepancies \((M_d/M_\delta > 1.5)\).

Formally speaking, it does not constitute a falsification. If the dynamical mass were generally less than the dynamical mass predicted by MOND, then in terms of mass-to-light ratios, was confirmed by radial velocity studies of rich clusters of galaxies. The discrepancy between the visible and Newtonian dynamical mass, quantified in terms of mass-to-light ratios, was more than a factor of 10. With the advent of X-ray observatories and the detection of hot gas in clusters, this discrepancy between dynamical and detectable mass was reduced to a factor of 10. Modified Newtonian dynamics reduces the discrepancy further, to a factor of 2–3, but it is clear that a discrepancy remains which cannot be explained by detected gaseous or luminous mass. It is also apparent that while the missing mass is primarily in the inner regions of clusters, it does extend beyond the core radius as defined by the gas density distribution.

How serious is this remaining mass discrepancy for MOND? Formally speaking, it does not constitute a falsification. If the dynamical mass predicted by MOND were generally less than the detected mass in stars and gas, then it would be a definite falsification, but this is not the case. More mass can always be found (as in the case of the hot gas), but it is difficult to make observed mass disappear.

We might reasonably argue that the implied presence of undetected mass runs against the spirit of MOND, which, in the view of most people, was suggested primarily as a replacement for dark matter. However, more generally, MOND should not be
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viewed simply as an alternative to dark matter; the systematic appearance of the mass discrepancy in astronomical systems with low internal accelerations is an indication that Newtonian dynamics or gravity may break down in this limit. MOND primarily addresses this issue: is physics in the low-acceleration regime Newtonian? The success of MOND in explaining the scaling properties and observed rotation curves of galaxies suggests that it may not be. MOND does not rest upon the principle that there is no undetected or dark matter. Indeed, comparing the density of luminous matter to the baryonic content of the Universe implied by considerations of primordial nucleosynthesis, we can only conclude that there is, as yet, undetected baryonic matter, probably in the form of diffuse gas in the intergalactic medium. Moreover, it is virtually certain that particle dark matter exists in the form of neutrinos; only its contribution to the total mass density of the Universe is unclear.

MOND would be incompatible with the widespread existence of dark matter which clusters on the scale of galaxies – CDM – but MOND is not inconsistent with hot dark matter such as 2-eV neutrinos, which can only aggregate on the scale of clusters of galaxies; indeed, I have presented evidence that this may be the case. Neutrinos, as particle dark matter candidates, are unquestionably well motivated, both from a theoretical point of view (they definitely exist) and from an experimental point of view (they have mass). No conjectured CDM particle shares these advantages. While I do not wish to state that the dark matter in clusters is definitely in the form of 2-eV neutrinos (there is more than enough remaining baryonic matter to make up the missing mass), there are indications that point this way. The largest discrepancies are found in the clusters with the largest core radii as would be the case with neutrinos. The indicated densities of dark matter are comparable to the maximum possible density of 2-eV neutrinos.

The fact remains that there exists an algorithm, MOND, which allows galaxy rotation curves to be predicted in detail from the observed distribution of matter, and it is for these systems that the kinematic observations are most precise. This fact challenges the current CDM paradigm, and demands explanation if dark matter lies behind the discrepancy. The factor of 2 remaining discrepancy in clusters is less challenging for MOND, particularly given that MOND makes no claims about the full material content of the Universe.

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