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Saltation Layer of Particles in Water Flows Related to Transport Stage

B. S. Mazumder and D. C. Dalal
Flume Laboratory, Physics and Earth Sciences Division,
Indian Statistical Institute, Calcutta 700 035, INDIA

A theoretical model has been developed to determine the maximum saltation layer thickness of sediment particles in water associated with the migration velocity of particle in the bed layer. This is consistent with Owen's (1964) hypothesis for saltation of uniform grain in air. The equation for mean particle velocity at the bed is derived by balancing the horizontal forces acting on the particle in the bed. The modified expression for mean particle velocity includes the effects of drag and lift coefficients, bed shear stress, coefficient of dynamic friction, settling velocity and pivoting angle. The saltation layer model presented here extends a reasonable physical assumption by converting the average horizontal particle velocity to a vertical component of velocity due to collisions with particles resting on the bed. This explicitly shows a functional dependence of saltation height on mean particle velocity and take-off angle. The proposed model has been tested using available experimental data and the agreement with particle velocities and saltation heights is excellent. An interesting outcome is that a quadratic relationship is suggested between the higher transport stage (upper plane bed) and the take-off angle of particle. This shows that the take-off angle decreases with increase in transport stage.

Introduction

Movement of non-cohesive sediments in water has received much attention in terms of basic problems related to the process of grain sorting. During transportation in the sand bed, sediment movement is generally classified into two categories: bed load and suspended load. In bed load, there are three modes of movement observed along

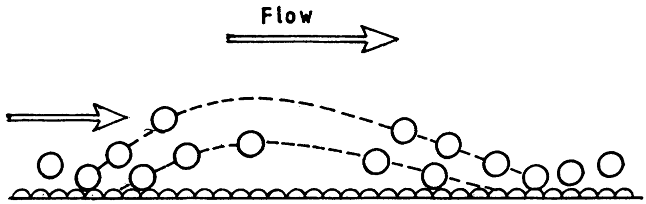

Fig. 1. Definition sketch for particle saltation.

the bottom - sliding, rolling and saltating. A grain at the surface of a bed will begin to move when boundary shear stress just exceeds the critical shear stress. At low transport stages, sand grains move by sliding and rolling over the surface of the bed; but with a small increase in shear stress, these grains will jump from the surface of the bed (called saltation), and follow the trajectories like ballistic (Fig. 1). This process is due to the combination of lift and drag forces, and submerged weight of the particle in the region at high shear stress near the bed. The changes from saltation to suspension occur in a flow condition when the vertical component of fluctuating velocity is approximately equal to the settling velocity of grain.

A number of researchers have studied various aspects of saltation of particles over a bed surface under unidirectional flow conditions. Owen (1964) presented a theoretical model based on wind tunnel experiments to estimate the thickness of saltation height of particle by balancing the potential energy at its highest elevation of the trajectory with maximum kinetic energy on the bed. He defined the thickness of saltation layer as $h_s = \rho_s u_0^2/2 (\rho_s - \rho) g$, where ρ_s is the density of sediments, ρ is the density of fluid, g is the acceleration due to gravity, and u_0 is the maximum vertical velocity of the saltating particle. u_0 was assumed to be proportional to friction velocity u_*. He did not consider the physical mechanism of particle velocity. Smith and McLean (1977) used a similar argument to estimate the thickness of saltation layer in water. Grant and Madsen (1982) developed an analogous equation for saltation height in an oscillatory flow using the concept of particle momentum equation.

Theoretical developments describing saltating heights of particles under the influence of fluid flow have been studied by Bagnold (1973), Francis (1973), Abbott and Francis (1977), Murphy and Hooshiari (1982), Wiberg and Smith (1985), Lee and Hsu (1994), Nino and Garcia (1994) and Mazumder (1995). Although there have been a number of attempts on saltation heights, there is no general consensus in developing the functional behaviour of saltation height with bed shear stress or friction velocity. Here we have tried to develop a model where the functional relationship of saltation layer explicitly includes the drag, lift and dynamic friction coefficients, bed shear stress, particle settling velocity, added mass coefficient and take-off angle of particle, which has not been reported earlier. These various parameters influence the physical aspects of the problem.

The purpose of this paper is to develop a theoretical model to determine the scale of saltation layer thickness of immersed particle at the near-bed based on the energy balance equation. This formulation explicitly depends on the modified particle velocity, added mass coefficient and take-off angle. The particle velocity in the bed layer is calculated by considering the equilibrium of agitating and stabilizing forces acting on it. The result in this paper is primarily for average thickness of moving sediment layers at the near-bed flows. A new empirical relationship between the take-off angle of saltating particle and the transport stage has been established for upper flow regime. The computed average particle velocity and average thickness of saltation layer are then compared with the available data of Luque and Beek (1976), Dietrich (1982), Wiberg and Smith (1985), Wiberg and Rubin (1989) and Sumer *et al.* (1996). The overall performance of the results obtained by the present models is comparable with the values estimated by other existing models. The present model can be claimed to be superior to others because inclusion of the physical parameters, such as ratio of lift and drag coefficients, dynamic friction coefficient, bed shear stress, particle settling velocity, added mass coefficient and take-off angle, in the model essentially provides a more realistic picture to the complex phenomena. The model serves as a framework to investigate the nature of sediment movement in the bed layer. It is also of interest in studying the total bed roughness of near-bed sediment transport due to moving sediments from the saltation layer thickness that will be studied in a separate paper.

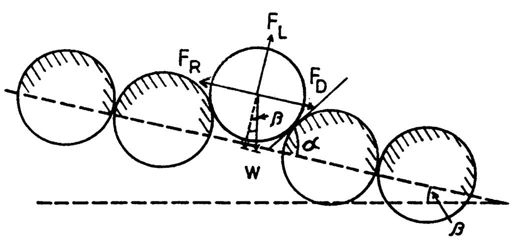

Fig. 2. Forces balance on a particle at the surface of a bed.

Theoretical Model

In order to develop a theoretical model for estimating saltation layer thickness of sediment particles at the near-bed flow, the initial particle velocity u_p is determined primarily based on the horizontal balance of maximum forces acting on a particle in the bed. A sediment grain at the bed surface will begin to move when the friction force for the particle moving along the bed is exactly balanced by the sum of the drag force and the appropriate component of gravitational force in a slopping bed (Fig. 2). Mathematically, the equation for the horizontal balance of forces can be expressed as

$$(W\cos\beta - F_L)\tan\alpha = F_L + W\sin\beta \quad (1)$$

where W is the submerged weight of the particle, F_L and F_D are respectively the fluid lift and drag forces on the bed load grains per unit bed area, and tan a is the dynamic friction coefficient, and b is the bed slope. The drag force F_D acting on a grain in the direction of the flow is expressed as

$$F_D = \frac{\pi}{8}\rho C_D D^2 (au_* - u_p)^2 \quad (2)$$

where the term $(au_* - u_p)$ is relative fluid velocity, u_* is the friction velocity, u_p is the mean particle velocity, ρ is the fluid density, au_* is the fluid velocity at the height y_D of effective drag on the spherical grain of diameter D, a is the coefficient depending on height of the effective drag, and C_D is the drag coefficient.

The equation for the lift force proposed by Anderson and Hallet (1986) is used in this study as

$$F_L = \frac{\pi}{8}\rho C_L D^2 (u_T^2 - u_B^2) \quad (3)$$

where C_L is the lift coefficient; and u_T and u_B are respectively the flow velocities at the top and bottom of the bed load grain. The lift force will be non-zero as long as $u_T \neq u_B$, *i.e.* there must be a velocity gradient across the grain. The lift force approaches zero as the velocity gradient tends to become zero, regardless of the value of the lift coefficient (Wiberg and Smith, 1985). Due to the asymmetry of the flow around a grain resting on the bed, there is an asymmetrical pressure distribution with higher pressure on the lower surface of the grain acting upward than on the upper surface of the grain acting downward. According to Chepil (1958) and Wiberg and Smith (1985), the velocity u_B at the bottom of the grain is much smaller than u_T and it can be neglected. Therefore, the mean lift force on the bed load grain can be expressed as

$$F_L = \frac{\pi}{8}\rho C_L^2 D \ u_T^2 \quad (4)$$

The equation of motion for a spherical particle falling vertically in water with its terminal settling velocity v_s under the influence of gravity is given by

$$W = \frac{\pi}{6} gD^3(\rho_s - \rho) = \frac{\pi}{8}\rho C'_D D^2 v_s^2 \quad (5)$$

where C'_D is the fluid drag coefficient on a grain at its terminal settling velocity, ρ_s is the sediment density and g is the acceleration due to gravity. Following Francis (1973), we have assumed $C'_D = C_D$ in our analysis. Such an assumption may be justified because both represent a drag acting in the direction parallel to the relative velocity with the difference being the potential effect of proximity to a boundary.

Substituting Eqs. (2), (4) and (5) in Eq. (1), the mean velocity u_p of any bed load grain is thus obtained as

$$u_p = au_* - u_* \left[\frac{4}{3C_D} \frac{1}{\theta_0} \left((\cos\beta - \frac{\lambda u_T^2}{v_s^2}) \tan\alpha - \sin\beta \right) \right]^{\frac{1}{2}} \tag{6}$$

where $\lambda = C_L/C_D$ is the ratio of lift coefficient to the drag coefficient on a grain, $\theta_0 = \tau_B / (\rho_s - \rho)\, g\, D$ is the dimensionless shear stress, τ_B is the bottom shear stress, and the drag coefficient C_D is to be determined from the threshold conditions.

At the initiation of sediment motion $u_p = 0$, $u_* = u_{*c}$, $a = a_c$, and $u_T = u_{Tc}$ in Eq. (6), the drag coefficient C_D is given as

$$C_D = \frac{4}{3a^2} \frac{1}{\theta_{0c}} \left((\cos\beta - \frac{\lambda u_{Tc}^2}{v_s^2}) \tan\alpha_c - \sin\beta \right) \tag{7}$$

From Eqs. (6)-(7), the expression for mean velocity u_p of the bed load grain yields as

$$u_p = au_* \left[1 - \left(\frac{\theta_{0c}}{\theta_0} \frac{(\cos\beta - \frac{\lambda u_T^2}{v_s^2}) \tan\alpha - \sin\beta}{(\cos\beta - \frac{\lambda u_{Tc}^2}{v_s^2}) \tan\alpha_c - \sin\beta} \right)^{\frac{1}{2}} \right] \tag{8}$$

as long as $\cos\beta > \lambda\, u_T^2 / v_s^2$, where $\theta_{0c} = \tau_c / (\rho_s - \rho) g D$ is the dimensionless critical shear stress, and subscript c denotes value of a parameter evaluated at the threshold of motion. Eq. (8) serves as a more general expression of mean velocity u_p of the bed load grains than those obtained from the previous studies. This equation will be used in the model for determining the average saltation layer thickness.

When bed slope β tends to become zero and $(\lambda u_T^2 / v_s^2) << 1$, the Eq. (8) reduces to

$$u_p \equiv au_* \left(1 - \left(\frac{\theta_{0c} \tan\alpha}{\theta_0 \tan\alpha_c} \right)^{\frac{1}{2}} \right) \tag{9}$$

which is similar to the expressions given by Engelund and Fredsoe (1976), Bridge and Dominic (1984), Bridge and Hanes (1985) and Mazumder (1995). Corresponding expression for C_D from Eq. (7) is given by

$$C_D = \frac{4}{3a^2} \frac{1}{\theta_{0c}} \tan\alpha_c \tag{10}$$

which agrees well with Engelund and Fredsoe (1976), and Mazumder (1995).

If $(u_T/v_s)^2 \equiv 1/\lambda$ at $\beta \rightarrow 0$, Eq. (8) reduces to $u_p \equiv au_*$ which is the ambient fluid velocity. According to Eq. (2), this implies zero drag, which is unlikely to occur in nature. The expression for the mean velocity of bed load grain is valid as long as $(1 - \lambda u_T^2/v_s^2) > 0$ as $\beta \rightarrow 0$ in Eq. (8).

The effective fluid velocity au_* in Eq. (8) can be described with a knowledge of the near bed velocity profile and roughness Reynolds number $R_* = u_* k_s / \nu$, where k_s is the equivalent sand roughness, and ν is the kinematic viscosity. For hydraulically smooth flows $(u_* k_s/\nu \leq 11.6)$, the grains are totally submerged in the viscous sub-layer. The fluid velocity profile is linear within the sub-layer and is given by

$$u(y_D) = a u_* = \frac{u_*^2 y_D}{\nu} \quad (11)$$

If the flow is hydraulically rough $(u_* k_s/\nu > 70)$, the velocity is given by the law of the wall

$$\frac{u(y_D)}{u_*} = a = \frac{1}{\kappa} \ln\left(\frac{y_D}{k_s}\right) + 8.5 \quad (12)$$

where the value of a will normally be in the range 6.8-9.5. For hydraulically transitional flow between the viscous sub-layer and the logarithmic flow $(11.6 < u_* k_s/\nu \leq 70)$, Reichardt (1951) derived an equation for velocity profile of a smooth unidirectional flow that extends through both the viscous sub-layer and the logarithmic layer, and provides a smooth transition between the two regions. Thus the Reichardt's velocity profile is given by

$$\frac{u}{u_*} = a = \frac{1}{\kappa} \ln(1+\kappa y^+) - \frac{1}{\kappa} \ln(y_0^+ \kappa) \left(1 - e^{-y^+/11.6} - \frac{y^+}{11.6} e^{-0.33 y^+}\right) \quad (13)$$

where $y^+ = u_* y_D/\nu$, $y_0^+ = u_* y_0/\nu$ and $y_0 = \nu/9u_*$ is the roughness height for hydraulically smooth flow. For transitional flows, $y_0 = k_s \exp(-\kappa B)$, where B is given by the quadratic relation as

$$B = -1.77L^2 + 3.9L + 7.34 \quad (14)$$

with $L \equiv \log_{10}(u_* k_s/\nu)$. Eqs. (11)-(14), therefore, will be used to determine the value of a according to the roughness Reynolds number, $u_* k_s/\nu$.

A similar argument, like Owen (1964), has been adopted here to estimate the maximum thickness of bed-load layer (scale of saltation layer) in water assuming the average horizontal particle velocity u_p (given by Eq. (8)) as the initial vertical velocity. Owen assumed the initial velocity of saltating particles to be proportional to the friction velocity u_*. A reasonable physical mechanism may be assumed which converts the average horizontal velocity to a vertical component of velocity ($u_p \sin \sigma$) through collisions with particles resting on the bed. Here σ is the angle of inclination of particle velocity with respect to the horizontal surface. In the literature (Middleton and Southard 1984), it has been stated that many of the saltating particles rise up *almost* vertically ($\sigma \approx 90^0$) from the bed surface, which may not be true. In fact, it is suggested that sand particles depart from the bed with a velocity u_p at an angle σ with respect to the bed surface rather than assuming the particles rise up vertically. Due to the collisions with other particles resting on the bed, migrating particles might bounce back into the flow, but continue to move again with the flow. Here the particle-particle collisions are assumed to be small because the fluid velocity is much higher than the impact velocity. Unlike any other models, it would be more reasonable to argue that the maximum thickness of saltation layer h_s may be expressed as

$$h_s = \frac{u_p^2 \sin^2\sigma}{2g_e}, \quad g_e = \frac{(\rho_s - \rho)g}{\rho_s - \rho C_m} \tag{15}$$

where u_p is the particle velocity given by the Eq. (8), and is a function of the ratio of lift and drag coefficients, dynamic friction coefficients, dimensionless bed shear stress and particle settling velocity; g_e is the equivalent gravitational acceleration of a particle, derived from the resulting equation of dynamics of motion of a particle with respect to the surrounding fluid (Murphy and Hooshiari, 1982) and σ is the take-off angle to be determined by adjusting the value with the experimental data of saltation height of particles.

The saltation layer equation is written as Eq. (15) with the realization that the particles lift up from the bed with a vertical velocity $u_p \sin\sigma$, where $\sin\sigma$ may also be treated as a correction factor at higher transport stage. The forces that the fluid exerts on the moving grain are due to inertial force, weight, buoyancy, and added mass. The 'added mass' force is a pressure that opposes the acceleration of an object if that object accelerates through the fluid. Its magnitude is frequently assumed to be half of the mass of the displaced fluid times the acceleration of the sphere. Here the added mass coefficient $C_m = 1/2$ for spherical particles (Wiberg and Smith, 1985) is assumed. The developed concept given in Eq. (15) will be tested to determine how best it compares with the actual historical experimental data.

Comparison with Experimental Data

Data used for verification of theoretical-empirical models given in Eqs. (8) and (15) for estimation of average particle velocity and thickness of saltation layer discussed above are taken from Luque and Beek (1976), Ghosh *et al.* (1981), Wiberg and Smith (1985), Wiberg and Rubin (1989) and Sumer *et al.* (1996). Models are tested only for grain sizes ranging from 0.013 to 0.33 cm. of density $\sigma_s = 2.64$ g/cm^3 and different transport stages $T_* = \tau_b/\tau_c$ varying from 1.24 to 30.00 in which values of u_* lie between 2.5 and 7.50 cm/sec. Transport stage T_* is the ratio of bed shear stress τ_b to the critical shear stress τ_c.

In order to compute the migration velocity (u_p) of any bed load grain from Eq. (8), it is important to know the values of parameter a, dimensionless shear velocities (θ_0 and θ_{0c}), ratio of lift and drag coefficients (λ), dynamic friction coefficients ($\tan\alpha$ and $\tan\alpha_c$), and settling velocity of particle v_s. The dimensionless critical bed shear stress q_{0c} is taken from the theoretical curve of Wiberg and Smith (1985) for different grain Reynolds numbers, $Re_* = u_* D/v$. The parameter a is evaluated from

Eqs. (11)-(14) at the height of effective fluid thrust acting on the grain (y_D) depending on the values of roughness Reynolds number $u_* k_s/\nu$. In these calculations the height of effective fluid thrust on the bed-load grain is scaled at the nominal grain diameter D.

Direct evaluation of fluid velocity u_T at the top of the grain from Eqs. (11)-(13) is not feasible, because the calculated fluid velocity at the top of the grain always exceeds the settling velocity of that grain. So these equations are not valid for such small value of y at the top of a grain. In fact, it is very difficult to determine the actual bed level (where the velocity is zero) and the near-bed variation of velocity with distance from the bed; and hence, for sediment beds, the level of the top of the bed grain is not easy to define, leading to unstable estimates of fluid velocity u_T at the top of the grain. Depending on the flow conditions, bed grains are assumed to lie within the viscous sub-layer, or the buffer layer or the logarithmic region. Both turbulent and non-turbulent lift forces may act upon sand grains, depending on the grain Reynolds number, whether or not grains immersed in the viscous sub-layer. To avoid difficulties of evaluation of fluid velocity u_T at the top of the grain, we have adopted the idea of anisotropy of turbulence which decreases as the bed is approached from the buffer layer, since it is zero at the bed. It is thus reasonable to assume the fluid velocities u_T and u_{Tc} at the top of the grain as the upward directed turbulent velocities on the bed-load grain, and are given by

$$u_T = bu_* , \quad u_{Tc} = bu_{*c} \tag{16}$$

where b is a measure of the vertical turbulence intensity and has to be determined from a functional relationship developed by Grass (1971) and Nakagawa and Nezu (1978) between the dimensionless distance $y^+ = yu_*/\nu$ and the measure of vertical turbulence intensity b (= $rms\ v'/u_*$) from experimental data. The approximate values of b are given by

$$b \approx 0.8 \quad \text{for} \quad \frac{u_* y_T}{v} > 50 , \approx \frac{0.8}{50}\left(\frac{u_* y_T}{v}\right) \quad \text{for} \quad \frac{u_* y_T}{v} < 50 \tag{17}$$

The turbulent intensity ($rms\ v'/u_*$) is determined from the model of turbulent bursting which occurs most violently near the wall ($y^+ < 50$). It is observed that b increases from zero at the bed as y^+ increases from zero, and v'/u_* is almost constant (0.8) for $y^+ > 50$. For different values of $y^+ < 50$ calculated from our data, the corresponding values of b (= v'/u_*) are taken from Fig. 7 of Bridge and Bennett (1992) provided from experimental results. This physical idea is a good representation of v'/u_* near the wall for various values of grain Reynolds number $u_* y_D/n$.

It is obvious from the physical fact that the threshold shear velocity u_{*c} for initiation of sediment motion is less than the shear velocity u_*. In similar way, from the same functional relationship we have taken the values of v'/u_{*c} for the corresponding values of ($y^+_{Tc} \equiv u_{*c} y_T/\nu$). It is observed from our calculations that $u_{Tc} <<$

v_s for each grain size in the bed. Hence, $\lambda u_{Tc}^2/v_s^2 < \lambda u_T^2/v_s^2 < 1$. So it is clear that $\lambda u_{Tc}^2/v_s^2$ is negligibly small compared to unity. Therefore, the migration velocity of any bed-load grain from Eq. (8) can be written as

$$u_p = au_*\left[1 - \left(\frac{\theta_{0c}\tan\alpha}{\theta_0\tan\alpha_c}\right)^{\frac{1}{2}}\left(1 - \frac{\lambda u_T^2}{v_s^2}\right)^{\frac{1}{2}}\right] \quad \text{for} \quad \beta \to 0 \tag{18}$$

Experiments of Luque and Beek (1976) provide some detailed measurements of average velocities for particles having diameters of D = 0.09, 0.18, 0.33cm. and density ρ_s = 2.64g/cm^3. For computational purpose, the average value of λ is taken as 0.85 from Chepil (1958) and Wiberg and Smith (1985). In order to match the computed values of particle velocities from the theoretical model with the experimental data, the appropriate values of dynamic friction coefficient (tanα) and pivoting angle (tanα_c) are obtained. For lower stage plane beds, the average fitted values of tanα and tanα_c for different grain sizes are 0.5 and 0.6 respectively, whereas for higher stage plane beds, the average value of tanα is 0.84 and tan$\alpha_c \equiv$ 0.6. The exact values of these coefficients may vary, but it is likely that values are reasonable (see Francis 1973). The data used for the computation of migration velocities of particles for lower and upper transport stages are listed in Table 1. The calculated particle velocities for different sizes (D = 0.09, 0.18, 0.33 cm.) are plotted against the measured values of Luque and Beek (1976) and the computed values of Wiberg and Smith (1985) in Fig. 3. From the theoretical results it is observed that the migration velocity of bed-load grain increases linearly with an increase in friction velocity u_* or transport stage T_*, that is comparable with Lee and Hsu (1994).

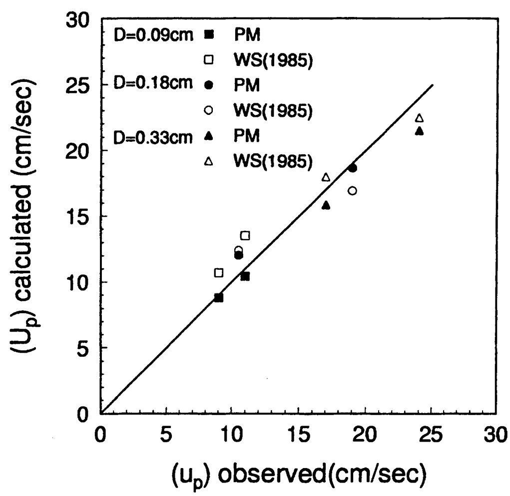

Fig.3. Calculated values of average particle velocity plotted against the measured velocities by Luque and Beek (1976) (filled symbols) and the computed velocities by Wiberg and Smith (1985) (open symbols).

Table 1 – Useful parameters for computation of migration velocities and saltation heights of particles.

Exp.	Run	D cm	τ_b / τ_c	u_* cm/sec	u_{*c} cm/sec	$\tan\alpha$	$\tan\alpha_c$	a
				Lower transport stage				
LB(1976)		0.090	1.32	3.56	3.10	0.50	0.65	9.20
		0.090	1.72	4.10	3.10	0.50	0.54	9.16
		0.180	1.24	4.45	4.00	0.50	0.85	8.50
		0.180	1.71	5.23	4.00	0.50	0.85	8.50
		0.330	1.35	4.65	4.00	0.45	0.85	8.30
		0.330	1.64	5.12	4.00	0.45	0.85	8.50
WS(1985)		0.050	1.50	2.70	2.20	0.50	0.65	8.29
		0.050	2.00	3.10	2.20	0.50	0.50	8.50
		0.050	4.00	4.40	2.20	0.50	0.85	8.80
		0.100	1.50	2.45	2.00	0.45	0.85	9.14
WR(1989)	21	0.093	2.60	3.00	1.86	0.65	0.50	8.70
	26	0.093	1.95	2.60	1.86	0.50	0.60	9.14
	27	0.093	2.43	2.90	1.86	0.50	0.50	9.18
	31	0.093	2.27	2.80	1.86	0.50	0.50	9.17
				Upper transport stage				
WR(1989)	01	0.033	13.44	6.60	1.80	0.84	0.60	8.97
	15	0.019	18.32	5.10	1.19	0.84	0.60	9.70
	25	0.028	26.60	6.50	1.26	0.84	0.60	8.83
	28	0.028	23.40	6.10	1.26	0.84	0.60	8.74
	—	0.025	10.00	3.80	1.20	0.84	0.60	9.50
SKFD	140	0.013	13.85	4.18	1.12	0.84	0.60	5.43
(1996)	142	0.013	18.33	4.81	1.12	0.84	0.60	6.25
	144	0.013	20.94	5.14	1.12	0.84	0.60	6.68
	147	0.013	23.45	5.44	1.12	0.84	0.60	7.07
	151	0.013	29.78	6.13	1.12	0.84	0.60	7.97

where LB, WS, WR and SKFD indicate Luque and Beek (1976), Wiberg and Smith (1985), Wiberg and Rubin (1989) and Sumer *et al.* (1996) respectively.

The maximum values of thickness of saltation layer h_s are computed from Eqs. (15) and (18) using the available data from Luque and Beek (1976), Wiberg and Smith (1985), Wiberg and Rubin (1989) and Sumer *et al.* (1996). The computation depends on particle velocity and take-off angle, where the value of the take-off angle σ is adjusted until the calculated saltation heights agree with those obtained from the existing models (Dietrich 1982; Wiberg and Smith 1985; and Wiberg and Rubin 1989) for different values of transport stage. From the theoretical calculations, it is shown that for the lower transport stage ($T_* \leq 3$), the saltating particles take-off almost vertically from the bed *i.e.*, the angle of inclination σ is almost 90^0, whereas

Table 2 – Comparison of thickness of saltation layers with different models.

Exp.	Run no.	D cm	τ_b/τ_c	$(u_p)_{pm}$ cm/sec	σ^0	$(h_s)_D$ cm	$(h_s)_{WR}$ cm	$(h_s)_{WS}$ cm	$(h_s)_{pm}$ cm
				Lower transport stage					
WS		0.050	1.50	6.62	90	0.0350	0.0398	0.0500	0.0432
(1985)		0.050	2.00	8.20	90	0.0429	0.0494	0.0552	0.0647
		0.100	1.50	9.36	90	0.0692	0.0855	0.0904	0.0851
LB		0.090	1.32	8.90	90	0.0564	0.0685	0.0719	0.0768
(1976)		0.090	1.72	10.28	90	0.0691	0.0853	0.0813	0.1032
		0.180	1.24	12.03	90	0.1073	0.1361	0.1440	0.1411
WR	21	0.093	2.60	8.01	90	0.0954	0.1220	—	0.1026
(1989)	26	0.093	1.95	8.54	90	0.0783	0.0978	—	0.0713
	27	0.093	2.43	9.88	90	0.0912	0.1160	—	0.0954
	31	0.093	2.27	8.97	90	0.0871	0.1101	—	0.0842
				Upper transport stage					
WR	15	0.019	18.32	37.98	10	0.0448	0.0360	—	0.0421
(1989)	—	0.025	10.00	25.29	17	0.0500	0.0463	—	0.0534
	28	0.028	23.40	48.06	10	0.0692	0.0644	—	0.0678
	25	0.028	26.60	53.16	9	0.0707	0.0655	—	0.0669
	01	0.033	13.44	42.22	15	0.0722	0.0729	—	0.0870
SKFD	140	0.013	13.85	17.47	15	0.0287	0.0204	—	0.0250
(1996)	142	0.013	18.33	23.41	14	0.0306	0.0213	—	0.0311
	144	0.013	20.94	27.05	13	0.0315	0.0217	—	0.0362
	147	0.013	23.45	30.47	11	0.0321	0.0219	—	0.0329
	151	0.013	29.78	40.17	10	0.0334	0.0225	—	0.0473

where subscript D and pm indicate Dietrich (1982) and present method respectively.

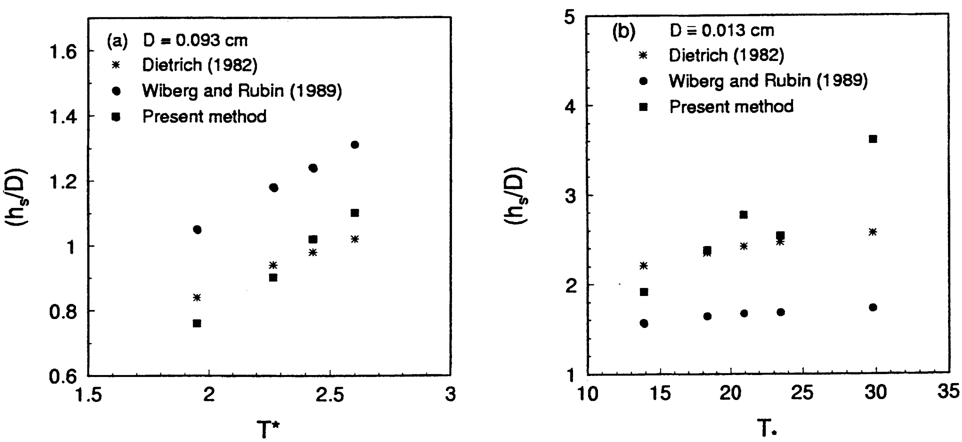

Fig. 4. Normalized saltation heights (h_s/D) of particles against transport stage T_* ; (a) for D = 0.093 cm at lower transport stage and (b) for D = 0.013 cm at upper transport stage.

for the upper transport stage ($T_* > 3$) the particles take-off from the bed with velocity u_p inclined at an angle σ within the range from $10^0 - 30^0$ with respect to the horizontal bed surface.

Table 2 shows the calculated values of saltation heights by the present model and those obtained by using other models for lower and upper transport stages. In this table, pm indicates present model, last column in Table 2. The range of normalized saltation thickness $(h_s/D)_{pm}$ is from 0.7 to 4.6 depending on the grain size and transport stage. Sumer *et al.*'s (1996) data were used only for higher transport stage to estimate saltation heights assuming sand grain of density $\rho_s = 2.64 \mathrm{g/cm}^3$. It is also observed from the Fig. 4(a, b) that the dimensionless thickness of saltation layer $(h_s/D)_{pm}$ linearly increases with the transport stage T_* for D = 0.093 cm for lower flow regime and for D = 0.013 cm for upper flow regime, which agree well with Wilson and Nnadi (1990).

Mixed-Size Bed Materials

Experimental data of higher transport stage from Sengupta (1979) and Ghosh *et al.* (1981) are also utilized to compute the average particle velocity and the thickness of saltation layers of particles from the present models. Two closed circuit laboratory flumes (Ghosh *et al.* 1986; Sengupta *et al.* 1991), one designed at Uppsala University and other at the Indian Statistical Institute, Calcutta were used for conducting controlled experiments. Data from three different distributions of sand beds (Beds – 2, 3 and 5 in Ghosh *et al.* 1981) under high velocity conditions were selected for the present investigation. Fig. 5 shows the particle size distributions of sands used in the experiments. The three sand beds have the following type of grain size distributions: nearly uniform (bed 2), bimodal (bed 3) and positively skewed with slight bimodality (bed 5). Hydraulic parameters from the experiments used for verification of the proposed models are given in Table 3.

In a similar manner as above, values of the parameter a are calculated by utilizing Eqs.(11)-(14) for four friction velocities (u_* = 5.44, 6.14, 6.36 and 7.45 cm/s.) over three different sand beds (Bed – 2, 3 and 5 of Ghosh *et al.* 1981) for different grain sizes. Using the fitted values of $\tan\alpha = 0.84$ and $\tan\alpha_c = 0.6$ in the upper plane bed runs, the migration velocities of particles are calculated by Eq. (18) for different grain sizes (D = 0.099, 0.070, 0.049, 0.035 cm) in the beds. Similarly, the take-off angle σ for each transport stage T_* is obtained in the range $10^0 - 30^0$ by comparing the values of saltation layer thickness computed by the present model (Eq. (15)) with the values computed by Dietrich (1982) and Wiberg and Rubin (1989) for each size above the three sand beds. The computed values of saltation layer are shown in Table 4 for comparison purpose with the values of Dietrich (1982) and Wiberg and Rubin (1989). Fig. 6(a, b) shows the plots of dimensionless thickness of saltation layer $(h_s/D)_{pm}$ along with other two existing models against transport stage T_* for D = 0.099, 0.035 cm.; and it is seen that the normalized saltation height almost linearly increases with transport stage T_*. Fig. 7(a, b) shows the plots of the dimensionless

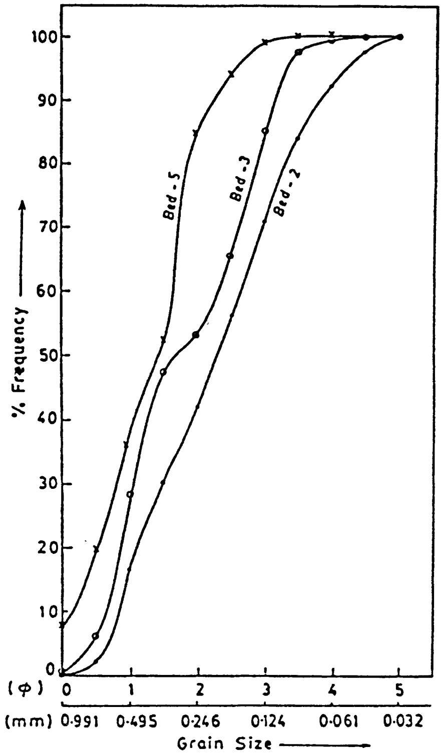

Fig. 5. Grain size distributions of sand beds 2, 3, and 5.

Table 3 - Flow parameters used for computation of migration velocities and saltation heights of particles. Data collected by Ghosh *et al.* (1981)

	Bed 2	Bed 3	Bed 3	Bed 5
Water depth d(cm)	30.000	30.0000	30.000	30.0000
Sand roughness ($k_s = D_{65}$) (cm)	0.031	0.0450	0.045	0.0520
Slope S	0.002	0.0016	0.003	0.0023
Temperature (^{0}C)	19.000	19.0000	21.000	27.0000
Shear velocity u_* (cm/sec)	6.140	5.4400	7.450	6.3600
Roughness Reynolds number $u_* k_s / \nu$	19.030	24.4800	33.530	33.0700
y_D (cm)	0.7D	0.5D	0.5D	0.5D

saltation height versus transport stage for different grain sizes by all three methods including the present method. Here it is observed that $(h_s/D)_{pm}$ computed by the present method compares well with that of Dietrich (1982).

It is here suggested that at the higher transport stage, whenever the particle leaves the bed, it lifts up at an angle σ and moves in trajectory with a velocity $u_p \sin\sigma$. Following the argument mentioned above, an important functional relationship between

Table 4 – Comparison of particle saltation layers by various methods including the present method (pm) for different beds' grain size distributions. Data collected by Ghosh *et al.* (1981).

D cm	τ_b / τ_c	a	u_p cm/sec	σ^0	$(h_s)_D$ cm	$(h_s)_{WR}$ cm	$(h_s)_{pm}$ cm
Bed - 2 at $u_* = 6.14$ cm/sec.							
0.0991	5.20	10.52	34.43	25	0.1516	0.2097	0.1732
0.0700	8.31	9.36	36.24	20	0.1311	0.1718	0.1489
0.0490	13.45	8.14	36.27	17	0.1072	0.1259	0.1090
0.0351	22.14	10.77	50.83	11	0.0860	0.0880	0.0840
Bed - 3 at $u_* = 5.44$ cm/sec.							
0.0991	4.70	9.21	23.44	30	0.1441	0.1971	0.1420
0.0700	6.50	8.38	25.00	25	0.1190	0.1533	0.1081
0.0490	10.50	7.15	25.12	22	0.1000	0.1168	0.0862
0.0351	15.80	9.55	37.26	14	0.0799	0.0825	0.0780
Bed - 3 at $u_* = 7.45$ cm/sec.							
0.0991	6.92	9.57	41.02	20	0.1730	0.2463	0.2120
0.0700	10.86	8.59	42.72	17	0.1440	0.1914	0.1521
0.0490	17.31	7.50	42.53	15	0.1141	0.1344	0.1179
0.0351	27.91	6.54	40.89	13	0.0893	0.0911	0.0945
Bed - 5 at $u_* = 6.36$ cm/sec.							
0.0991	5.60	9.20	32.04	25	0.1571	0.2190	0.1780
0.0700	8.50	8.60	34.72	20	0.1322	0.1735	0.1372
0.0490	14.10	7.14	40.02	15	0.1090	0.1276	0.1151
0.0351	23.00	11.10	54.57	10	0.0865	0.0885	0.0869

the transport stage T_* and the take-off angle σ has been established for the higher transport stage. This relationship is shown in Fig. 8 and it is valid for all the grain sizes *selected.* The best-fit equation predicted from the data with correlation co-efficient $r = 0.955$ is given by

$$\sigma = 0.038\, T_*^2 - 1.955\, T_* + 35.262 \quad \text{for} \quad T_* \geq 3 \qquad (19)$$
$$\sigma \simeq 90^0 \quad \text{for} \quad T_* < 3$$

which reveals that as the transport stage T_* increases, the take-off angle decreases. The decreasing trend of take-off angle with T_* is consistent with Nino *et al.*'s (1994) experimental results for gravel saltation. Using Eq. (19) it is possible to determine the take-off angle of particles of any size within the range 0.013 to 0.33cm from the

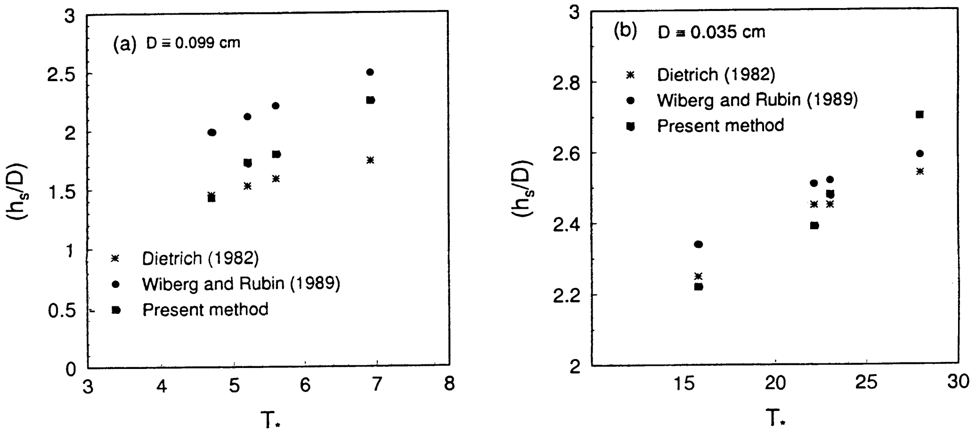

Fig. 6. Normalized saltation thickness (h_s/D) of particles against transport stage T_*, from all three beds; (a) for D = 0.099cm, and (b) for D = 0.035cm.

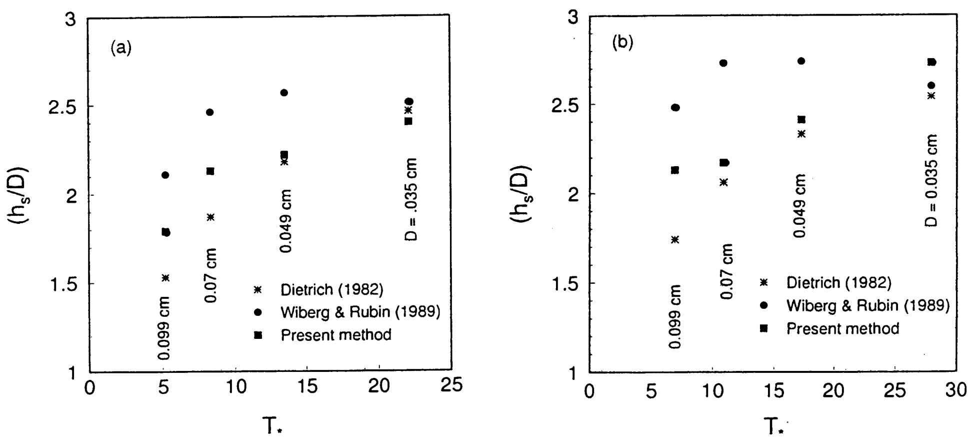

Fig. 7. Normalized saltation thickness (h_s/D) of particles against transport stage T_*, for different grain sizes; (a) for bed–2 at u_* = 6.14 cm/sec. and (b) for bed-3 at u_* = 7.45 cm/sec.

bed, if the transport stage T_* is known. The experimental observations over a larger range of grain sizes and transport stages would be valuable in studying more details of the variation of take-off angle σ with the transport stage. The saltation height of a particle is estimated from Eq. (15) when the velocity of that particle in the bed layer and take-off angle corresponding to the transport stage from the Eq. (19) are known.

We have developed a new method for estimation of saltation height of particles based on particle migration velocity in an open channel flow. This method is superior to other methods because it allows us to incorporate the effects of various physical parameters in the computation unlike the existing methods.

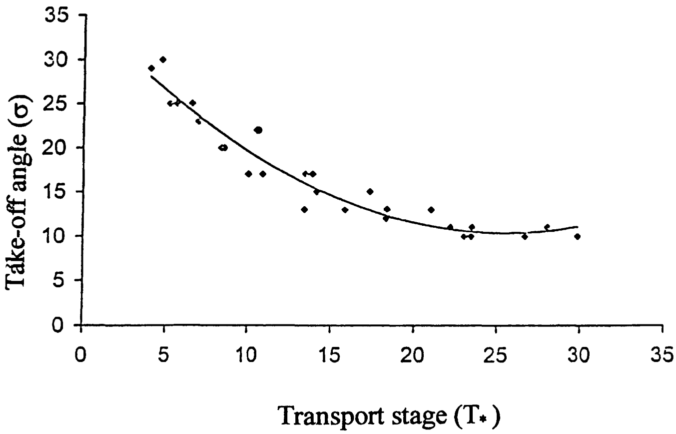

Fig. 8. Take-off angle of particle lifting from the bed plotted against the transport stage T_* for any grain size within the range from 0.013 to 0.33cm. Continuous line represents fitted Eq. (19).

Conclusions

The migration velocities of sediment particles in the bed layer are computed from the equilibrium condition of drag and stabilizing forces of particles in the bed. The mean particle velocity at the bed is a function of shear velocity, critical shear velocity corresponding to Shield's grain movement, ratio of drag and lift coefficients and settling velocity of particle. The computed velocity of a representative size is comparable to the fluid velocity at the bed layer which is computed by Mazumder (1994) using an extrapolation method. The real merit of the present theoretical model to estimate the thickness of saltation layer is that the concept of mean horizontal velocity of any bed load grain is adopted to the energy balance equation instead of friction velocity unlike the existing models. Moreover, a significant contribution of this paper is a development of new functional dependence of saltation height on particle velocity, take-off angle and added mass coefficient. A quadratic relationship is suggested between the upper transport stage ($T_* > 3$) *vs*. the particle's take-off angle σ, and it reveals that for a given grain size, the take-off angle decreases with increase in transport stage, and it varies between 10^0 to 30^0 for the transport stage ($T_* > 3$). Particles with larger migration velocities tend to produce larger bed layer thickness. The results obtained for particle velocities and saltation layers by the present model compare well with the observations and computations made by other researchers. This research will have significant effect on determining the total roughness of near-bed sediment transport due to moving sediments in open channel flow.

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Address:
Flume Laboratory,
Physics and Earth Sciences division,
Indian Statistical Institute,
203 Barackpore Trunk Road,
Calcutta 700 035,
India.
Email: bijoy@isical.ac.in