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APPENDIX A

Spectral-Density-Averaging for Rotordynamic-Coefficient Identification

Spectral density averaging techniques are used to obtain the frequency-response functions (FRF) of Eq. (7), which defines a multi-input multi-output (MIMO) system. The inputs are D_{xx}, D_{xy}, D_{yx}, and D_{yy}, and the outputs are $F_{xx} - M_s A_{xx}$, $F_{xy} - M_s A_{xy}$, $F_{yx} - M_s A_{yx}$, and $F_{yy} - M_s A_{yy}$.

Before calculating the FRF for this MIMO system, first consider the best solution for the FRF of a single-input single-output (SISO) system. A SISO system with a clean input a and a noise contaminated output b will be examined. According to Bendat and Peirsol (1986), an unbiased estimate H, of the FRF for this type of SISO system is given by,

$$H = \frac{G_{ab}}{G_{aa}}, \tag{A.1}$$

with G_{ab} being the cross spectral density and G_{aa} the auto spectral density. For discrete data, (which we have) an FFT algorithm is used to calculate the Discrete Fourier Transforms (DFT's) of Eq. (A.1). In particular, the spectral densities are calculated by,

$$G_{ap} = \frac{2}{n_d N \Delta t} \sum_{i=1}^{n_d} A_i^*(\omega) P_i(\omega) \tag{A.2}$$

where n_d is the number of statistically independent blocks of data, N is the number of data points in each data block, Δt is the time increment between sampling of each data point, $A^*(\omega)$ = the complex conjugate of $\mathfrak{F}\{a(t)\}$ and $P(\omega) = \mathfrak{F}\{p(t)\}$ ($\mathfrak{F}$ is used to represent the DFT here).

We wish to apply the unbiased estimator for the SISO system described above to the MIMO system of Eq. (7). First, recall that we have 32 statistically independent blocks of data ($n_d = 32$), each block containing 1024 ($N = 1024$) data points for each of the dynamic loads, accelerations, and relative motions whose $\mathfrak{F}$ appear in Eq. (7). Next, we identify each independent dynamic load of Eq. (7) (F_{xx} and F_{yy}) as a clean input a, and each of the resulting elements of the force and displacement matrices as noise contaminated outputs b. Eq. (A.1) is then applied separately to each individual element of the force and displacement matrices, yielding an unbiased estimate for each input and output element contained in these matrices. The resulting equation,

$$\begin{bmatrix} 1 - M_s \dfrac{G_{f_{xx}a_{xx}}}{G_{f_{xx}f_{xx}}} & \dfrac{G_{f_{yy}f_{xy}}}{G_{f_{yy}f_{yy}}} - M_s \dfrac{G_{f_{yy}a_{xy}}}{G_{f_{yy}f_{yy}}} \\ \dfrac{G_{f_{xx}f_{yx}}}{G_{f_{xx}f_{xx}}} - M_s \dfrac{G_{f_{xx}a_{yx}}}{G_{f_{xx}f_{xx}}} & 1 - M_s \dfrac{G_{f_{yy}a_{yy}}}{G_{f_{yy}f_{yy}}} \end{bmatrix} = \begin{bmatrix} H_{xx} & H_{xy} \\ H_{yx} & H_{yy} \end{bmatrix} \begin{bmatrix} \dfrac{G_{f_{xx}d_{xx}}}{G_{f_{xx}f_{xx}}} & \dfrac{G_{f_{yy}d_{xy}}}{G_{f_{yy}f_{yy}}} \\ \dfrac{G_{f_{xx}d_{yx}}}{G_{f_{xx}f_{xx}}} & \dfrac{G_{f_{yy}d_{yy}}}{G_{f_{yy}f_{yy}}} \end{bmatrix}, \tag{A.3}$$

yields improved estimates for the FRF when solved.

Once Eq. (A.3) is solved, the rotordynamic coefficients are extracted from the FRF definitions of Eqs. (6) by performing a least-squares curve fit on the real and imaginary parts of each element of the FRF matrix. Plots of typical real and imaginary parts for Hxx are shown in Fig. 6. The stiffness, damping, and added-mass coefficients of Eqs. (1) and (3) have now been identified. A more detailed discussion is provided by Rouvas and Childs (1992).

DISCUSSION

J. Scharrer[4]

The authors are to be congratulated on their development of a first class hydrostatic bearing test facility. The data presented in this publication and that presented in future publications will provide a valuable database for the validation of computer models and design tools. While this paper is very complete in its presentation, I would like to focus on several points.

The paper focuses on testing hydrostatic bearings. Are there limitations inherent in the design of the tester which would prevent it from being used for annular seals, foil bearings, or even magnetic bearings?

Also, the authors state that the noise generated by hydrostatic bearing is on the same order of magnitude as the excitation force. Have the authors considered an approach which uses this noise to excite the stator instead of using an external excitation source? The transfer function data shown in Fig. 6 seems very clean and tightly grouped. Did the authors perform any tests or analyses to arrive at the 32 excitation average or was this simply a limitation of the system?

It is admirable that the authors have included an estimate of the experimental uncertainty with their results. Could the authors expand the discussion and explain in more detail the process used to obtain uncertainties for the stiffness, damping, and inertia coefficients? Were uncertainties related to the data acquisition hardware accounted for in addition those of the transducers? Also, it has been my experience that eddy-current proximity probes similar to those used for this effort have frequency dependent amplitude and phase characteristics. Was this accounted for in the uncertainty analysis and/or the calibration of the test apparatus?

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Authors' Closure

Concerning alternate measurement applications for the test apparatus, rotordynamic coefficients have been measured for high-speed annular liquid seals, Lindsey (1993). The tester could be used to measure rotordynamic coefficients of a range of force elements including laminar oil seals for compressors, foil bearings, magnetic bearings, conventional hydrodynamic bearings, or rolling-element bearings. The hydrostatic shaker heads can be used in "force" mode as described here or in "displacement" mode. In the displacement mode, a prescribed displacement (or relative displacement across the test element) is commanded. Hence the stator can be "held" and excited which is very useful in measuring rotordynamic coefficients of "soft" elements such as foil bearings. For "hard" elements such as rolling-element bearings, the force mode is preferable and has been used for hydrostatic bearings with high radial stiffnesses 350 KN/mm (2×10^6 lb/in).

Concerning the use of incidental noise within the bearing to excite the rotor, the authors have considered this option on a *theoretical* basis and are divided concerning the outcome. From a practical viewpoint, the use of noise would be a lengthy and difficult approach. Our experience with low-energy spectra (developed by impulse hammers, Rouvas and Murphy (1992)) yielded usable rotordynamic coefficients but only after protracted and lengthy testing. Noise input would invalidate the PSD basis of our current identification approach, given the absence of a "clean" signal.

The 32 excitation averages are not a limitation of the system. Before the coefficient uncertainty analysis was added to the identification algorithm, the number of averages was selected by visual inspection of the "tightness" of the FRF curves and the proximity of interchannel coherences to 1. The uncertainty analysis now identifies the precision (random) error contribution to the total uncertainty. This precision error contribution can be adjusted by adjusting the number of averages. However, the statistical calculations are simplified if a minimum of 30 averages is maintained.

Concerning expanding the discussion on experimental uncertainties, the detailed groundwork necessary to convey the process by which the uncertainties were identified is outside the scope of this paper. The authors plan to submit a separate paper which addresses this issue solely. Uncertainties related to the data acquisition hardware as well as the transducers were accounted for. Accelerometers, load cells, and motion transducers were attached to the bearings to aid in identifying the rotordynamic coefficients. All these transducers have frequency dependent amplitude and phase characteristics. While the static amplitude characteristics are well defined by pre-test calibrations, only the accelerometer manufacturer publishes amplitude frequency response data with each transducer. To account for this, the worst case published amplitude specifications for the other transducers were taken to extend over the entire bandwidth of the tester. No manufacturer published phase frequency response data with their transducer, but all claimed "insignificant" phase errors at the testers bandwidth. However, in view of information we had received from unpublished tests by users of the motion probes, and in accordance with prior experience from [5]TAMU tests, a phase accuracy of 1° was taken to extend over the bandwidth of the tester. If published data from independent tests of any of these type transducers are available, the authors would certainly appreciate access to it.

Additional Reference

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[5]A transducer conformity test was conducted out to 200 Hz only. A preloaded load cell, an accelerometer, and an eddy-current motion probe were all mounted to the same "target" on a shaker system. The phase measuring instrument was only accurate to 1° and all three transducers stayed within 1° phase relative to each other throughout a 30 Hz to 200 Hz bandwidth.