Assessment of uncertainty in soil test phosphorus using kriging techniques and sequential Gaussian simulation: implications for water quality management in southern Quebec

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ABSTRACT

Missisquoi Bay, located in southern Quebec at the north-eastern extremity of Lake Champlain, is subject to eutrophication arising from excess nutrients, predominantly phosphorus (P), contributed by agricultural watersheds. Factors such as land use pattern, management practices, soil properties and geomorphology have an impact on soil P levels. Land patches under different management practices introduce a cyclic pattern, especially when fitting the variogram. Geostatistics procedures were used to model soil test phosphorus (STP) within the 11 km² Castor Watershed in southern Quebec, Canada. An ordinary kriging (OK) method was used to estimate STP at unsampled points, but this had a smoothing effect, resulting in an underestimation of high values and overestimation of low values. Therefore, a more efficient technique was needed to draw predictions from the conditional probability distribution at the simulation grid nodes. A sequential Gaussian simulation (SGS) was adopted for this purpose, and used to create 50 equal probable realizations. Uncertainty was modelled using the E-type (mean) of the realizations, which ranged from 12.5 to 223 mg P kg⁻¹ soil. The adequate spatial probability pattern for STP is a valuable criterion when seeking to delineate areas of high STP for site-specific best management practices (BMPs).

Key words | ordinary kriging, phosphorus, probability map, sequential Gaussian simulation, soil test, uncertainty

INTRODUCTION

Globally, non-point source pollution (NPS), such as nutrient losses from agricultural watersheds, can lead to the degradation of freshwater resources (Carpenter et al. 1998; Hegman et al. 1999; Beauchemin & Simard 2000). Freshwater pollution can come from point sources (PS) such as community treatment plants, agricultural dairy effluents or industrial setups such as fertilizer manufacturing plants. Over the years, the government policies on water pollution control have helped to minimize PS pollution. On the other hand, nutrient losses – mostly nitrogen (N) and phosphorus (P) – from agricultural fields are very difficult to control or monitor (Sharpley 2007). N and P losses from agricultural fields that exceed certain pre-defined limits lead to the enrichment of surface waters causing what is termed as eutrophication.

In recent years, the Missisquoi Bay (north-eastern portion of Lake Champlain) in southern Quebec, Canada, has experienced eutrophication due to excess nutrient losses from agricultural watersheds (Hegman et al. 1999; Jamieson et al. 2003; Deslandes et al. 2004; Medalie & Smeltzer 2004; Michaud 2004; Michaud et al. 2004, 2005). This has been attributed to the consistent application of fertilizers and organic manures and a ‘lack of prior knowledge’ about the nutrient richness and status of soils in this area over the years (Hegman et al. 1999; LCBP 2004; Beaudin et al. 2005). This has been reported to have very serious impacts on tourism, recreation and aquatic life in the basin (Priskin 2008).

Differences in soil types and properties generally lead to the differences in the nutrient status across a field, although...
other external factors such as agricultural practices, management practices, geomorphology, etc. may also be involved. Crop growth and yield depend heavily on the agricultural field’s nutrient status, which may vary widely across the field. This information is very important to the farmers in order to apply the ‘exact’ quantity of nutrient to meet the needs of the crop, particularly in precision agriculture. This variability occurs in both space and time (Geypens et al. 1999), which poses a challenge in finding an accurate covariance function for variogram in spatial modelling. In general, soil N and P are considered to be highly variable compared to other soil properties (Geypens et al. 1999; Bennett et al. 2004; Grunwald et al. 2004).

The land use pattern is another factor that strongly affects the spatial distribution of soil test phosphorus (STP). Arable lands often receive large mineral and organic fertilizer inputs, which make large quantities of P available for loss through erosion or leaching.

Spatial probability mapping and quantification of uncertainty are very important criteria when one wants to delineate areas of high and low STP in order to implement site-specific best management practices (BMPs). Spatial modelling involves both estimation and simulation. Ordinary kriging (OK) is one of the most efficient methods in spatial estimation, while a sequential Gaussian simulation (SGS) can be used for simulation. SGS involves using a Monte Carlo technique to draw predictions from the conditional probability distribution at each of the grid nodes. In other words, SGS involves generating realizations. According to Remy et al. (2006), each realization can be obtained by: (i) defining a random path visiting each node of the grid, (ii) obtaining, for each node along the path, a data set for the area in the neighbourhood that includes the observed data set and prior simulated values, (iii) estimating the local conditional cumulative distribution function (ccdf) as a Gaussian distribution with mean given by simple kriging and its variance by the kriging variance, (iv) drawing a value from the Gaussian ccdf and adding the simulated value to the data set, and (v) repeating for another realization.

One of the major disadvantages of the kriging method is that it has a smoothing effect (Goovaerts 1997; Juang et al. 2004), resulting in small values being overestimated and large values being underestimated (Goovaerts 1997; Juang et al. 2004). The smoothening effect obstructs the understanding of the variability and detection of patterns of extreme values, such as areas with high STP (Goovaerts 1997; Juang et al. 2004). Thus the kriging output only represents a simplistic pattern and does not reproduce spatial detail. This shortcoming can result in a misleading interpretation when the goal of the research is to characterize the areas that have a large amount of STP for site-specific BMPs in the form of management zones. Juang et al. (2004) reported that the kriging value at each unsampled location includes an estimation variance, which can cause uncertainty in mapping. Considering this uncertainty is very important, especially when determining ‘hot spots’ of high STP, which, if linked with P transport pathways, have the potential to cause excess P transport into fresh water.

SGS is a method that is able to determine the multi-point or global uncertainties of STP using the conditional probability distribution of simulated and observed data at the simulation grid nodes (Juang et al. 2004; Bivand et al. 2008). SGS allows the quantification of uncertainty at several locations simultaneously. This is required in order to assess the reliability of an area in the form of management zones for site-specific BMP applications, as opposed to the estimated output from kriging methods (Goovaerts 1997; Grunwald et al. 2004; Juang et al. 2004). SGS has been reported to overcome the smoothing limitations of ordinary kriging (Goovaerts 1997; Grunwald et al. 2004; Juang et al. 2004). SGS takes into consideration not only the spatial variation of observed data at sampled locations, but also the variation in estimation at unsampled locations, which kriging ignores (Grunwald et al. 2004; Juang et al. 2004). Evaluations of uncertainty for all possible simulation nodes are plausible. Finally, SGS can collect mapping uncertainties at several locations simultaneously, which is ‘masked’ in ordinary kriging.

In this paper, however, the OK method was used to estimate the STP at unsampled locations. The shortcomings or limitations of OK in terms of the smoothing effect were evaluated. SGS was used to quantify the uncertainties at unsampled locations by generating realizations that have equal probabilities. These realizations honour the conditional data set in each of the simulation nodes. Probability maps above recommended thresholds were developed using SGS and OK. Such a study is very important, especially for the Missisquoi Bay where there are recurring issues of cyanobacteria blooms from the influx of P from agricultural fields.
MATERIALS AND METHODS

Study area

Situated near Bedford, QC, the 11 km² Castor watershed is the smallest of a number of sub-watersheds in the Pike River Watershed (Figure 1) (Beaudin et al. 2005; Deslandes et al. 2007). It feeds into the downstream portion of the Pike River, which has its outlet in the Missisquoi Bay, in the north-eastern portion of Lake Champlain. The Castor watershed is in an area of intensive agricultural activity, where land is cropped 44% to corn (Zea mays L.), 28% to grass (hay) and 20% to cereals (Beaudin et al. 2005). Other agricultural activities include swine, poultry and dairy production. Ranging from 36 to 49 m AMSE, the Castor watershed is relatively flat. This watershed has a total of six soil classes according to the Quebec classification (Michaud 2004), of which the largest proportion (by area) are fine textured (Figure 2). This results in low water infiltration and poor drainage, which along with wet antecedent moisture conditions and snow melt have contributed to flooding events in recent years. Consequently, sub-surface drains are installed on most farms to remove excess water from the agricultural fields. The water table in this region is reported to be very shallow (Beaudin et al. 2005; Deslandes et al. 2007). The mean long-term annual precipitation of this study area is about 1,057 mm (MDDEPQ 2003; Michaud 2004; MDDEPQ 2005; MENV 2006).

The major soil series of the watershed are (Cann et al. 1946; Cattal 2004; Michaud 2004):

- Ste Rosalie: poorly drained lacustrine and marine clays
- Bedford an Orthic Humic Gleysol, fine-loamy mixed calcareous
- Ste Brigide: an Orthic Humic Gleysol, coarse-loamy mixed calcareous
- St Sebastien: Gleyed Sombric Brunisol, loamy-skeletal, mixed, monoacid, mesic.

Field sampling technique

A 100 m × 100 m grid (being the smallest that could be used) was imposed over the entire area of the Castor watershed, and each cell had a geo-referenced coordinate at its centre (Figure 3). The study area was divided into three sub-basins (Figure 3) (Beaudin et al. 2005). The soil series served as strata for a stratified random sampling, which covered the entire spatial extent of the
study area. Due to limited financial resources and time constraints, it was only feasible to obtain a total of 211 sampling points, geo-referenced and imported into a Garmin Oregon 450 Global Positioning System (GPS). Webster & Oliver (1992) reported that 100–150 sampling points are sufficient to compute a reliable variogram, especially in the area where the spatial variation is isotropic. This (variogram) aids in developing and estimating spatial structure and dependency of the variable of interest and also is a requirement for spatial interpolation using a kriging technique. ArcGIS 10 software (Environmental Systems Research Institute, Redlands, CA) was used to conduct the geostatistical estimation, simulation and visualization. Each of the points was geo-referenced and imported into a GPS, which has a positional accuracy of ±2 m. A sampling depth of 0–0.30 m was used in this study. This is the most active layer in terms of P fate and transport. The soil samples were all air-dried, and sieved through a 2 mm size sieve.

Laboratory analysis

STP was extracted from soil using the Mehlich-3 (Mehlich 1984) extractant. The samples were analysed following the procedure of Sims (2000). An air-dried soil sample (2.5 g) was mixed with 25 mL of Mehlich-3 extractant and shaken for 5 min. at 200 rpm, then filtered through Whatman No. 42 filter paper. The sample filtrates were transferred into the auto-sampler rack of an inductively coupled plasma-optical emission spectrometer (ICP-OES) to assess STP levels. The ICP can analyse about 120 samples for 10 elements within 7 hours.

Variogram analysis

Matheron (1970) described geostatistics as a field that deals with a random field $X$ that is linked to a multi-dimensional locus $u$, and there exists a relationship between $X(u)$ and $X(u + h)$, where $h$ is the lag-distance (Ploner & Dutter 2000; Barnes 2011). A variogram, a term used here synonymously with semi-variogram, characterizes the spatial continuity or spatial roughness of a spatial data set (Bivand et al. 2008). The variogram plots the semi-variance as a function of distance $h$.

One must make a stationary assumption in order to estimate the spatial correlation of the observation data (Bivand et al. 2008). A very basic form of assumption of stationarity is that of intrinsic stationarity. This assumes that the process that generated the observation data is a random function $X(u)$, which can be expressed as:

$$X(u) = \mu + \epsilon'(u)$$

where:

- $X(u)$ is a random function at a location $u$
- $\mu$ is the constant mean
- $\epsilon'(u)$ is the residual or error term.
A constant mean, $\mu$, is the expectation of $X(u)$:

$$E(X(u)) = \mu$$ (2)

Therefore, under this assumption, the variogram, $\gamma(h)$ can be defined as:

$$\gamma(h) = \frac{1}{2} E[X(u) - X(u + h)]^2$$ (3)

where $h$ is the separation distance.

From Equation (3), the variance of $X$ is constant and the spatial correlation of $X$ depends not on location $u$, but on the separation distance $h$ (Bivand et al. 2008).

If ‘isotropy’ is assumed in Equation (3), $h$ can be replaced with $||h||$. This means that the variogram has directional independence (Bivand et al. 2008). Using this isotropic assumption, the variogram from $N_l$ sample with data pairs $X(u_i + h)$, $X(u_i + h)$ for a number of distances $h_j$ (distance intervals) can be estimated as (Bivand et al. 2008):

$$\Gamma(h_j) = \frac{1}{2N_l} \sum_{i=1}^{N_l} [(X(u_i) - X(u_i + h)]^2, \forall h \in h_j$$ (4)

where $\Gamma$ is the estimate of the sample variogram.

In many geostatistics software products, there are four basic covariance functions (variogram models) along with any positive linear combinations of them that generate the linear model of regionalization (Goovaerts 1997). These four models are (Remy 2004):

(i) Nugget effect model

$$\gamma(h) = \begin{cases} 0 & \text{if } ||h|| = 0 \\ 1 & \text{otherwise} \end{cases}$$ (5)

(ii) Spherical model with actual range $a$

$$\gamma(h) = \begin{cases} \frac{3}{2a} ||h|| - \frac{1}{2} \left( \frac{||h||}{a} \right)^3 & \text{if } ||h|| \leq a \\ 1 & \text{otherwise} \end{cases}$$ (6)

(iii) Exponential model with practical range $a$

$$\gamma(h) = 1 - \exp \left( -\frac{3||h||}{a} \right)$$ (7)

(iv) Gaussian model with practical range $a$

$$\gamma(h) = 1 - \exp \left( -\frac{3||h||^2}{a^2} \right)$$ (8)

These four covariance functions (Equations (5)–(8)) are those most commonly used in geostatistics analysis. These models have a monotonic property (Pyrcz & Deutsch 2003). However, there are some situations where the experimental variogram exhibits some cyclic pattern. Presenting a non-monotonic continuity structure, this kind of variogram is often called a hole effect variogram (Pyrcz & Deutsch 2003). Neglecting these non-monotonic structures in the analysis may result in misinterpretations in heterogeneity models, such that they do not reproduce the observed patterns of variability (Pyrcz & Deutsch 2003).

The mathematical expression that represents a hole effect can be given as (Pyrcz & Deutsch 2003):

$$\gamma(h) = c_n \left[ 1 - \cos \left( \frac{h\pi}{a_n} \right) \right]$$ (9)

where:

$c_n$ is the partial sill (variance contribution) and $a_n$ is the range parameter, and other parameters are as defined above.

**Ordinary kriging**

Ordinary kriging assumes that the means ($\mu$) are constant in the local neighbourhood of each estimation point rather than the entire domain, that is (Isaaks & Srivastava 1989; Goovaerts 1997; Bohling 2005; Bivand et al. 2008):

$$\mu(u_o) = \mu(u)$$ (10)

where $u_o, u$ are location vectors for estimation point and one of the neighbouring data points, indexed by $a$.

The kriging estimator can be written as (Bohling 2005):

$$X(u) = \mu(u) + \sum_{a=1}^{n(u)} \lambda_a(u)[X(u_a) - \mu(u)]$$ (11)
where:

\( n(u) \) is the number of data points in the local neighbourhood used for estimation of \( X(u) \).

\( \lambda_a(u) \) is the kriging weight assigned to datum \( X(u_a) \) for estimation location \( u \).

Equation (11) can further be expressed in terms of the expected values of \( X(u) \) and \( X(u_a) \) as Bohling (2005):

\[
X(u) - m(u) = \sum_{a=1}^{n(u)} \lambda_a(u) [X(u_a) - m(u_a)]
\]

(12)

where \( m(u), m(u_a) \) are expected values (means) of \( X(u) \) and \( X(u_a) \), respectively.

**Sequential Gaussian simulation**

The procedure for generating a sequential Gaussian simulation consists in defining a regularly spaced grid that covers the entire watershed area and thereby establishes a random path for the entire grid. The process follows the steps enumerated below (Chilès & Delfin 1999; Bourennane et al. 2007; Bivand et al. 2008; Delbari et al. 2009).

(a) **Transform variable:** This constitutes a statistical transformation of the variable of interest to normality using a multi-Gaussianity assumption. It is possible to transform the marginal distribution of the variable into a Gaussian distribution and simulation is performed on the transformed variable (Remy 2004). The transformation of the variable \( X(u) \) with the cumulative distribution function \( F_z \) into a standard normal variable \( Y(u) \) with the cdf \( G \) is given as (Remy 2004):

\[
Y(u) = \frac{F[X(u)]}{G}
\]

(13)

(b) **Estimate covariance:** Calculation of the covariance of the transformed values in (a).

(c) **Grid path analysis:** Definition of a grid path so as to visit all the nodes to be simulated and so that no empty node is visited along the path.

(d) **Estimate using Simple Kriging (SK):** Estimation of simple kriging mean and variance conditioned by the available data and prior predicted values.

(e) **Apply Monte Carlo technique:** The Monte Carlo technique will be used to draw a value from the conditional Gaussian distribution as defined by the SK mean and variance.

(f) **Add simulated value:** Add the simulated value to the conditioning data set.

(g) **Iterate all nodes:** Return to (c) until all nodes are visited.

Back transform the simulated values: Transformation to the original values to obtain a realization (Remy 2004), e.g. transforming the simulated values \( y_1, y_2, \ldots y_N \) into \( x_1, x_2 \ldots x_N \) such that:

\[
x_i = F^{-1}(G(y_i)) \quad i = 1, 2, \ldots N
\]

(14)

**RESULTS AND DISCUSSION**

**STP data analysis**

Descriptive statistics of STP levels across the entire the Castor watershed are shown in Table 1. Over the full watershed there was a wide range of STP values (4.29 to 306.55 mg kg\(^{-1}\)). As shown in Figure 4, where the whiskers indicate the 10th and 90th percentiles of measured STP values, these varied according to land use. Pastures showed

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Full watershed</th>
<th>Pasture</th>
<th>Low-P (residential, tall grass, forest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>71.0</td>
<td>91</td>
<td>76</td>
</tr>
<tr>
<td>Standard error</td>
<td>3.9</td>
<td>15.8</td>
<td>4.8</td>
</tr>
<tr>
<td>Median</td>
<td>55</td>
<td>69</td>
<td>58</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>57</td>
<td>61</td>
<td>57</td>
</tr>
<tr>
<td>Sample variance</td>
<td>3356</td>
<td>3762.4</td>
<td>3299.9</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.70</td>
<td>-0.59</td>
<td>3.37</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.86</td>
<td>0.88</td>
<td>1.77</td>
</tr>
<tr>
<td>Range</td>
<td>302</td>
<td>189.5</td>
<td>301.2</td>
</tr>
<tr>
<td>Minimum</td>
<td>4.29</td>
<td>21</td>
<td>5.2</td>
</tr>
<tr>
<td>Maximum</td>
<td>306.55</td>
<td>211.54</td>
<td>306.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Arable (corn and soy)</th>
<th>Low-P (residential, tall grass, forest)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>71.0</td>
<td>49</td>
</tr>
<tr>
<td>Standard error</td>
<td>3.9</td>
<td>7.3</td>
</tr>
<tr>
<td>Median</td>
<td>55</td>
<td>37</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>57</td>
<td>53</td>
</tr>
<tr>
<td>Sample variance</td>
<td>3356</td>
<td>2830.1</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.70</td>
<td>12.14</td>
</tr>
<tr>
<td>Skewness</td>
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<td>3.21</td>
</tr>
<tr>
<td>Range</td>
<td>302</td>
<td>298.6</td>
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<tr>
<td>Minimum</td>
<td>4.29</td>
<td>4.2</td>
</tr>
<tr>
<td>Maximum</td>
<td>306.55</td>
<td>302.97</td>
</tr>
</tbody>
</table>
the highest STP levels, while forested areas showed the lowest. This makes sense since forests generally do not receive P inputs as either manure or fertilizer.

Kriging is based on the stationarity assumption for the mean value. Given the wide variability in STP levels it would be a challenge to develop an efficient interpolated surface for these unstructured and noisy data using this method; however, these data clearly showed a distinct and less variable mean value according to the land use pattern. Accordingly, for the kriging assumption to be valid for this study, the data were divided into three land use categories, based on their estimated STP values. Descriptive statistics for these categories are presented in Table 1:

- Pasture fields
- Arable lands: corn and soy fields
- Low P land use areas: residential, grasslands (prairie tall grasses) and forested areas.

The highest STP values were found in areas falling into the arable lands category. Therefore using this land use class as a representative of the study will not mask high STP areas. Also, there are similarities between the descriptive statistics of STP for the lumped land use classes and the arable lands (Table 1). This will allow us to assume stationarity of OK. Therefore, the data used in this research were reduced to arable lands only.

A one-way analysis of variance (ANOVA) (unequal sample sizes) was applied for the groups since the annual phosphorus application and its effects were considered as the only factor across the categories. The mean values for the pastures, arable lands and low-P land categories were 91, 77 and 49 mg kg⁻¹, respectively. There was evaluation done on whether differences between these means were statistically significant or simply based on random sampling error. The test result showed an F-value of 5.673, while the critical F was 3.039 (α = 0.05). This resulted in a P-value of 0.00399. The null hypothesis (equal means) was rejected, indicating that the difference between these groups was not due to random sampling error. Therefore, it would not be correct to combine and model the three groups together. In fact, it would be a challenge to get a good covariance function fit for such combinations. Arable lands (corn and soy) were therefore chosen for spatial modelling.

**Variogram and spatial modelling**

Expert knowledge (through the authors’ experience in geostatistics estimation of soil attributes) was used to generate the covariance function for the experimental variogram (Figure 5). There is evidence of a hole effect in the experimental variogram. Therefore a hole effect covariance function was fitted to the data.

The spatial patterns of STP obtained using the OK and STP (E-type, i.e. mean of realizations) are shown in Figures 6(a) and 6(b), respectively. The higher values were close to the areas associated with intensive agriculture and animal husbandry. The STP map generated by OK can be seen to be much smoother compared with that derived from the SGS realization technique. SGS method generated 50 realizations where a random path was defined and the algorithm visited each node of the grids using the measured values as conditioning data. In terms of model performance, the OK method resulted in coefficients of correlation and determination of 0.85 and 0.73, respectively (Figure 7).

The spatial uncertainty obtained using SGS is shown for two realizations (Figures 8(a) and 8(b)). Figures 9(a) and 9(b) capture the best and worst scenarios (minimum and maximum) developed using SGS. Stochastic simulation using SGS generated multiple equiprobable realizations of STP compared to the mean estimation using OK (Bohling...
In essence, SGS technique added some noise to the simulation in order to overcome the smoothing effect using OK (Bohling 2005). From the foregoing, SGS gives a better representation of the spatial variability of STP and provides a means for quantifying the uncertainty. Therefore, coupled with the E-type map, these SGS maps provide more realistic estimates of actual conditions than a single output map using the OK technique (Grunwald et al. 2004).
A threshold of 60 mg kg\(^{-1}\) STP was recommended by Sharpley et al. (2003) as being the necessary level of P in the soil for plant uptake and sustainable growth. Above this value, there is a propensity for P to be washed into fresh water during runoff or erosion (if the land is exposed).

This threshold value was used to develop probability maps using both OK and SGS methods (Figure 10). SGS estimated lower probabilities in the risk class [0.8 to 1.0] with 18% of all pixels above threshold, while OK estimated probabilities in the risk class [0.8 to 1.0] in 14% of all pixels. This can be attributed to OK having overestimated low values or underestimated high values (Isaaks & Srivastava 1989; Webster & Oliver 2001; Grunwald et al. 2004). Probabilities of P loss exceeding 0.50 accounted for 48 and 49% of all pixels when using the SGS and OK methods, respectively. Some areas classified as exceeding the threshold in OK were below the threshold in SGS. Clearly, OK and SGS methods arrived at different results from the same data set.

Implications for water quality and site-specific nutrient management in Quebec

These results have some impact on water quality management, especially for Quebec. The usual method of
Figure 9  | Scenarios for soil test phosphorus developed using sequential Gaussian simulation, (a) best (minimum) scenario, (b) worst (maximum) scenario.

Figure 10  | Probability of exceeding the soil test phosphorus threshold of 60 mg kg\(^{-1}\) determined using (a) ordinary kriging and (b) sequential Gaussian simulation.
mapping soil attributes (kriging) has inherent uncertainty and a smoothing effect, such that decision-making using this method may be misleading. Low and high values of STP may be masked. The arable lands in the Beaver watershed have a mean STP of 70 mg kg\(^{-1}\), which exceeds the recommended threshold of 60 mg kg\(^{-1}\) necessary for plant growth. The excess 10 mg kg\(^{-1}\) has the potential of being washed into fresh water systems. From a lake management perspective, there is a standing agreement between the province of Quebec and the State of Vermont for a target reduction of 97.2 Mg yr\(^{-1}\) in the total P load entering Missisquoi Bay. The reduction goal for Vermont is 58.3 Mg yr\(^{-1}\) (60%), while that for Quebec is 38.9 Mg yr\(^{-1}\) (40%) (Vermont, New York and Quebec, 1996; Medalie & Smeltzer 2004). Quebec has yet to meet this target, as recurrent algal blooms in the bay support this claim (MDDEPQ 2009; CBS 2013). The P and sediment fluxes are coming from intensive agricultural fields (including this study site).

Therefore, there is a need to intensify the implementation of various BMPs that can reduce nutrient flux into fresh water. Some of these include conservation tillage and buffer strips, as well as crop nutrient management. Farmers in these areas should be encouraged to apply only the right amount of fertilizer and manures. Excessive applications of these build up in the soil and have the potential to flow into surface and ground water. Also, conservation buffer strips have the ability to slow down runoff and trap nutrient and sediment from entering fresh water. The riparian vegetation of at least 3 m (Bradley et al. 2011; LCBP 2012) from the stream banks has the potential to reduce the movement of nutrients and sediments into streams and lakes.

Overall, this research provides a salient guide for site-specific BMPs for high STP areas identified in Figures 8(a) and 8(b), using the aforementioned BMPs, recommendations and much more.

Overall, the uncertainty quantification of highly variable soil parameters like STP requires careful statistical consideration. Whenever the desire is to classify the watershed into areas greater than some pre-defined limit in terms of a threshold STP concentration, SGS gives more realistic maps and scenarios instead of a single map derived in OK.

The major advantage of the SGS technique over methods such as OK or a Thiessen Polygon (TP) approach is that SGS provides an estimate of both the mean and the standard deviation of the variable at each grid node (Lin 2008). This implies that the variable can be represented at each grid node as a random variable following a Gaussian distribution (Lin 2008). Instead of using the mean as the estimate at each node, SGS chooses a random deviate (through a Monte Carlo technique) from this Gaussian distribution, selected according to a uniform random number representing the probability level (Lin 2008). SGS generates a set of realizations that have statistics similar to that of the conditioning data set and equally quantify the spatial uncertainty (Lin 2008).

Another important aspect of this paper is its relevance in the areas of nutrient status of this watershed. Soil types and properties generally induce the differences in the nutrient status across the field, although there are other external factors such as agricultural practices. Recently, there has been great interest in site-specific agriculture (Geypens et al. 1999; Adamchuk et al. 2002). Plant growth and yield depend on the nutrient status of the field. This status varies across the field. The variability also exists in space and time. Generally P is considered to be highly variable compared to other properties (Geypens et al. 1999; Bennett et al. 2004; Grunwald et al. 2004). Farmers’ benefits lie on the ability to apply a variable rate of fertilizer or manure. This is only possible if the experts are able to give the correct site-specific information and recommendations (Geypens et al. 1999). This paper has been able to quantify precise information about the nutrient status of the watershed through geostatistical techniques, and identify vulnerable areas where the risk of surface runoff when combined with the presence of excess P has the potential to erode P into fresh water. This risk is high in areas where STP exceeds the recommended threshold of what plant really needed.

**CONCLUSION**

Many soil property maps are based on OK. This technique masks areas of high or low values because of its smoothing effect. Although kriging techniques are still used in digital soil mapping, their results can be misleading because of this smoothing effect (underestimation of high values and overestimation of low values). SGS generates multiple realizations,
including an error component, which are absent in classical interpolation techniques such as OK (Lin 2008). This study demonstrates the application of OK and sequential SGS. The OK technique is based on the procedure of kriging interpolation, which provides the best linear unbiased prediction of a value in terms of minimum error variance (Chilès & Delfiner 1999). As a consequence of management practices, high STP levels were evident in pasture lands. A high coefficient of variation illustrates the lack of spatial pattern or structure in the STP. When the watershed is divided into arable and non-arable lands, there is evidence of an undamped non-monotonic covariance function. This shows that in spatial modelling of an environmental variable that is impacted by management practices, it should be divided into those factors that have influence upon it.

Non-monotonic variogram structures, or hole effects, provide a better variogram fit. The hole effect helps in capturing the impact of the management practices based on the land-use pattern. This form of hole effect is often neglected in geostatistical modelling, but its usage in this paper provides valuable information concerning spatial uncertainty. Neglecting this important information in variogram modelling often gives misleading model results. The OK method provides an interpolated surface that captures some of the hotspots, but the SGS method provides a better result using a Monte Carlo technique to draw from the conditional probability distribution of the nodes of the simulation grids. The realizations from SGS have equal probabilities. The realizations represent a spatial pattern without the smoothing effect of OK. Several arable land parcels exceeded the STP threshold of 60 mg kg\(^{-1}\) needed for plant growth. Therefore, for Quebec to meet the targeted reduction of 38.9 Mg yr\(^{-1}\) in total P load entering the Missisquoi Bay there is more work to do in terms of manure and fertilizer applications. In fact, as of 17 August 2012 (reported by NIWR 2012), blue-green algae had caused the death of thousands of fish in Missisquoi Bay. The excess P that caused the bay’s nutrient enrichment came from intensive agricultural fields (including this study site).

From the foregoing, in terms of management decision, SGS provides several scenarios and situations with several maps instead of only one map as obtained with the OK method. Thus, for site-specific BMP applications to reduce P flow into fresh water, the SGS technique, which takes into account uncertainties, gives a more reliable result. This information is very important for deciding where to target BMPs. The greatest utility of this research is that agro-nomists can use the maps and information developed to target sites and various P reduction measures to specific areas where they will achieve the most benefit.

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