An approximate solution for two-dimensional groundwater infiltration in sewer systems

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ABSTRACT

Estimating groundwater infiltration into sewer systems is important for wastewater treatment operators and municipalities. This paper presents an approximate solution for steady-state groundwater infiltration into sewer systems through line defects. The groundwater table was assumed to be horizontal and the aquifer homogeneous and isotropic. Mobius transformation technique and equivalent circumference method were introduced to solve the governing equation. The infiltration rate is found to be controlled by the hydraulic conductivity of the surrounding soil, the total hydraulic head above the sewer pipe, the size of the sewer pipe, the position of the defect, and the size of the defect.

Key words | groundwater, infiltration, line defect, sewer systems, steady state

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INTRODUCTION

Groundwater infiltration is a common problem in sewer systems. As sewer pipes age, groundwater can infiltrate into these sewer pipes through various types of defects (e.g., circumferential defect, line defect, holes). This extraneous water increases the cost of wastewater treatment; it causes an increase in energy consumption and operating costs of pumping stations; it reduces the residual hydraulic capacity of the sewer network but increases the frequency of combined sewer overflow; the discharged non-treated wastewater is detrimental to the receiving water bodies and environment. In addition, this groundwater infiltration can also wash the surrounding soil into the sewer system and accelerate the structural deterioration of the sewer pipes, and may even cause sinkhole problems. Therefore, estimating the amount of infiltration is important for wastewater treatment operators and municipalities.

Several methods for infiltration rate estimation have been developed, e.g., traditional methods (Brombach *et al.* 2002; Weiss *et al.* 2002), pollutant loads analysis method (Kracht & Gujer 2005), and tracer method (Kracht *et al.* 2007; Prigiobbe & Giulianelli 2009). De Benedittis & Bertrand-Krajewski (2005) presented a comprehensive review and conducted comparative studies on the traditional methods. However, these methods are applicable only in specific situations, and are relatively expensive. On the other hand, as the assessment technologies for sewer systems develop rapidly (Wirahadikusumah *et al.* 1998; Costello *et al.* 2007), more and more information on the internal structural condition of the sewer pipe (the defect type, size, and position, etc.) and the surrounding soil (the groundwater table, whether or not the erosion void exists) is obtained and collected by the municipal engineers. Therefore, a method to connect the routine expenditure on inspection with infiltration estimation would be attractive and beneficial.

Recently, in the tunneling field, some researchers obtained analytical solutions for estimating groundwater infiltration into tunnels (Lei 1999; El Tani 2003; Kolymbas & Wagner 2007; Park *et al.* 2008). However, their analytical procedures only dealt with the situation in which the entire tunnel body is pervious by applying a constant pressure on the tunnel. Therefore they cannot be used to solve the infiltration problem of sewer systems, as groundwater can only infiltrate in through the defect; the other part of the sewer pipe is intact and impervious.

In this paper, we study groundwater infiltration into a sewer pipe through a line defect and develop an

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approximate solution for estimating the infiltration rate. Assuming that the groundwater table above the embedded sewer pipe is horizontal, and the surrounding soil is homogeneous and isotropic, the infiltration process can be simplified into a two-dimensional potential flow problem. The Mobius transformation and an equivalent circumference method are introduced to solve the governing equation. The parameters influencing the infiltration rate are then discussed, and some advice is provided for the municipal engineers doing infiltration investigation and estimation.

METHODS

Governing equation

We consider a circular sewer pipe of radius r embedded in a fully saturated semi-infinite porous aquifer. There is a line defect along the pipe wall. In practice, this longitudinal type of defect usually extends a long distance in the underground aquifer. In such cases, the groundwater flow into the defect can be approximated as a two-dimensional potential flow problem. To further simplify the problem, these following assumptions are made: (1) the surrounding soil is homogeneous and isotropic with a constant hydraulic conductivity K; (2) the groundwater table is horizontal. We take the groundwater table as the reference datum, and then a coordinate system is obtained (x–y plane). All the relative information is illustrated in Figure 1, where h is the hydraulic head above the center of the sewer pipe, and α and β represent the location and the open size of

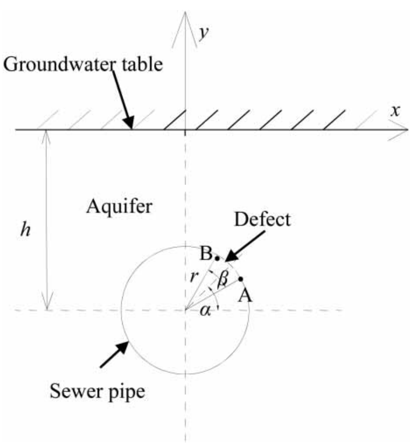

Figure 1 | Sectional view of a sewer pipe with a line defect embedded in semi-infinite aquifer.

the line defect, respectively. The value range of α is 0 to 2π, while β is expected to be in a small value range, for example, 0 to $\pi/18$. This is because if the width of the defect (the product of r and β) is too large, the soil particle is going to be washed into the pipe. This case is beyond our consideration in this work.

The groundwater flow follows Darcy's law. By combining Darcy's law and mass conservation, the two-dimensional groundwater flow around a defective sewer pipe is essentially governed by the following Laplace equation:

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0 \tag{1}$$

where φ is the hydraulic head, which is the sum of the pressure head and elevation head:

$$\varphi=\frac{p}{\gamma_w}+y \tag{2}$$

with p being the pore-water pressure, γ_w the unit weight of water, y the elevation head. With the horizontal groundwater table as the elevation reference datum, the boundary condition at the groundwater surface can be expressed as:

$$\varphi|_{y=0}=0 \tag{3}$$

In addition, the boundary condition at the defect can be expressed as:

$$\varphi|_{\text{defect}}=P_i+y \tag{4}$$

where P_i is the pressure head inside the pipe. P_i depends on the content level of the sewer pipe. If the defect is above the content level, P_i equals the atmospheric pressure and can be taken as zero; otherwise, P_i is the pressure due to the depth of the water above the defect. As the defect is the only entrance for groundwater infiltration, i.e., the rest of the pipe wall is waterproof, we have the third boundary condition at the other part of the pipe wall as:

$$\frac{\partial\varphi}{\partial r}|_{\text{pipewall}}=0 \tag{5}$$

To obtain the absolute analytical solution for two-dimensional infiltration, the above Equations (1)–(5) have to be resolved. However, the difficulty in handling the last two boundary conditions (4) and (5) prevents its existence at present. Therefore, an approximate solution is more reasonable and practical. To achieve this purpose, an equivalent circumference method and Mobius transformation technique are introduced in this paper.

Equivalent circumference method and Mobius transformation

The principle of the equivalent circumference method is to transfer the line defect (represented by arc AB in the x–y plane) into a permeable column which locates at the center of the defect (represented by the small circle in the x–y plane, see Figure 2(a)), making sure that the perimeter of the circumference of the small circle equals the length of arc AB, which means:

$$h' = h - r \cdot \sin\alpha \tag{6}$$

$$r' = \frac{\beta}{2\pi} \cdot r \tag{7}$$

Then, construct a new x'–y' coordinate system (Figure 2(a)), and use the following Mobius transformation to transfer it into ζ'–η' complex plain (Figure 2(b)):

$$\xi' = \frac{x'^2 + y'^2 - h'^2 + r'^2}{x'^2 + \left(y' - \sqrt{h'^2 - r'^2}\right)^2} \tag{8}$$

$$\eta' = \frac{2x'\sqrt{h'^2 - r'^2}}{x'^2 + \left(y' - \sqrt{h'^2 - r'^2}\right)^2} \tag{9}$$

This transformation preserves the Laplace equation, and it maps the conceived circle and the horizontal groundwater table onto two concentric circles of radius $\lambda' = (h'/r') - ((h'^2/r'^2) - 1)^{1/2}$ and 1, respectively (Figure 2(b)). Therefore, in the ξ'–η' plane, the governing equation and boundary conditions can be reorganized, using polar coordinate, into the following forms:

$$\frac{\partial^2\varphi}{\partial\rho'^2} + \frac{1}{\rho'}\frac{\partial\varphi}{\partial\rho'} + \frac{1}{\rho'^2}\frac{\partial^2\varphi}{\partial\theta'^2} = 0 \tag{10}$$

$$\varphi|_{\rho'=1} = 0 \tag{11}$$

$$\varphi|_{\rho'=\lambda'} = P_{\mathrm{i}} - h'\frac{(\lambda'^2 - 1)^2}{\lambda'^2 + 1}\frac{1}{1 - 2\lambda'\cos\theta' + \lambda'^2} \tag{12}$$

The general solution for the Laplace equation in a circular domain (e.g., Arfken & Weber 1995) can be expressed as:

$$\varphi = C_1 + C_2\ln\rho' + \sum_{n=1}^{\infty}(C_3\rho'^n + C_4\rho'^{-n})\cos n\theta' \tag{13}$$

where C_1, C_2, C_3 and C_4 are constants to be determined by boundary conditions.

Substituting the boundary conditions (11) and (12), we obtain:

$$\varphi = \frac{P_{\mathrm{i}} + h\dfrac{\lambda'^2 - 1}{\lambda'^2 + 1}}{\ln\lambda'}\ln\rho' + 2h\frac{\lambda'^2 - 1}{\lambda'^2 + 1}\sum_{n=1}^{\infty}\frac{\lambda'^{2n}}{\lambda'^{2n} - 1}(\rho'^n - \rho'^{-n})\cos n\theta' \tag{14}$$

Therefore, the approximate solution of groundwater infiltration rate Q into a sewer pipe with a line defect

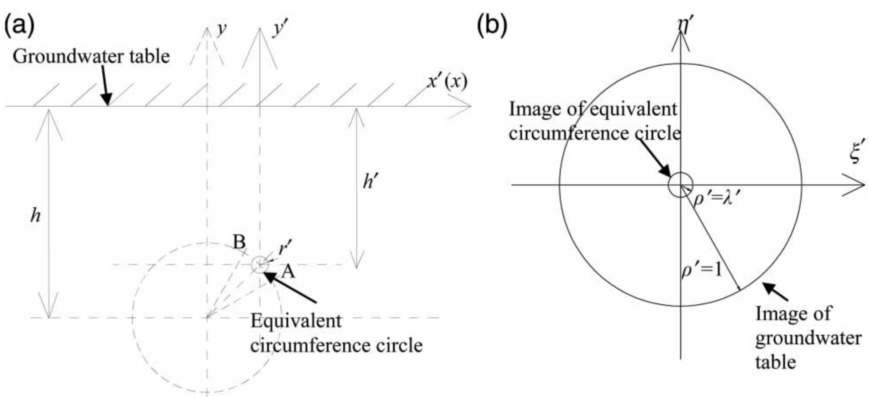

Figure 2 | Equivalent circumference method. (a) Images in the x'–y' plain; (b) images in the ζ'–η' complex plain.

per unit pipe length can be expressed as:

$$Q = \int_0^{2\pi} K \frac{\partial \varphi}{\partial \rho'} \rho' \mathrm{d}\theta' = 2\pi K \frac{P_\mathrm{i} + h\dfrac{\lambda'^2 - 1}{1+\lambda'^2}}{\ln \lambda'} = 2\pi K \frac{h\dfrac{1-\lambda'^2}{1+\lambda'^2} - P_\mathrm{i}}{\ln \lambda'^{-1}}$$
$$= 2\pi K \left(h\frac{1-\lambda'^2}{1+\lambda'^2} - P_\mathrm{i} \right) \ln^{-1}$$
$$\times \left(\frac{2\pi}{\beta}(h/r - \sin\alpha) \left(1 + \sqrt{1 - \left(\frac{\beta}{2\pi(h/r - \sin\alpha)} \right)^2} \right) \right) \quad (15)$$

As the defect open size β is small,

$$\lambda' = \frac{2\pi}{\beta}\left(\frac{h}{r} - \sin\alpha\right) - \sqrt{\left(\frac{2\pi}{\beta}\left(\frac{h}{r} - \sin\alpha\right)\right)^2 - 1} < < 1,$$
$$\left(\frac{\beta}{2\pi(h/r - \sin\alpha)}\right)^2 < < 1,$$

the above solution can therefore be simplified to the following form:

$$Q = 2\pi K(h - P_\mathrm{i}) \ln^{-1}\left(\frac{4\pi}{\beta}(h/r - \sin\alpha)\right) \quad (16)$$

RESULTS AND DISCUSSION

Parametric study

In studying the infiltration through the entire perimeter of a tunnel, El Tani (2003) and Park *et al.* (2008) obtained an analytical solution for predicting the infiltration rate for zero internal pressure condition. By letting $\beta = 2\pi$, Equation (15) reproduces their solution.

The derived expression (Equation (16)) indicates that the infiltration rate is controlled by the following five factors: the hydraulic conductivity of the surrounding soil K; the total hydraulic head above the sewer pipe h; the radius of the sewer pipe r; the position of the defect α and the open size of the defect β. To understand how the infiltration rate is affected by various parameters, we conduct parametric analysis in Figure 3, considering $P_\mathrm{i} = 0$, $K = 1.0 \times 10^{-5}$ m/s, $h = 2$ m, $r = 0.3$ m, $\alpha = \pi/2$, $\beta = \pi/90$.

The influence of soil permeability

The analytical solution shows a linear relationship between the hydraulic conductivity and the infiltration rate. However, the permeability of the natural soil varies significantly; for example, the permeability coefficient of the coarse sand can reach 10^7 times that of the clay. This indicates that if the sewer pipe was embedded in a type of soil with good permeability, e.g., sandy soil, there would be a severe infiltration. In contrast, if the sewer pipe was embedded in watertight soil, e.g., clay soil, the infiltration might be less of a concern. Therefore, it is suggested that when doing infiltration estimation, the permeability of the surrounding soil should be investigated, and more attention should be given to the defective sewer pipes under sandy soil.

The influence of total hydraulic head above the sewer pipe

The total hydraulic head above the sewer pipe is determined by the embedded depth and the location of the groundwater table. Generally, the groundwater table varies seasonally; therefore, the infiltration rate in an urban area has different conditions at different times of the year. Figure 3(a) shows an approximately linear relationship between the total hydraulic head and the infiltration rate. It indicates the significant influence of the groundwater table on the infiltration rate; e.g., when the groundwater table increases by 1 m, the infiltration can increase by about 0.55 m^3 per meter sewer pipe in one day.

The influence of pipe size

Figure 3(b) shows that larger sewer pipes have larger infiltration rates. However, the influence of the sewer pipe size on the infiltration rate is not as significant as the groundwater table. This phenomenon shows municipal engineers that when doing infiltration investigation and estimation, attention should not be given only to the main pipe, e.g., interceptor sections, but also to the small size pipes, e.g., the service connection.

The influence of defect position

The influence of the position of the defect on infiltration shows a very interesting phenomenon, though its influence is relatively small on the infiltration rate. Figure 3(c) shows an approximate sinusoidal variation of the infiltration rate as the position of defect changes along the pipe wall. When the defect locates at the top, it will have the largest infiltration rate, while at the bottom, it will have the smallest infiltration rate. However, this sinusoidal behavior is possible only when the sewer pipe is empty ($P_\mathrm{i} = 0$). The existing content in the pipe will decrease infiltration when the defect locates below its level. According to the structural analysis results (Davies *et al.* 2001; Tan 2007), the type of line

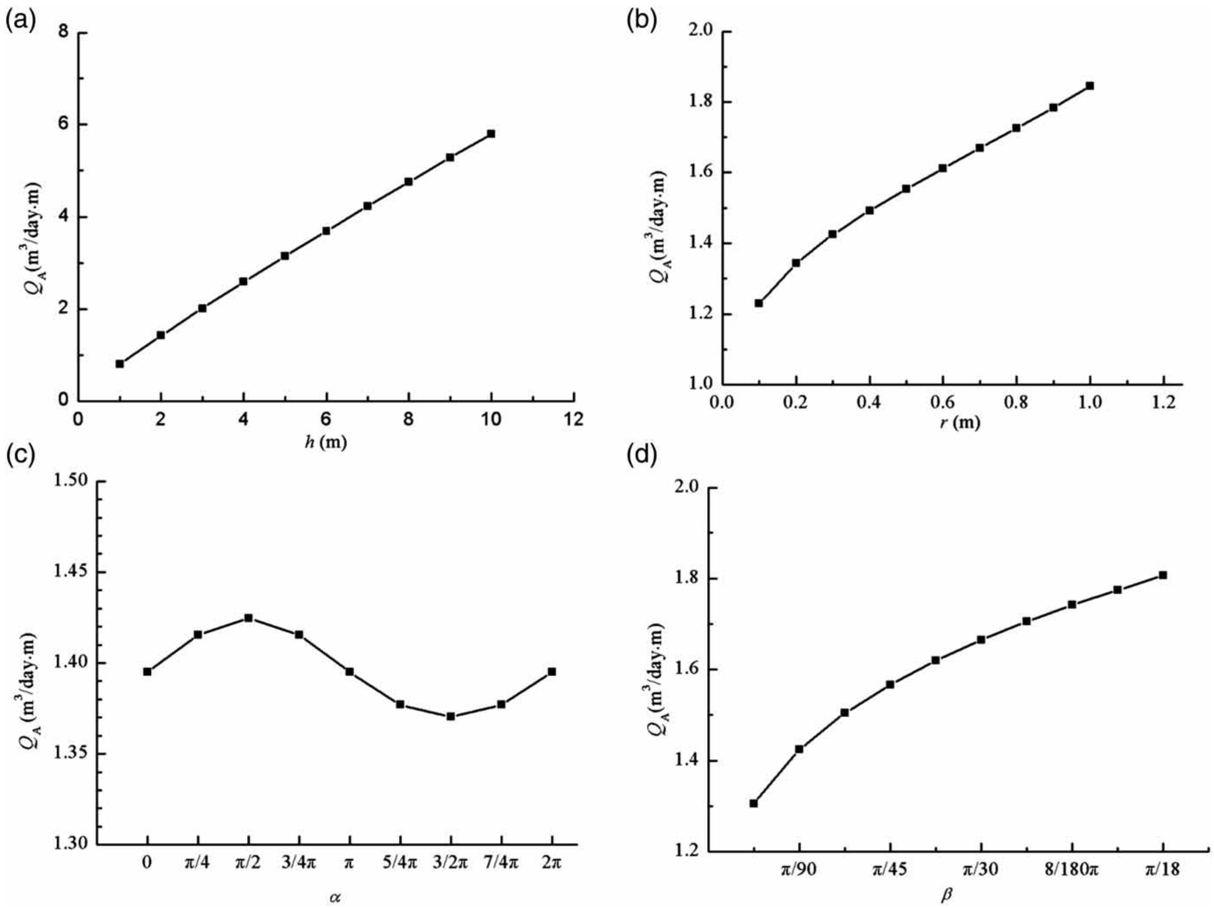

Figure 3 | Parametric studies. (a) Influence of the hydraulic head; (b) influence of the sewer pipe size; (c) influence of position of the defect; (d) influence of the open size of the defect.

defect is usually generated at the following positions: crown, invert and springlines. These four positions represent $\alpha = \pi/2$, $3\pi/2$, 0 and π, respectively.

The influence of defect open size

Infiltration rate shows an increasing trend with β as shown in Figure 3(d). It is easy to understand that larger defect will cause larger infiltration. β only represents the open angle of the defect in the two-dimensional plain; the size of the defect is actually determined by the product of r and β. However, the line defect usually has a small range of variation to sustain a steady-state infiltration.

CONCLUSIONS

Groundwater infiltration through various defects on the pipe wall is a complex three-dimensional problem. Analytical analysis or numerical simulation is rarely found in literature because of its complexity. At present, groundwater infiltration of the sewer system is usually assessed as an annual volume at the inlet of a treatment plant or as a daily volume at the scale of sub-catchments. Municipal agencies must have specific budget allocation for infiltration estimation, as most of the existing assessment methods are expensive. This paper aims to present a new methodology for infiltration calculation related to the individual sewer pipe segment.

This paper first introduces and discusses the governing equation and boundary conditions for the groundwater movement around a defective sewer pipe. Then, an approximate solution was presented for assessing two-dimensional infiltration to a sewer pipe through a line defect. To use the proposed model in this paper, we must know the values of the following five parameters: hydraulic head, pipe radius, defect position and its size, and hydraulic conductivity of the surrounding soil. However, the routine inspection provides all that we need.

Municipal agencies worldwide are conducting structural assessment on their sewer systems for the rehabilitation planning. Advanced inspection technologies capture not only the internal structural condition of the sewer pipe, such as the defect type, size, and position, but also the external groundwater table, soil type, etc. The data collected during a sewer diagnostic study are usually stored in an integrated database such as GIS. It is convenient for municipal engineers to extract the data needed for calculating the infiltration rate of the individual sewer pipe segment with a line defect by the proposed model in this work, therefore saving the budget allocation for the sub-catchments scale infiltration estimation.

However, the work presented in this paper is limited to the line defect and two-dimensional problem. The more practical and applicable three-dimensional infiltration problem around other types of defect, e.g., a hole defect, is worthy of more attention.

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