DISCUSSION

Henry A. Becker

This study, compared to past work, emphasizes the post-jet-mixing zone of transition from the initial confined-jet flow to the final equilibrium pipe flow. The most valuable new contribution to knowledge consists in the extensive and apparently excellent data on turbulence intensity and shear stress.

The authors mention the work of Curtet, but they have not utilized the important concept introduced by the Craya-Curtet analysis: the confined-jet similarity criterion. Use of this criterion would set the results in order and facilitate a higher degree of generalization. The effects of nozzle/duct diameter ratio and velocity ratio would be rationalized as a strong effect of the similarity criterion and a weak or negligible effect of the parameter ratio.

The Craya-Curtet criterion is a dimensionless similarity parameter expressing the effects of the input momentum flux distribution on the confined jet system. I have, since my first involvement with it, thought about ways in which the concept might be formulated with greatest generality. I have arrived at the following revised definition of the Craya-Curtet number

\[ C_t = \left( \frac{m_1}{m_0} \frac{U_2}{U_1} \right)^{1/2} \]

where \( m_1 \) and \( F_1 \) are, respectively, the primary (nozzle) stream mass flux and momentum flux, and \( m_0 \) and \( F_0 \) are, respectively, the secondary stream mass flux, density, and flow area (all measured in the nozzle mouth). When the primary stream velocity is uniformly \( U_1 \) and the secondary uniformly \( U_2 \), get the simple form

\[ C_t = \left( \frac{m_1}{m_0} \frac{U_2}{U_1} \right)^{1/2} \]

This has the uniquely defined limit behaviour that \( C_t = 0 \) when \( m_1 = 0 \) and \( C_t = \infty \) when \( U_1 = U_2 \). When the streams are moreover equal in density,

\[ C_t = \left( \frac{U_2}{U_1} \frac{A_2}{A_1} \right)^{1/2} (1 - \frac{U_2}{U_1})^{1/2} \]

We normally have \( A_3 \approx A - A_1 \), where \( A \) is the duct cross-sectional area in the nozzle plane. When nozzle and duct are round, \( A_3/A_1 \approx (D^2 - D_2^2)/D_2^2 \) where \( D_1 \) and \( D_2 \) are the diameters of the nozzle and the duct, respectively.

\[ C_t = \left( \frac{U_2}{U_1} \frac{D^2 - D_2^2}{D_2^2} \right)^{1/2} (1 - \frac{U_2}{U_1})^{1/2} \]

The virtues of the new definition of \( C_t \) will be argued elsewhere.

For the present argument it suffices to say that \( C_t \) calculated from equation (4) resolves the Razinsky and Bright data convincingly. The direct effect of the diameter ratio \( D_1/D_2 \) is thus shown to be feeble or negligible, depending on the variable in question. Take for example the axial pressure distributions, Figs. 3 and 4 in the paper: the curves are fairly well ordered on the basis of \( C_t \) alone. (For this purpose, the calculated values of \( C_t \) corresponding to the operating data in Table 1, reading from top to bottom are: 0.62, 1.08, 2.41, 4.18, 6.83, 0.30, 0.52, 1.15, 2.00, and 3.20.)

Following the foregoing rationale, Figs. 5 to 30, showing various radial profiles, are best labelled with the values of \( C_t \) and the diameter ratio \( D_1/D_2 \) rather than with the velocity ratio and the diameter ratio.

R. Curtet

The entrainment of an air stream by a coaxial jet in a duct depends on the length of the duct. This influence is important, but not yet known in detail.

As shown in Fig. 31, most of the classical experimental works on round confined jets have been carried out in pipes, whose length \( L \) is less than ten times \( r_j \), where \( r_j \) is the radius of the pipe. Mikhail [10], however, measured as far as 18 diameters, while Deal [16, 24] took measurement within a length of 20 diameters.

The authors obtained mean velocity and fluctuations distributions up to almost 60 diameters, i.e., at lengths much greater than all previous works. This study represents a contribution to the understanding of confined jets, particularly in the region III of mixing (Fig. 1).

We would however like to make the following comments about the region I. During preliminary studies of the work described in [11], we discovered that the turbulence level \( \alpha \) at the entrance, in the secondary flow, has an appreciable influence on the mixing of the streams. We studied two different admission systems, which permitted us to achieve \( \alpha = 1 \) percent and 5 percent, respectively, in the initial zone of the secondary flow. Figs. 32 (a) and (b) show the evolution along the duct of the velocity \( u_{vel} \) on the axis, and of the effective width \( \beta \), which is analogous to the width \( \beta_0 \) defined by the authors. On the Figs. 32 (a) and (b), \( u_{vel} \) with respect to mean velocity \( u_{x} \) and width \( \alpha \) with respect to radius \( r_j \) of the jet are shown as a function of \( x/r_j \).

It can be observed that, if the turbulence level is increased, the spreading of the mixing zone is increased, and the velocity on the axis decreases more quickly. This logically means that a high level of turbulence in the secondary flow favours mixing.

We should like to know if the authors have observed such phenomenon during their experiments, where the secondary
Additional References


H. Weinstein

This paper is a detailed experimental treatment of confined jet mixing. While the title claims that the work is done only at nonseparating conditions, several cases with separating conditions are treated. This is certainly not to be considered a deficiency of the work. However, it is for these cases in which separation occurs and the cases in which it is approached that this discussion would like to raise some questions. First, the authors are to be complimented on the care with which the measurements were taken and the effort expended in taking such complete and detailed measurements.

The mean velocity profiles for four cases are given in Figs. 5 to 8. Of these, three show adverse pressure gradients measured at the wall. For each of the cases with the adverse pressure gradient, the flat portion of the outer jet velocity falls slightly after the initial face (profile B) which is in keeping with the adverse pressure gradient. In one case, for 1/6 < r/r_j < 2, the centerline velocity does not exceed the initial centerline velocity and separation does not occur for this flow. However, for the other two cases of adverse pressure gradient, 1/6 < r/r_j < 2, the centerline velocity at station B exceeds the initial centerline velocity and recirculation does occur. These changes in velocity can be thought of as being due to the Biot-Savart induction from the concentrated cylindrical vortex sheet initially between the two jets. This effect is felt before there is enough time for the turbulent diffusion of vorticity to decrease the strength of the sheet [26, 27]. This centerline behavior is not in keeping with an adverse pressure gradient there, indicating that substantial radial pressure gradients exist for the first few radii downstream of the initial face.

Shavit [28] recently numerically solved the Navier-Stokes equations for confined laminar coaxial jet mixing and showed that for high velocity ratios, the centerline pressure first decreased and then increased in the flow direction. While his work is for the laminar case, the effects should be similar to the turbulent case in a gross fashion.

Since the authors measured velocity with an impact tube and a wall static pressure probe, a radial pressure gradient would cause an error in their mean velocity measurement. This error may exist in profile B for all three cases with adverse pressure gradient.

There is another apparent inconsistency in the data presented in this paper. Both cases with velocity ratio 9 show a recirculating region. A velocity profile through this recirculating region shows two extrema. One of these is at the centerline and the authors carefully point out that the u'v' correlation changes sign here. In the center of the recirculating region there is a velocity minimum in a region of negative velocity. One would also expect the u'v' correlation to show a zero and change sign in this region. However, the authors report positive values of u'v' throughout this region. They do, however, call attention to the approximate nature of the data here.

The above comments do not affect the observations and conclusions of the authors. However, they should rename the pressure gradient the wall pressure gradient in region I. They should also define the potential core regions as regions where the flow is a potential one rather than as regions where the velocity is constant.

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8 Numbers in brackets designate Additional References at end of discussion.
**Additional References**


**S. Wolf**

The authors have enlightened the problems associated with confined jet mixing by developing a significant, useful amount of experimental information. The turbulence velocities and Reynolds stress distributions should prove most useful in development of a real prediction method for the mean flow.

A possible restriction of the Reynolds stress data might be imposed because the authors have obtained their data only for a small variation in the similitude parameter intended to characterize the jet inlet flows, as given by several authors (references [11, 13, and 14]). For example, the variation in the similitude parameter given by Hill is from 0.80 to 0.997 for u0/u^ = 2 and r^/r, = 1/5 to 1/113 for u0/u^ = 6 and r^/r, = 1/5. Perhaps the authors would discuss what governed their choice of flows and what variation from the present data might be expected at lower values of the similitude parameter. Comparison of the Reynolds stress data to data for models developed by other investigators should also be useful. For instance, why is the maximum Reynolds stress term of about 0.002 as given for fully developed flow in Figs. 18, 20, 22, and 24, approximately 50 percent smaller than the value of about 0.003 deduced from data given by Schlichting.

In reading the paper I noted several errors that probably occurred in the publication process, namely: (1) In Table 1, the two columns labeled r and r give diameters rather than radii, (2) In Fig. 5 the curve not labeled should probably be for station 1, r/r, = 113 1/5, and (3) the Reynolds stress term in Figs. 17-24 should be negative, u/v/u_2 to result in positive values.

**Authors' Closure**

The authors would like to thank the discussers for their comments.

Professor Becker's comment on the employment of the revised Craza-Curtet number and diameter ratio to define a set of flow conditions is concurrent with our feelings that the use of the Craza-Curtet number alone is not sufficient for complete specification of the initial flow conditions. Further discussion on the need for a two parameter flow specification can be found in the work of Exley and Brighton.

The particular set of velocity and radius ratios were selected so as to cover a range of flows in which early flow development was dominated by the jet (high velocity ratio and high radius ratio) to those primarily influenced by the wall boundary layer (low velocity ratio and low radius ratio). As mentioned in the text, the velocity ratio had a large influence in determining the pressure distributions and the possibility of flow reversal, while the diameter ratio was of smaller consequence. The axial locations of separation and reattachment indicated a definite radius ratio effect. A more extensive investigation of the latter behavior has been reported in the paper by Exley and Brighton.

Regarding Professor Curtet's remarks on the influence of the initial turbulence level of the secondary stream; we attempted to keep the level of turbulence low and no attempts were made to determine the effect of different levels of turbulence. Professor Curtet's observations are important and similar influences of the free stream turbulence on the rate of spread of shear zones have been observed in free jet and boundary layer flows.

Professor Weinstein suggests the possibility of a significant radial pressure variation near the inlet, this causing some error in velocity measurements using wall static pressure. Some preliminary readings using a wedge probe indicated that the static pressure variation was small and could not significantly influence the velocity readings. However, these results are not conclusive and we are going to make some additional measurements.

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