

## DISCUSSION

### C. H. Venner<sup>1</sup> and A. A. Lubrecht<sup>2</sup>

The authors are to be congratulated for their detailed analysis of the different numerical techniques that can be used to solve the problem of ElastoHydrodynamic Lubrication. However, after careful reading of the paper, the discussers had a number of questions that persisted:

The authors' new relaxation scheme does not seem to use distributive relaxation which the discussers found to be necessary to ensure stability at high loads and (very) fine grids.

The discussers have never encountered the necessity for any continuation technique for highly loaded conditions, using the Full MultiGrid (FMG) technique. Assuming that the discretised equations are the same and are solved to sufficient accuracy, the results obtained in Tables 4 and 8 should be identical, why are they different?

Venner's results quoted in Table 8 were obtained using a slightly different set of viscosity parameters. The film thickness results for the authors' set are given in Table 9. From this table it can be concluded that the first-order and second-order

**Table 9** Dimensionless minimum film thickness  $H_m$  and central film thickness  $H_c$  as a function of the mesh size for  $M = 20$ ,  $L = 10$ , with  $\eta_0 = 8.48 \cdot 10^{-3}$ ,  $\alpha = 1.7 \cdot 10^{-8}$ ,  $Z = 0.68$ ,  $p_0 = 1.96 \cdot 10^8$ . "First order" refers to standard first-order upstream discretization of the wedge term as used by the authors. "Second order" refers to second-order upstream discretization of the wedge term. Computational domain:  $-4.5 \leq X \leq 1.5$  and  $-3 \leq Y \leq 3$ . Solutions computed with FMG with 3 W (2, 1) cycles per gridlevel.

$n_x \times n_y$	First order		Second order	
	$H_m$	$H_c$	$H_m$	$H_c$
17 × 17	0.2830	0.4412	0.1667	0.2503
33 × 33	0.3090	0.4600	0.2612	0.3749
65 × 65	0.3054	0.4493	0.2843	0.4109
129 × 129	0.2998	0.4383	0.2889	0.4202
257 × 257	0.2955	0.4313	0.2903	0.4226
513 × 513	0.2931	0.4274	0.2905	0.4232
1025 × 1025	0.2918	0.4254	0.2906	0.4233

schemes show indeed first and second-order convergence, and both schemes converge to the same values.

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### Authors' Closure

The authors would like to thank the discussers for their thoughtful comments. In reply to their first point, while it is true that the authors' scheme does not use distributive relaxation, the scheme that is used, as described by Nurgat and Berzins, does seem to work even for highly-loaded problems. We plan to compare the two techniques in future work.

Although it is possible to obtain solutions without using continuation, the homotopy methods used in the paper provide an automatic way to reduce the residuals to machine roundoff level. As such the method, notwithstanding its expense, is a good benchmark method for the assessment of faster but more problem-specific methods, such as the other methods considered in the paper.