Blocking Effect and Coriolis Anti-Pairing Effect on Deformed Odd Nuclei

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Effects of rotational motion on the pairing correlation in odd mass nuclei and their various nuclear properties affected by these effects are investigated. The rotational motion weakens the pairing interactions (Coriolis anti-pairing effect), and further in odd mass nuclei the odd nucleon also reduces the effect of the pairing interaction by blocking the scattering of the nucleon pairs into the orbit occupied by the odd nucleon (blocking effect). The blocking effect is also affected by the rotational motion and weakened by dispersing the odd nucleon into other orbits by configuration mixing.

The Hartree Bogoliubov equation is solved assuming the pairing and quadrupole interactions in consideration of the rotational motion. The second order perturbation approximations are used and the self-consistent calculations are carried out in obtaining the energy gap parameter and chemical potential. The moments of inertia and the gyromagnetic ratio are numerically calculated and compared with experimental data. Agreements are generally good.

§ 1. Introduction

Two characteristic quantities of the rotational motion of deformed nuclei, the moment of inertia $\mathcal{J}$ and the collective gyromagnetic ratio $g_R$, have been investigated by many authors based on the various nuclear models. However, it was found that the moment of inertia cannot be quantitatively explained by the simple models: the unified model of Bohr\textsuperscript{1} with the assumption of the irrotational flow and the cranking model of Inglis\textsuperscript{2} without the residual interactions.

With the success of Bardeen, Cooper and Schriefer's\textsuperscript{3} (BCS) theory of superconductivity, Bohr, Mottelson and Pines\textsuperscript{4} proposed the application of this theory to nuclei. The pairing correlation gives rise to the energy gap $2\Delta$, which was found to reduce the value of the moment of inertia calculated by the cranking model to almost the experimental values.\textsuperscript{5}

The effect of the rotational motion on the pairing correlation was first observed by Mottelson and Valatin.\textsuperscript{6} They considered an analogy between the effects of the magnetic field and the rotational motion on the pairing correlation. The latter effect is expressed as the Coriolis term which acts on the nucleons in the time reversed orbits with opposite way to the pairing force and weakens this interaction. As the angular velocity $\Omega$ of the rotation increases, the anti-
pairing effect increases, and the energy gap parameter $\Delta$ decreases, finally to zero. This fact corresponds to the phase transition from the superconducting state to the normal state.

The above discussions on deformed nuclei are generally known, but for odd mass nuclei we have to require a special treatment of the odd nucleon. The pairing interaction acts on a nucleon pair occupying the degenerate and time reversed orbits and makes this pair jump to other orbits. However, the presence of the odd nucleon prevents jump into the orbit occupied by the odd nucleon; this is called the blocking effect. This odd nucleon gives extra contribution to the moment of inertia and also weakens the pairing correlation as was shown by Soloviev. However, the effect of the rotation on the pairing correlation in odd mass nuclei has not yet been fully investigated.

In the presence of the Coriolis term a single particle orbit with the magnetic quantum number $m$ mixes with orbits with $m \pm 1$, and this configuration mixing makes the blocking effect imperfect and also weakens the effective pairing interaction. It is the purpose of this work how the rotation affects the pairing correlation in odd mass nuclei and how the rotational energy spectra and the gyromagnetic ratio are influenced by this effect.

In the following section the Hartree Bogoliubov equation for odd number of nucleons is introduced. The rotational motion is taken into account by a Coriolis like term and the equation is simplified by assuming the interaction as the pairing and quadrupole interactions. In § 3 the solution of the Hartree Bogoliubov equation is presented in the second-order approximation in the angular velocity of the rotation. The energy gap parameter $\Delta$ and the chemical potential $\lambda$ are determined by using the above results self-consistently, and formulae for the moment of inertia and the gyromagnetic ratio are given in § 4. In § 5 the numerical results for energy gap $\Delta$, the moment of inertia, and the gyromagnetic ratio are obtained and compared with experiments. In the last section discussions and conclusions are given.

§ 2. The generalized Bogoliubov transformation for the system with odd number of nucleons

We first consider a deformed nucleus: a system of nucleons moving in a deformed well and interacting with each other by the residual interactions. The Hamiltonian of this system consists of the single particle energies and potential energies. The single particle orbit is just Nilsson's own, and characterized by quantum numbers which are expressed by $\alpha, \beta, \cdots$ in short. The deformation is assumed to be axially symmetric so that the symmetry axis component of the angular momentum $m_a$ is a good quantum number. The single particle energy is denoted by $\varepsilon_a$ and is degenerate with respect to the two directions of the magnetic quantum number $\pm m_a$. 
The residual interaction is assumed to be smooth enough, and matrix element \( v_{\alpha \beta \gamma \delta} \) is finite and satisfies the following antisymmetry and Hermitian relations,

\[
v_{\alpha \beta \gamma \delta} = -v_{\alpha \beta \gamma \delta} = -v_{\beta \alpha \gamma \delta} = v_{\gamma \delta \alpha \beta}.
\]

(2.1)

The Hamiltonian is given in the second quantization form by

\[
H = \sum_{\alpha} (\epsilon_{\alpha} - \lambda) c_{\alpha}^+ c_{\alpha} + \sum_{\alpha \beta \gamma} v_{\alpha \beta \gamma \delta} c_{\alpha}^+ c_{\beta}^+ c_{\gamma} c_{\delta},
\]

(2.2)

where \( c_{\alpha}^+ \) and \( c_{\alpha} \) are the creation and annihilation operators of a nucleon with the quantum number \( \alpha \). The chemical potential \( \lambda \) is introduced as a Lagrange multiplier to keep the expectation value of the nucleon numbers to the assigned value.

Next the rotation is taken into account by adding a Coriolis like term to the Hamiltonian. If the nuclear system is rotated with the angular velocity \( \Omega \) around the \( x \) axis which is perpendicular to the symmetry axis, the Hamiltonian becomes

\[
\mathcal{H} = H - H_\theta,
\]

(2.3)

where

\[
H_\theta = \Omega \sum_{\alpha \beta} (j_\alpha)_{\alpha \beta} c_{\alpha}^+ c_{\beta}.
\]

(2.4)

In the above expression, \( j_\alpha \) is the component of the angular momentum along the \( x \) axis.

The Hamiltonian given above is now rewritten in terms of quasi-particle to incorporate the pairing correlation into the calculation. By the generalized Bogoliubov transformation the quasi-particle operators are defined as

\[
a_{i}^+ = \sum_{\alpha} \{ A_{i} \alpha^* c_{\alpha}^+ + B_{i} \alpha^* c_{\alpha} \},
\]

(2.5)

where \( A_{i} \alpha \) is the amplitude of quasi-particle’s particle part, \( B_{i} \alpha \) that of its hole part, and the index \( i \) denotes all quantum numbers of the quasi-particle. The requirement that the \( a \)'s should also form a set of fermion operators entails the orthogonality relations

\[
\sum_{\alpha} (A_{i} \alpha^* A_{\delta} \beta^* + B_{i} \alpha^* B_{\delta} \beta^*) = \delta_{ij} \delta_{\alpha \beta},
\]

(2.6)

\[
\sum_{i} (A_{i} \beta^* A_{\delta} \alpha^* + B_{i} \beta^* B_{\delta} \alpha^*) = 0,
\]

and the inverse relation

\[
c_{\alpha} = \sum_{i} \{ A_{i} \alpha^* a_{i} + B_{i} \alpha^* a_{i}^* \}.
\]

(2.7)
We now restrict ourselves to the ground states of odd mass nuclei. If the wave function for quasi-particle vacuum is denoted by $|\rangle$, then the ground states of the odd mass nuclei may be expressed by

$$|\rangle = a^+_l |\rangle,$$

(2.8)

where $l$ stands for all quantum numbers of the last occupying odd nucleon. It is clear that if operators $a_i (i \neq l)$ and $a_i^+$ operate on $|\rangle$ then it vanishes. It is apparent that the role of the creation and annihilation operators for the orbit occupied by the last nucleon is exchanged. So, we exchange the notation of the quasi-particle for this orbit in this work. It is noted that the wave function given by Eq. (2.8) is reduced to the one given by Soloviev\(^7\) when the rotational motion is removed.

Coefficients of the Hartree-Bogoliubov transformation may be obtained through a variational principle. First the expectation value of the Hamiltonian (2.3) is evaluated, by using the quantities quite analogous to the corresponding quantities defined by Baranger\(^8\) in the case of an even nucleus. The density matrix and the pairing matrix are defined by using the notation (2.8):

$$\langle c^+ \cdot c \rangle = \rho_{\alpha \beta} = \rho^\beta_{\alpha},$$

(2.9)

$$\langle c \cdot c^+ \rangle = \kappa_{\alpha \beta} = -\kappa_{\beta \alpha}.$$

The matrix elements are expressed by using Eq. (2.5) as

$$\rho_{\alpha \beta} = \sum_i B_{\alpha}^i B_{\beta}^{i*} - B_{\alpha}^{i*} B_{\beta}^i + A_{\beta}^i A_{\alpha}^{i*},$$

$$\kappa_{\alpha \beta} = \sum_i A_{\alpha}^i B_{\beta}^{i*} - A_{\beta}^{i*} B_{\alpha}^i + A_{\beta}^i B_{\alpha}^{i*},$$

(2.10)

which are easily shown to satisfy the following relations with the help of Eq. (2.6),

$$\rho^T - \kappa^* \kappa = \rho,$$

$$\kappa \rho = \rho^T \kappa,$$

(2.11)

where $\rho^T$ is the transposed matrix of $\rho$. The Hartree-Fock and pairing potentials are also defined in terms of the density and pairing matrices:

$$\Gamma_{\alpha \beta} = \sum_{\gamma \delta} \delta_{\alpha \gamma} \delta_{\beta \delta} \rho_{\gamma \delta} = \Gamma^*_{\beta \alpha},$$

$$\Delta_{\alpha \beta} = \sum_{\gamma \delta} 2 \delta_{\alpha \gamma \delta} \kappa_{\gamma \delta} = -\Delta_{\beta \alpha}.$$

(2.12)

The expectation value of the Hamiltonian (2.3) may be calculated as

$$W = (1/2) \sum_{\alpha \beta} \left[ \{(\varepsilon_{\alpha} - \lambda) \delta_{\alpha \beta} - \Omega(j_x)_{\alpha \beta} \} \rho_{\alpha \beta} + (1/2) \Gamma_{\alpha \beta} \rho_{\alpha \beta} \right.$$\n
$$+ (1/2) \Delta_{\alpha \beta} \kappa^*_{\alpha \beta} + \text{complex conjugate}].$$

(2.13)

Requiring that the expectation value (2.13) should be minimum under the constraints (2.6), we obtain the following coupled equations,
\[ A_{l} A_{l} = \sum_{\beta} \{ (\varepsilon_{\alpha} - \lambda) \delta_{\alpha\beta} - \Omega(j_{l} \lambda_{\alpha}) \} A_{l}^\alpha + A_{l}^\alpha B_{l}^\beta + \Gamma_{\alpha\beta} A_{l}^\alpha, \]
\[- A_{l} B_{l} = \sum_{\beta} \{ (\varepsilon_{\alpha} - \lambda) \delta_{\alpha\beta} - \Omega(j_{l} \lambda_{\alpha}) \} B_{l}^\beta + A_{l}^\alpha B_{l}^\beta + \Gamma_{\alpha\beta} B_{l}^\beta, \]  

(2·14)

for all \( i \)'s including \( l \). The above equations may also be obtained by requiring that the terms like \( a^+ a^+ \) and \( aa \) in the Hamiltonian (2·3) should vanish and the terms like \( a^+ a \) are diagonal. The chemical potentials \( \lambda \) are determined from the following equation,

\[ N = \sum_{\alpha} \rho_{\alpha}, \]  

(2·15)

where \( N \) is the neutron or proton number and the sum over \( \alpha \) is restricted to neutron or proton orbit respectively.

For later convenience the time reversed state is introduced here. The phase of the spherical harmonics is chosen as the Condon-Shortley's multipliers \( Y_{lm}(\theta\phi) \) multiplied by \( i \). Then the time reversed state \( \bar{\alpha} \) to the state \( \alpha \) is defined by

\[ c_{\alpha}^+ = (-)^{j_\alpha - m_\alpha} c_{\bar{\alpha}}, \]  

(2·16)

where \( j_\alpha \) is the total angular momentum of the single particle \( \alpha \). By using this definition, it is easy to show that

\[ (j_{\bar{\alpha}})_{\alpha} = -(j_{\alpha})_{\bar{\alpha}}, \]  

(2·17)

where the representation of \( j_{\bar{\alpha}} \) is chosen to be real.

For residual interactions we have made no assumption so far except the singularity. However, the following simple model which has been used in many occasions is now adopted; the residual interactions consist of two parts, one is the pairing interaction given by

\[ v_{\alpha\beta\gamma} = -(1/4) G \delta_{\alpha\beta} \delta_{\gamma\bar{\gamma}}, \]  

(2·18)

and the other the quadrupole interaction

\[ v_{\alpha\beta\gamma} = -(1/4) \chi \sum_{\kappa} q_{\kappa\gamma}^k q_{\kappa\beta}^l, \]  

(2·19)

where

\[ q_{\alpha\beta}^k = \langle \alpha | r^k Y_{l\kappa} | \beta \rangle. \]  

(2·20)

The strengths of these two parts are \( G \) and \( \chi \). For the quadrupole interaction the exchange term is neglected as shown by Eq. (2·19).

The Hartree-Fock and pairing potentials given by Eqs. (2·12) have contributions both from the pairing and quadrupole interactions, but we neglect the contribution from the quadrupole interaction to the pairing potential and the one from the pairing interaction to the Hartree-Fock potential.

According to the above assumption the form of the pairing potential becomes simple:

\[ \Delta_{\alpha\beta} = \delta_{\alpha\beta} A, \]  

(2·21)
where

\[ \Delta = \left( \frac{G}{2} \right) \sum_{\gamma} \kappa_{\gamma}. \]  

The coefficient \( B_{\alpha} \) is replaced by

\[ B_{\alpha} = C_{\alpha}. \]  

If we use Eqs. (2·21), (2·22) and (2·23) then Eq. (2·14) becomes

\[
\begin{align*}
A_\alpha C_\alpha &= (\varepsilon_\alpha - \lambda) C_\alpha + \mathcal{A} C_\alpha + \sum_\beta (\Gamma_{\alpha\beta} - \Omega(j_\alpha)_{\alpha\beta}) A_\beta, \\
-A_\alpha C_\alpha &= (\varepsilon_\alpha - \lambda) C_\alpha - \Delta A_\alpha + \sum_\beta (\Gamma_{\alpha\beta}^* + \Omega(j_\alpha)_{\alpha\beta}) C_\beta.
\end{align*}
\]  

These coupled equations may be written in a compact form

\[ A_i \mathcal{W}_i = M \mathcal{W}_i, \]  

where \( \mathcal{W}_i \) is a vector in the space with the dimension twice as many as the number of single particle states \( N_{\varepsilon, \rho} \) and \( M \) is a matrix. The elements of the vector are arranged in the following order,

\[
\mathcal{W}_i = \begin{pmatrix}
A_\alpha \\
\vdots \\
A_\alpha \\
C_\alpha \\
\vdots \\
C_\alpha
\end{pmatrix},
\]

and the matrix is written as

\[
M = \begin{pmatrix}
(\varepsilon - \lambda) + \Gamma - \Omega j_\alpha \\
\Delta \\
- (\varepsilon - \lambda) - \Gamma - \Omega j_\alpha
\end{pmatrix},
\]

where \( \varepsilon - \lambda \) is the diagonal matrix and \( \Gamma - \Omega j_\alpha \) is the matrices in \( N_{\varepsilon, \rho} \times N_{\varepsilon, \rho} \) dimensions. Their matrix elements are appeared in Eq. (2·24). The matrix \( \mathcal{A} \) has only matrix elements between \( \alpha \) and \( \bar{\alpha} \), and independent of \( \alpha \), as appeared in Eq. (2·24).

§ 3. Perturbation approximation

The coupled equations presented in the last section are now solved by the perturbation method. All quantities are expanded in power of \( \Omega \), and the \( i \)-th order term is shown by a superscript \((i)\). The zeroth-order equations are reduced to the limiting case of \( \Omega = 0 \), and solutions are well known. The eigenvalue is just the quasi-particle energy

\[ A_\alpha^{(0)} = \pm E_\alpha = \pm \sqrt{(\varepsilon_\alpha - \lambda)^2 + \Delta^2}. \]
The eigenvector is described by
\[
\begin{align*}
    u_a^2 &= \frac{1}{2} \left( 1 + \frac{\varepsilon_a - \lambda}{E_a} \right), \\
    v_a^2 &= \frac{1}{2} \left( 1 - \frac{\varepsilon_a - \lambda}{E_a} \right),
\end{align*}
\]
(3.2)
where \( \lambda \) and \( \Delta \) are determined by the following coupled equations,
\[
\begin{align*}
    \Delta &= \sum_{\sigma \neq \pm} \left( G/2 \right) (u_a \, v_a), \\
    N - 1 &= \sum_{\sigma \neq \pm} v_a^2. 
\end{align*}
\]
(3.3)

The notation \( \tilde{R} \) indicates the zeroth-order orbit for the last occupying nucleon which has been denoted by the quasi-particle quantum number \( \ell \). The contribution from \( q^0_n \) to the zeroth-order self-energy \( \Gamma^{(0)}_{\alpha\beta} \) was included in \( \varepsilon_a \) from the beginning, while that from \( q^{k}_n \) \((k \neq 0)\) is neglected because the axial symmetric deformation has been assumed.

The zeroth-order solutions satisfy the following equations,
\[
\begin{align*}
    M^{(0)} \Psi^{(0)}_a &= E_a \Psi^{(0)}_a, \\
    M^{(0)} \Psi^{(-)}_a &= -E_a \Psi^{(-)}_a, \\
    M^{(0)} \Psi^{(-)}_a &= -E_a \Psi^{(-)}_a, \\
    M^{(0)} \Psi^{(-)}_a &= -E_a \Psi^{(-)}_a,
\end{align*}
\]
and the eigenvectors are given by
\[
\begin{align*}
    \Psi^{(0)}_a &= \left( \begin{array}{c} u_a \\ 0 \\ \vdots \\ v_a \end{array} \right), \\
    \Psi^{(-)}_a &= \left( \begin{array}{c} -v_a \\ 0 \\ \vdots \\ u_a \end{array} \right).
\end{align*}
\]
(3.5)

Of course they satisfy the orthonormality relation.

The first-order solution is expanded in terms of the zeroth-order solution with both positive and negative energies;
\[
\Psi^{(1)}_a = \sum_\beta \left( c^{(1)}_{a\beta} \Psi^{(0)}_\beta + c^{(1)}_{a\beta} \Psi^{(-)}_\beta \right),
\]
(3.6)
where the coefficients are determined by the following first order equation,
\[
M^{(1)} \Psi^{(1)}_a = E_a \Psi^{(1)}_a + A^{(1)} \Psi^{(0)}_a.
\]
(3.7)

In order to obtain the solution it is necessary to evaluate the matrix elements of \( M^{(1)} \), which are given by
\[
\begin{align*}
    M^{(1)}_{a\beta} &= -\mathcal{Q}(j_{a\beta}) \tilde{R}^{(2)} + \Gamma^{(1)}_{a\beta} \tilde{R}^{(1)} + \delta_{a\beta} \left( J^{(1)} J^{(1)} + J^{(1)} \right) (\tilde{R}^{(2)}), \\
    M^{(1)}_{a\beta} &= -\mathcal{Q}(j_{a\beta}) \tilde{R}^{(2)} + \Gamma^{(1)}_{a\beta} \tilde{R}^{(1)} + \delta_{a\beta} \left( J^{(1)} J^{1} + J^{(1)} \right) (\tilde{R}^{(2)}),
\end{align*}
\]
(3.8)
where
In the above equations the first-order Hartree-Fock and pairing potentials
are calculated by replacing $A$ and $C$ by their first-order term which may be
obtained from Eq. (3.6):

$$A^{(1)}_{\alpha\beta} = c^{(1)}_{\alpha\beta} u_{\beta} - c^{(1)}_{\alpha\beta} v_{\beta},$$

$$C^{(1)}_{\alpha\beta} = c^{(1)}_{\alpha\beta} v_{\beta} + c^{(1)}_{\alpha\beta} u_{\beta}.$$ \hspace{1cm} (3.10)

If we put Eq. (3.10) into Eq. (3.7) and using Eqs. (3.8) and (3.9), we obtain

$$c^{(1)}_{\alpha\beta} = c^{(1)}_{\alpha\alpha} = 0,$$

$$c^{(1)}_{\alpha\beta} = -\frac{\mathcal{O}(J_{\alpha})}{E_{\alpha} - E_{\beta}} c^{(1)}_{\alpha\beta} - \frac{Z}{E_{\alpha} - E_{\beta}} \sum_{k} q^{k}_{\alpha\beta} x^{k},$$

$$c^{(1)}_{\alpha\beta} = \frac{\mathcal{O}(J_{\alpha})}{E_{\alpha} + E_{\beta}} c^{(1)}_{\alpha\beta} + \frac{Z}{E_{\alpha} + E_{\beta}} \sum_{k} q^{k}_{\alpha\beta} x^{k},$$ \hspace{1cm} (3.11)

where

$$x^{k} = \sum_{\gamma} q^{k}_{\alpha\beta} (\delta_{\epsilon_{\alpha} - \epsilon_{\gamma}} - \delta_{\epsilon_{\beta} - \epsilon_{\gamma}}) \left[ \frac{\eta^{(+)}_{\gamma\beta}}{E_{\alpha} + E_{\beta}} - \frac{\xi^{(+)}_{\gamma\beta}}{E_{\alpha} + E_{\beta}} \sum_{\gamma} q^{k}_{\beta\gamma} \xi^{(+)}_{\gamma\beta} \right] Q_{k}^{-1} \left[ -\mathcal{O}(J_{\alpha}) \right]_{\alpha\beta},$$

$$Q_{k} = 1 + Z \sum_{\gamma} q^{k}_{\gamma\gamma} \left[ (1 - \delta_{\epsilon_{\gamma} - \epsilon_{\alpha}} - \delta_{\epsilon_{\gamma} - \epsilon_{\beta}}) \left( \eta^{(+)}_{\gamma\beta} \xi^{(+)}_{\gamma\alpha} \right) \frac{E_{\alpha} + E_{\beta}}{E_{\alpha} + E_{\beta}} \right] Q_{k}^{-1} \left[ -\mathcal{O}(J_{\alpha}) \right]_{\alpha\beta}. \hspace{1cm} (3.12)$$

If the effect of the rotation on the Hartree-Fock and pairing potentials is
neglected, then $x^{k}$ vanishes. In the following this effect is neglected both in the
first- and second-order calculations. The wave functions in first order are ex­
pressed as

$$A^{(1)}_{\alpha} = -\frac{\mathcal{O}(J_{\alpha})}{E_{\alpha} - E_{\beta}} \left[ u_{\beta} (E_{\beta} u_{\beta} + \Delta v_{\beta} - (\epsilon_{\alpha} - \lambda) u_{\beta}) \right],$$

$$C^{(1)}_{\alpha} = -\frac{\mathcal{O}(J_{\alpha})}{E_{\alpha} - E_{\beta}} \left[ v_{\beta} (E_{\beta} v_{\beta} + \Delta u_{\beta} - (\epsilon_{\alpha} - \lambda) v_{\beta}) \right]. \hspace{1cm} (3.13)$$

The second-order calculation may be carried out quite analogously to the
first-order one, and the results are shown as follows,

$$A^{(2)}_{\alpha} = \mathcal{O}^{2} \sum_{\gamma} \frac{(J_{\gamma})_{\alpha\gamma}}{(E_{\beta} - E_{\beta})^{2}} \left[ u_{\beta} (E_{\beta} + \Delta v_{\beta} - (\epsilon_{\alpha} - \lambda) u_{\beta}) \right],$$

$$+ E \left( \epsilon_{\alpha} - \lambda \right) + (\epsilon_{\alpha} - \lambda)(\epsilon_{\gamma} - \lambda),$$

$$A^{(2)}_{\alpha} = -4 \left[ \eta^{(+)}_{\gamma\beta} \right]^{2} E_{\beta} E_{\gamma} \left( 4 \epsilon_{\gamma} - \lambda \right) \left( \epsilon_{\gamma} - \lambda \right),$$

$$C^{(2)}_{\alpha} = -4 \left[ \eta^{(+)}_{\gamma\beta} \right]^{2} E_{\beta} E_{\gamma} \left( 4 \epsilon_{\gamma} - \lambda \right) \left( \epsilon_{\gamma} - \lambda \right).$$
Although the state $\alpha$ and its time-reversed state $\bar{\alpha}$ are degenerate, the formulae based on the perturbation given previously are valid except the case of $m_\alpha = 1/2$, because $M_{\bar{\alpha}\alpha} = 0$ unless $m_\alpha = -m_\beta = 1/2$. In the case of $m_\alpha = \pm 1/2$, however, $\alpha$ and $\bar{\alpha}$ are degenerate and also $M_{\bar{\alpha}\alpha} \neq 0$. Besides, we encounter such cases in which two orbits with $m_\alpha$ and $m_\beta = m_\alpha \pm 1$ are almost degenerate. In these cases the perturbation approximation fails or becomes very bad, and we have to modify the starting basis. The matrix $\epsilon - \lambda - \Omega j_\alpha$ is diagonalized with respect to the degenerate or almost degenerate orbits and the resulting wave functions and eigenvalues are used as a starting basis. The results given in Eqs. (3·13) and (3·14) must be slightly modified, but the calculation is straightforward, so that these will not be presented here.

We remark here that the zeroth-order solution given by Eqs. (3·1) and (3·2) cannot be obtained for the orbit $l$ if Eq. (2·24) is derived from Eq. (2·13) assuming $\Omega = 0$ from the beginning, because Eq. (2·13) does not contain the term dependent on $l$ in the case of $\Omega = 0$.

### § 4. The moment of inertia and the gyromagnetic ratio

Calculations in the preceding sections are based on the simple perturbation theory. In this section the energy gap parameter $\Delta$ and the chemical potential $\lambda$ are self-consistently determined by using the results obtained in the last section.

If the wave functions given by Eqs. (3·13) and (3·14) are put into Eq. (2·22) which defines the energy gap parameter, then we get

$$
(2/G) \Delta = \sum_{m_\alpha > 0, \alpha \neq \bar{\alpha}} \frac{1}{E_\alpha} \left[ 1 - \Omega^2 \sum_{\beta \neq \bar{\beta}} \langle j_\beta \rangle_{\alpha \bar{\alpha}}^2 \left( \frac{2 \eta_{\alpha \bar{\beta}}}{E_\alpha + E_\beta} + \frac{2 \eta_{\alpha \bar{\beta}}}{E_\alpha + E_\beta} (\epsilon_\alpha - \lambda) \right) \right] + \Omega^2 \langle j_\alpha \rangle_{\alpha \bar{\alpha}}^2 \left( \frac{2}{E_\alpha^2 - E_\bar{\alpha}^2} \left( 1 + \frac{\eta_{\alpha \bar{\beta}}}{E_\alpha} \right)^2 \frac{E_\alpha}{E_\bar{\alpha}} + \frac{1}{E_\alpha (E_\alpha + E_\beta)} \right).$$

The energy gap parameter is calculated by using the pairing matrix $\kappa$ which is calculated based on the perturbation and contains the zeroth-order energy gap. Therefore the energy gap parameter $\Delta$ on the left- and right-hand sides of Eq. (4·1) are not equal. However, by many iterations they should converge, so that we put them equal and calculate their limiting values $\Delta$. The resulting equation for nontrivial $\Delta$ is
The second term on the right-hand side of Eq. (4·2) has an effect to reduce the effective strength of the pairing interaction because the quantity in the curly bracket in the second term is likely positive. The third term represents the blocking effect. Numerical calculation shows that the sum of the second and third terms is less than unity and has the total sign of minus.

The blocking effect is usually small, but if there exists a state \( \alpha \) near to the state \( \xi \), the effect becomes important. However, such a case corresponds to the failure of the perturbation theory and the original equations (2·24) must be solved more accurately according to the prescription given at the end of § 3.

It is easily seen that Eq. (4·1) does not have solution other than \( \Delta = 0 \) for \( \mathcal{Q} \geq \mathcal{Q}_c \), where \( \mathcal{Q}_c \) is the critical value. The solution with \( \Delta = 0 \) corresponds to the normal state, and the superconducting state with higher spin changes itself to the normal state at the critical spin value \( I_c \) corresponding to \( \mathcal{Q}_c \). This phenomenon is called the 2nd-order phase transition, and in this case it occurs by the rotation.

Next the self-consistent determination of the chemical potential is considered. If we put the wave functions (3·13) and (3·14) into Eq. (2·15), then we obtain the following equation,

\[
\begin{align*}
N - 1 &= \sum_{\alpha \neq \beta} 2\varepsilon_{\alpha}^2 + 2\mathcal{Q}^2 \left[ \sum_{\alpha \neq \beta} (j_{\alpha \beta})^2 \left\{ \frac{\eta_{\alpha \beta}}{(E_{\alpha} + E_{\beta})^2} \right\} (\varepsilon_{\alpha} - \lambda) \right. \\
&+ \sum_{\alpha \neq \beta} (j_{\alpha \beta})^2 \left\{ \frac{2E_{\alpha} - E_{\beta}}{E_{\alpha}^2 (E_{\alpha} - E_{\beta})} - \frac{1}{E_{\alpha}^2 (E_{\alpha} + E_{\beta})^2} \right\} \Delta_{\alpha \beta}^{-} \eta_{\alpha \beta}^{-} \Bigg] \\
&\quad + 2\mathcal{Q}^2 (j_{\alpha \beta})^2 \left\{ \frac{2\varepsilon_{\alpha}^2 - \eta_{\alpha \beta}}{(E_{\alpha} + E_{\beta})^2} \right\} (\varepsilon_{\alpha} - \lambda) \frac{\Delta_{\alpha \beta}^{-} \eta_{\alpha \beta}^{-}}{\Delta_{\alpha \beta} (E_{\alpha} + E_{\beta}) E_{\alpha}} \\
&+ \sum_{\alpha \geq \beta} (j_{\alpha \beta})^2 \left\{ \frac{2E_{\alpha} - E_{\beta}}{E_{\alpha}^2 (E_{\alpha} - E_{\beta})} - \frac{1}{E_{\alpha}^2 (E_{\alpha} + E_{\beta})^2} \right\} \Delta_{\alpha \beta}^{-} \eta_{\alpha \beta}^{-} \frac{\Delta_{\alpha \beta}^{-} \eta_{\alpha \beta}^{-}}{\Delta_{\alpha \beta} (E_{\alpha} + E_{\beta}) E_{\alpha}}.
\end{align*}
\]

The above equation should be solved coupled with Eq. (4·2) and self-consistently. Then we obtain the self-consistent \( \Delta \) and \( \lambda \). However, it is very difficult in practice, and the effect of the 2nd term in Eq. (4·3) is neglected in practical calculations.

We are now ready to calculate the moment of inertia and gyromagnetic ratio, which are characteristic quantities for the rotational motion. We already introduced the Coriolis term \( H_2 \) to take into account the effect of rotation on the intrinsic motion. This term may also be interpreted as a term which gives a constraint on the ground-state wave function so that the expectation value of the angular momentum may have a certain value. The angular velocity \( \mathcal{Q} \) is just a Lagrange multiplier. Therefore from the expectation value of \( H_2 \) we can obtain \( I_1 \), the angular momentum component perpendicular to the symmetry axis,
The moment of inertia perpendicular to the symmetry axis $I_\perp$ is defined by

$$I_\perp = \Omega f_1.$$  \hfill (4.4)

The moment of inertia may be calculated by inserting the coefficients $A$ and $C$ given by Eqs. (3.13) and (3.14) into the expression of the expectation value of $H_\perp$. The results are given by

$$I_\perp = \sum_{\alpha\beta, \gamma, \delta} \frac{2}{E_\alpha + E_\beta} \gamma_{\alpha\beta} \gamma_{\gamma\delta} + \sum_{\alpha, \beta, \gamma, \delta} \frac{2}{E_\alpha + E_\beta} \gamma_{\alpha\beta} \gamma_{\gamma\delta} + \frac{2}{E_\alpha - E_\beta} \gamma_{\alpha\beta} \gamma_{\gamma\delta} (1 - \gamma_{\alpha\beta}^2).$$ \hfill (4.6)

The second and third terms on the right-hand side of the above equation are concerned with the odd nucleon with quantum number $\xi$ and a consequence of the blocking effect. It is well known that the moment of inertia tends to the value for rigid body if the energy gap parameter vanishes, when the angular momentum exceeds the critical value.

The collective gyromagnetic ratio is defined by the relation

$$g_\perp = \frac{\langle \mu_\perp \rangle}{\langle I_\perp \rangle},$$ \hfill (4.7)

where $\mu_\perp$ is the component of the magnetic momentum operator perpendicular to the symmetry axis;

$$\mu_\perp = g_\perp s_\perp + g_1 l_\perp.$$ \hfill (4.8)

In the last equation, $s_\perp$ and $l_\perp$ are the components of the spin and orbital angular-momentum operators perpendicular to the symmetry axis. If the numerator is calculated by using the wave functions given by Eqs. (3.13) and (3.14), we obtain

$$g_\perp = \left[ g_\perp^p + (g_\perp^p - 1) W^p + g_\perp^n W^n \right] f_1^{-1},$$ \hfill (4.9)

where $g_\perp^p$ and $g_\perp^n$ are the spin gyromagnetic ratio of proton and neutron respectively, and their values are given by 5.585 and $-3.826$ in the unit of nuclear magneton respectively. Notations $W^p$ and $W^n$ are concerned with expectation values for the proton and neutron spin operators, and $W^p$ is defined by

$$W^p = \sum_{\alpha, \beta} \left( s_\alpha a_\beta (j_\alpha a_\beta) \gamma_{\alpha\beta} \gamma_{\beta\alpha} \right)$$ \hfill (4.10)

where the sum over $\alpha$ and $\beta$ are restricted to proton orbits, and $W^n$ is similarly defined.

The value for $W^p$ can be calculated by using the wave function $A$ and $C$ given by Eqs. (3.13) and (3.14), as

$$W^p = \sum_{\alpha, \beta, \gamma, \delta} \frac{(s_\alpha a_\beta (j_\alpha a_\beta) \gamma_{\alpha\beta} \gamma_{\beta\alpha})}{E_\alpha + E_\beta} + \sum_{\alpha, \beta, \gamma, \delta, \epsilon} \left\{ (s_\alpha a_\beta (j_\alpha a_\beta) \gamma_{\alpha\beta} \gamma_{\beta\alpha}) \gamma_{\delta\epsilon} \gamma_{\epsilon\delta} \right\} (1 - \gamma_{\alpha\beta}^2).$$ \hfill (4.11)
The energy of the ground rotational band is calculated from the expectation value of the Hamiltonian as

$$\langle H \rangle = \langle \mathcal{H} \rangle + \langle H_0 \rangle,$$  \hspace{1cm} (4.12)

which may be expressed as the sum of two parts: one is proportional to \( \mathcal{Q}^2 \) and the other does not contain \( \mathcal{Q} \) explicitly.

$$\langle H \rangle = U_0 + (1/2) (\mathcal{Q}_x I_x + \mathcal{Q}_y I_y),$$

$$U_0 = (1/2) \sum_i \left[ (\varepsilon_i - A_i) (B_i \cdot B_i^* + \mathcal{Q}_a (B_i B_i^* - A_i A_i^* )) - 2A_i B_i \cdot B_i^* \right],$$ \hspace{1cm} (4.13)

where \( I_x \) and \( I_y \) are the expectation values of the angular momentum perpendicular to the symmetry axis. \( \mathcal{Q}_x \) and \( \mathcal{Q}_y \) are the angular velocity components along \( x \) and \( y \) axis, and we have the following relations,

$$I_x = \mathcal{Q}_x J_1, \quad I_y = \mathcal{Q}_y J_1,$$ \hspace{1cm} (4.14)

as we assumed that the deformation is axial symmetric. If we write the expectation value of the symmetry component of the angular momentum by \( K \), which is the contribution from the odd nucleon, the expression (4.13) becomes

$$\langle H \rangle = U_0 + \frac{I^2 - K^2}{2J_1},$$ \hspace{1cm} (4.15)

where \( I \) is the total angular momentum and \( I^2 = I_x^2 + I_y^2 + K^2 \).

The first term of the right-hand side of the above equation also depends on the spin \( I \) through the energy gap parameter \( \Delta \) and chemical potential \( \lambda \). Therefore the excitation energy of the spin \( I \) state is given by

$$W(I) - W(I_0) = \frac{I^2 - I_0^2}{2J_1(I)} + U_0(I) - U_0(I_0),$$ \hspace{1cm} (4.16)

where \( I_0 \) is the ground-state spin. When \( K = 1/2 \), the above excitation energy must be modified as

$$W(I) - W(I_0) = \frac{1}{2J_1(I)} \left[ I^2 - I_0^2 + a \left( (-)^{I+1/2} (I+1/2) \right) - (I_0 + 1/2) (-)^{I_0+1/2} \right] + U_0(I) - U_0(I_0),$$ \hspace{1cm} (4.17)

where \( a \) is the decoupling parameter.

\section*{Numerical calculation}

Before presenting the results of numerical computations, the values of parameters used in computations are explained. First the single particle energy levels are taken from Nilsson's\(^{10}\) work using the following values of parameters...
Fig. 1. The change of gap as a function of angular velocity $\Omega$. The two cases, odd neutron and odd proton nuclei, are exhibited.

Fig. 2. The influence of $G$ on $\beta$, and $\Delta$ as a function of $\Omega$. The nucleus is Yb$^{171}$.

The interaction is not necessary because the interaction is neglected in the practical computation.

Practical calculation is carried out as follows. First the single particle energy in deformed well are calculated by just the same method as Nilsson's. Most of the results ($\delta = 0.2-0.3$) are provided by Mikoshiba, but in some cases ($\delta = 0.31, 0.3$) NEC 2206 computer in Osaka University is used to obtain the results.

Using the single particle energy and Nilsson's wave function, the energy gap parameter $\Delta$ and the chemical potential $\lambda$ are obtained by solving Eqs. (4.2) and $z = 0.05$ for all shells and $\mu = 0.45$ except for $N = 4$ and 5 proton shells ($N$ is the total number of oscillation quanta), where $\mu = 0.55$ is used. Levels taken in computation are $N = 3, 4, 5$ shells for protons (46 levels) and $N = 4, 5, 6$ shells for neutron (64 levels), as our computations are restricted to rare earth nuclei. The mass number dependence of positions of these levels is neglected. For the energy scale of the single particle spectra we use $\hbar \omega_0 = 41 \times \Delta^{-1/3} \text{MeV}$.

The deformation parameter $\delta$ is taken from the work of Mottelson and Nilsson$^{11}$ except Tm$^{165}$, for which $\delta = 0.3$ is used following Grin et al.$^{20}$ The value of pairing strength $G$ is taken from the work of Nilsson and Prior,$^{13}$ who fix values of $G$ so that the energy difference between even and the neighbouring odd mass nuclei agrees with experiments. The strength of the quadrupole interaction is neglected in the practical computation.
Table I. The parameters are used for odd proton nuclei, and the theoretical values are compared with experimental values. In column 5 the gap parameter for $\Omega=0$ is exhibited for both neutron and proton shells. In columns 6 and 7 the theoretical values $\varepsilon$ for the first excited levels are compared with experimental values. The critical energy and the critical spin are also exhibited in columns 8 and 9. Experimental values are quoted from references 16), 17), 18) and 19).

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_x$(Mev)</th>
<th>$I_c$/$I_0$</th>
<th>$G_pA$</th>
<th>$G_nA$</th>
<th>$\varepsilon_n(0)$</th>
<th>$\varepsilon_p(0)$</th>
<th>theory $(1/h^2)\varepsilon$ (Mev)$^{-1}$</th>
<th>exp. $(1/h^2)\varepsilon$ (Mev)$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}_{69}$Eu$^{153}$</td>
<td>411</td>
<td>3/2</td>
<td>0.30</td>
<td>17</td>
<td>23</td>
<td>1.010</td>
<td>0.520</td>
<td>41.40</td>
</tr>
<tr>
<td>$^{94}_{55}$Tb$^{159}$</td>
<td>411</td>
<td>3/2</td>
<td>0.31</td>
<td>17</td>
<td>23</td>
<td>0.820</td>
<td>0.510</td>
<td>43.10</td>
</tr>
<tr>
<td>$^{98}_{57}$Ho$^{165}$</td>
<td>523</td>
<td>7/2</td>
<td>0.30</td>
<td>17</td>
<td>24</td>
<td>0.740</td>
<td>0.510</td>
<td>43.20</td>
</tr>
<tr>
<td>$^{100}_{69}$Tm$^{169}$</td>
<td>411</td>
<td>1/2</td>
<td>0.30</td>
<td>17</td>
<td>24</td>
<td>0.635</td>
<td>0.535</td>
<td>41.10</td>
</tr>
<tr>
<td>$^{104}_{71}$Lu$^{175}$</td>
<td>404</td>
<td>7/2</td>
<td>0.28</td>
<td>17.5</td>
<td>23.5</td>
<td>0.640</td>
<td>0.530</td>
<td>36.60</td>
</tr>
</tbody>
</table>

and (4·3) for the assigned value of $\Omega$. In practice, however, the values of $\Omega$ and $\lambda$ are calculated by changing values of $\delta$ from 0.1 MeV to 1 MeV with 0.1 MeV's step on the average. The relations between odd nucleus and neighbouring even nucleus are shown in Fig. 1. In Fig. 2 it is shown how the relation changes when the strength of the pairing interaction $G$ is changed a little. After the values of $\delta$ and $\lambda$ are calculated, the wave function may be calculated by using Eqs. (3·10) and (3·11). These values are utilized to calculate various quantities such as the moment of inertia. All these calculations are performed by NEC 2203 computer in Osaka University.

All nuclei for which the calculations are made are listed in Tables I and II, together with parameters used and results. For nuclei Dy$^{161}(642, 5/2)$, Tb$^{159}$, Ho$^{165}$, Eu$^{153}$ and Gd$^{157}$, the partly diagonalizing procedure given at the end of § 3 are taken because the perturbation approximation fails. Let us first compare the moment of inertia: the agreements between theory and experiment are good except Dy$^{161}(642, 5/2)$, Gd$^{157}$, Ho$^{165}$ and Er$^{167}$, In these cases the experimental moment of inertia is larger than the theoretical one. The disagreement may be due to our insufficient knowledge of various parameters especially on single particle energies. Moreover our approximation used in solving the basic equa-
Table II. The same parameters and results as Table I for odd neutron nuclei.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>last particle level</th>
<th>$\delta$</th>
<th>$E_0$(Mev)</th>
<th>$I_\sigma-I_\alpha$ (n. p.)</th>
<th>theory $(1/\hbar^2)\delta J$ (Mev)$^{-1}$</th>
<th>exp. $(1/\hbar^2)\delta J$ (Mev)$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{91}_{84}$Gd$^{155}$</td>
<td>521 3/2</td>
<td>0.31</td>
<td>3.2</td>
<td>21 15</td>
<td>12.15</td>
<td>17.27</td>
</tr>
<tr>
<td>$^{93}_{84}$Gd$^{157}$</td>
<td>6.5 2.2</td>
<td>15 20</td>
<td>5.1</td>
<td>10 25</td>
<td>10.00</td>
<td>10.85</td>
</tr>
<tr>
<td>$^{95}_{84}$Dy$^{161}$</td>
<td>1.3 5.6</td>
<td>22 26</td>
<td>2.2</td>
<td>20 34</td>
<td>2.25</td>
<td>19.73</td>
</tr>
<tr>
<td>$^{97}_{84}$Dy$^{163}$</td>
<td>2.5 10</td>
<td>9 23</td>
<td>2.5</td>
<td>9 23</td>
<td>6.50</td>
<td>5.94</td>
</tr>
<tr>
<td>$^{101}_{84}$Er$^{167}$</td>
<td>1.1 3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{103}_{84}$Yb$^{171}$</td>
<td>0.8 4.1</td>
<td>7 23</td>
<td>0.8</td>
<td>7 23</td>
<td>4.80</td>
<td>6.35</td>
</tr>
<tr>
<td>$^{105}_{84}$Hf$^{177}$</td>
<td>2.8 1.8</td>
<td>18 13</td>
<td>2.8</td>
<td>18 13</td>
<td>6.25</td>
<td>5.72</td>
</tr>
</tbody>
</table>

An extremely large moment of inertia in the case of Dy$^{161}$ (642, 5/2) is difficult to be explained. However, the good agreement in other cases is rather surprising.

The critical energy and spin at which the transition from superconductor to normal states occurs are shown in columns 8 and 9. For odd $N$ nuclei the neutron critical points are lower than those of proton, and for odd $Z$ nuclei those of proton are lower than those of neutron. This may be interpreted to be due to the blocking effect, which helps the reduction of effective pairing strength. It is difficult to guess a general trend of the dependence of the critical points on mass number.

In column 10 the differences between the moment of inertia of odd mass nucleus and the neighbouring even nucleus are shown. Various quantities for even nuclei are obtained from the corresponding ones for odd mass nuclei by dropping the term for the orbit $l$ and removing the restriction on sum over $\alpha$, $\beta$, ... So, the difference in the moments of inertia may be calculated from
Eq. (4·6). The numerical calculations are performed using this equation. However, the sign of the difference \( \delta \mathcal{J} = \mathcal{J}^{\text{odd}} - \mathcal{J}^{\text{even}} \) can be roughly estimated so long as we neglect the change in self-energy.

\[
\delta \mathcal{J} = \sum_{\sigma = \uparrow, \downarrow} \sum_{\alpha, \beta} 2(j_x)^2 \left( \frac{\eta_{\alpha \beta}^{(-)}(\text{odd})}{(E_\alpha + E_\beta)} - \frac{\eta_{\alpha \beta}^{(-)}(\text{even})}{(E_\alpha + E_\beta)} \right) \\
+ \sum_{\sigma = \uparrow, \downarrow} \sum_{\alpha, \beta} 2(j_x)^2 \left( \frac{1}{E_\alpha - E_\beta} - \frac{\eta_{\alpha \beta}^{(-)}(\text{odd})}{E_\alpha + E_\beta} - \frac{\eta_{\alpha \beta}^{(-)}(\text{even})}{E_\alpha + E_\beta} \right),
\]  

(5・1)

where the superscripts (odd) and (even) denote that the values of \( \Delta \) and \( \lambda \) belong to either odd or even system. So long as the change of the chemical potential is small, the order of the first term in Eq. (5・1) is

\[
\sum_{\sigma = \uparrow, \downarrow} \sum_{\alpha, \beta} 2(j_x)^2 \frac{\eta_{\alpha \beta}^{(-)}(\text{odd})}{E_\alpha + E_\beta} \left( - \frac{\Delta}{E_\alpha E_\beta} \right),
\]

where \( \delta \mathcal{J} \) is the change of energy gap and this value is negative. Thus the first term in Eq. (5・1) is positive and the second term is also positive, because the last occupied level is very close to the Fermi surface in comparison with any other coupled levels. Thus we infer that \( \delta \mathcal{J} \) is positive.

The gyromagnetic ratio \( g_R \) is shown in Fig. 3. The deviation from the classical value \( Z/A \) is explained very well by the theoretical calculation. However, the effect of the rotation on the gyromagnetic ratio is very little for the ground state.

Fig. 3. The theoretical \( g \) values compared with the experimental values. Symbols 1 denotes \((642, 5/2)\) band of Dy\(^{164}\), while 2 \((523, 5/2)\) band. The dotted line indicates the value \( Z/A \). Experimental values are quoted from J. de Boer and J. D. Rogers.\(^{20}\)

A more detailed comparison of the energy spectra is made in Figs. 4 and 5. Only nuclei for which many levels in the rotational band have been observed are shown. As a general trend it is observed that the theory predicts slightly higher energies for the beginning of the rare earth and slightly lower energies
Figs. 4 and 5. The comparison between theoretical energy levels and experimental values. The levels with constant $\delta_{\text{exp}}$ which is determined by the first 3 levels are also shown to make clear the effect of Coriolis anti-pairing effect.
at the end of the region.

The rapid increase of $\mathcal{J}$ reverses the order of the higher spin state like $15/2$ spin state and $17/2$ spin state in Yb\textsuperscript{171}, which are not yet observed experimentally.
In Fig. 6 the spin dependence of the energy gap parameter is shown. The discontinuity of $\delta$ denotes that there exists 1st-order phase transition. The 1st-order phase transition is the sudden change of the ground state wave function when some parameter passes through a certain value, while the 2nd-order phase transition which have appeared in § 4 is the sudden change of the derivative of the wave function. The spin dependence of $\delta$ is less than that in the neighbouring even nuclei.

In Fig. 7 the change of the gyromagnetic ratio with spin is shown. The dips of the curves in Fig. 7 are the consequences of the neutron phase transition where $\delta_n$ increases much more than $\delta_p$ does. For Er$^{168}$ and Tm$^{169}$ the gyromagnetic ratio increases with spin at first and decreases after the maximum point, because the phase transition point of protons is lower than that of neutrons.

The changes of moments of inertia with spin are compared with the case of neighbouring even nuclei in Figs. 8 and 9. Near the ground state the moment of inertia for odd mass nuclei is larger than the one for even nuclei and increases with the spin, but the increase of the moment of inertia for even nuclei is more rapid and soon takes over the one for odd mass nuclei.

§ 6. Discussion

The good fit to the experimental values is obtained on the levels of the ground band and gyromagnetic ratio by our model in § 5. This fact indicates that our assumption is reasonable. Now we examine the approximation and the limitation of this theory. At first the interaction between neutrons and protons is not considered in our theory. This treatment may be acknowledged in the ground state of the rare-earth nuclei, as neutrons and protons fill different shells, and the radial wave functions are very different for protons and neutrons. However, this effect will become appreciable when the rotation increases, pairing force decreases, and the nuclei are excited to higher-energy levels.
Secondly the values $U_0(I)$ in Eq. (4.13) is also neglected in the practical calculations because they are very laborious without a large computer. This term is just of the same order as the second term in Eq. (4.3), which is neglected.

The rotation and vibration coupling is not taken into account in our theory, but this may have more influences on the moments of inertia than the Coriolis antipairing effect. As for the levels of Dy$^{164}$ (642, 5/2), we must consider this effect adding to careful examination of dependence on the deformation parameter and the single particle energy levels.

When we calculate the phase-transition energy, the solutions for diminishing values of the energy gap parameter are used. In such a critical case, the approximation of the non-conservation of nucleon number becomes important, and it may be expected that the critical energies become higher. In the worst case the transition may not be so distinct as predicted by this theory.

At last there are very few high spin collective rotational state observed in the deformed odd $A$ nuclei. Those states are necessary to investigate the Coriolis antipairing effect. We are eager to have more accurate experimental data about these states.

It may be concluded that although there are various effects, the Coriolis decoupling and blocking effects are important to explain the rotational levels of the ground band in the deformed odd nuclei, and our assumptions have been proved to be reasonable.

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