Baryon Decays and Unitary Symmetry

Yoshimatsu YOKOO

Department of Physics, Osaka University
Toyonaka, Osaka

(Received December 1, 1965)

Non-leptonic and leptonic decays of the baryons are quantitatively discussed on the basis of the current-current picture by introducing the axial vector current belonging to the 27-dimensional representation of SU(3) instead of the $F$-type one used in Cabibbo's theory. It is shown that the decay $\Sigma^+ \to \eta \pi^+$ proceeds through a pure $P$ wave and that the $P$ wave is larger than the $S$ wave for the decay $\Sigma^+ \to p \pi^0$. The interaction types of the decays $\Lambda \to p + l^+ + \nu$ and $\Xi^- \to \Lambda + l^+ + \nu$ are predicted to be $V - 1.1A$ and $V + 1.1A$, respectively.

Another interesting result is the predominance of the 27-dimensional representation part for the axial vector current. It is concluded that Cabibbo's currents are not the only ones possible to explain the semi-leptonic processes.

§ 1. Introduction

The leptonic decays of strongly interacting particles have been fairly well explained by Cabibbo on the basis of unitary symmetry. Many efforts have subsequently been made to extend the concepts to the non-leptonic processes. Sugawara, Lee and others have obtained a new triangle relation

$$2\Sigma^- = \Lambda^0 - \sqrt{3}\Sigma^+_0,$$

assuming particular transformation properties of the weak effective Hamiltonian under $SU(3)$ symmetry and $R$ conjugation. This relation (1) can also be derived by pole dominance methods which have no obvious connection with pure symmetry.

It seems, however, difficult to understand quantitatively the hyperon non-leptonic decays on the simple current-current picture composed of Cabibbo's currents. Various dynamical mechanisms have therefore been considered for the non-leptonic processes in addition to the current-current picture. We have so far no satisfactory quantitative explanation of hyperon non-leptonic decays. This fact prompts us to reexamine Cabibbo's currents from a new standpoint in treating the non-leptonic processes.

The purpose of the present paper is to give a possible explanation of both non-leptonic and leptonic decays based upon the current-current picture by modifying Cabibbo's currents within the framework of $SU(3)$ symmetry. In the Cabibbo theory of semi-leptonic processes, the vector currents are $F$ type whereas the axial vector currents are predominantly $D$ type. From this fact it
will be reasonable for us to adopt a postulate that the vector currents have an odd property under $R$ conjugation while the axial vector ones are even.\(^4\) According to the above assumption, we shall introduce the $D$ type and $27$-dimensional representation currents for the axial vector parts. On the other hand the CVC hypothesis implies that only the $F$ type currents exist for the vector parts.

The weak interaction Hamiltonian will be constructed from these currents in the current-current form. In the strangeness-changing decays, we have so far the approximate $|\Delta I|=1/2$ rule, violation of which will not be discussed here.\(^5\) Therefore the Hamiltonian responsible for these decays will be assumed to transform as an iso-spinor. We shall further postulate the octet enhancement of the Hamiltonian in the non-leptonic decay processes.

In § 2 the non-leptonic hyperon decays will be analyzed on the basis of the above proposals and the results will be shown to be in good agreement with the present experimental evidence. Section 3 will be devoted to a discussion of the semi-leptonic processes on the same footing.

§ 2. Non-leptonic decays of hyperons

We shall begin by describing the effective interactions responsible for the non-leptonic hyperon decays.

2-A. The effective Hamiltonian

Before going into the detailed discussion, we briefly summarize the basic assumptions stated in the Introduction as follows:

(i) The vector current is only $F$ type, $V^F$.

(ii) The axial vector current is composed of $D$ type, $A^D$ and $27$-currents $A^27$ belonging to the $27$-dimensional representation of $SU(3)$.

(iii) The effective Hamiltonian of the current-current form transforms as the $K^0$ meson under the $SU(3)$ group, satisfying the $|\Delta I|=1/2$ rule.

From the above assumptions (i) and (ii), we can describe the effective Hamiltonian responsible for the mesonic decays of hyperons in the form

\[(V^F + A^D + A^27) \otimes \partial \mu M, \tag{2}\]

where $M$ denotes the pseudoscalar meson octet.

We select the octet parts from the Hamiltonian (2) in accordance with the assumption (iii). The product $8 \otimes 8$ gives two octets $8_1$ and $8_2$, while the product $8 \otimes 27$ gives one octet. Therefore the octet meson current $\partial \mu M$ produces two effective interactions, coupled with the baryonic vector currents $V^F$ and three interactions together with $D$ type and $27$-axial vector currents.

We have five possible interaction types from the expression (2). The $SU(3)$ Clebsch-Gordan coefficients for each type of interaction are obtained by
using the tables of de Swart and his phase convention, which are listed in Table I.

Table I. Clebsch-Gordan coefficients for each interaction.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Vector</th>
<th>Axial vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \rightarrow p\pi^- )</td>
<td>((-\sqrt{30}/20) \times 1)</td>
<td>((-\sqrt{6}/20) \times 1)</td>
</tr>
<tr>
<td>( A \rightarrow m\pi^0 )</td>
<td>(-\sqrt{2}/2 \times 1)</td>
<td>(-\sqrt{2}/2 \times 1)</td>
</tr>
<tr>
<td>( \Sigma^+ \rightarrow p\pi^0 )</td>
<td>(0 \times 0)</td>
<td>(\sqrt{3} \times 0)</td>
</tr>
<tr>
<td>( \Sigma^+ \rightarrow n\pi^- )</td>
<td>(\sqrt{2}/3 \times \sqrt{6}/3)</td>
<td>(-\sqrt{6}/2 \times -\sqrt{2}/2)</td>
</tr>
<tr>
<td>( \Sigma^- \rightarrow n\pi^0 )</td>
<td>(1 \times 1)</td>
<td>(-1 \times -1)</td>
</tr>
<tr>
<td>( \Sigma^0 \rightarrow A\pi^- )</td>
<td>(1/\sqrt{2} \times 1/\sqrt{2})</td>
<td>(-1/\sqrt{2} \times -1/\sqrt{2})</td>
</tr>
</tbody>
</table>

Table I shows that for these processes coupling coefficients of the \(8_i\) type are those of the \(8_1\) type multiplied by the constant \((-\sqrt{5}/3)\) for each of the vector and axial vector parts. One can therefore express the effective Hamiltonian responsible for the non-leptonic hyperon decays by means of three coupling parameters as follows:

\[
A \rightarrow p\pi^- : \quad G \bar{p} \gamma_\mu (1 - a_3) \partial_\mu \pi^+,
\]

\[
A \rightarrow m\pi^0 : \quad -\frac{G}{\sqrt{2}} \bar{m} \gamma_\mu (1 - a_3) \partial_\mu \pi^0,
\]

\[
\Sigma^+ \rightarrow n\pi^+ : \quad \sqrt{3} b \bar{p} \gamma_\mu \gamma_5 \Sigma^+ \partial_\mu \pi^-,
\]

\[
\Sigma^+ \rightarrow p\pi^0 : \quad -\frac{G}{\sqrt{3}} \bar{p} \gamma_\mu (1 + 3a_3) \Sigma^+ \partial_\mu \pi^0,
\]

\[
\Sigma^- \rightarrow n\pi^- : \quad \sqrt{\frac{2}{3}} G \bar{m} \gamma_\mu \gamma_5 \Sigma^- \partial_\mu \pi^+,
\]

\[
\Xi^- \rightarrow A\pi^- : \quad G \bar{A} \gamma_\mu (1 + a_3) \Xi^- \partial_\mu \pi^+,
\]

\[
\Xi^0 \rightarrow A\pi^0 : \quad \frac{G}{\sqrt{2}} \bar{A} \gamma_\mu (1 + a_3) \Xi^0 \partial_\mu \pi^0.
\]

2-B. Comparison with experiments

We consider seven non-leptonic hyperon decay modes, among which are independent since the validity of the \(|d| = 1/2\) rule is assumed here. Accordingly there are eight independent physical quantities, namely, the four decay rates and the four asymmetry parameters, while we have three independent parameters \(G, a\) and \(b\).

It is seen from the effective Hamiltonians (3) that the decay \(\Sigma^+ \rightarrow n\pi^+\) proceeds via a pure \(P\) wave, which is compatible with recent experiments. The triangle relation (1) is satisfied for each of the \(S\) and \(P\)-wave amplitudes,
irrespective of the coupling parameters, if the mass differences are neglected.

Adjusting the three parameters $G$, $a$ and $b$ in order to make them fit with experiment, the various decay quantities have been calculated; they are tabulated in Table II where we have used the following values:

$$a = 0.53, \quad b = -0.51,$$

and

$$\left( \frac{G^2}{4\pi} \right) = 2.56 \times 10^{-12} / m_p^4.$$

It is to be noticed that in our calculation we have assumed $CP$ invariance and ignored the final state interaction giving rise to $\beta = 0$.

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>$\Lambda \to p\pi^-$</th>
<th>$\Sigma^+ \to n\pi^+$</th>
<th>$\Sigma^+ \to p\pi^0$</th>
<th>$\Sigma^- \to n\pi^-$</th>
<th>$\Xi^- \to \Lambda\pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>theory</td>
<td>0.60</td>
<td>0</td>
<td>-0.96</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>experiment</td>
<td>0.62±0.07(3)</td>
<td>-0.05±0.08(3)</td>
<td>-0.79±0.09(3)</td>
<td>-0.16±0.21(3)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>theory</td>
<td>0.80</td>
<td>-1</td>
<td>-0.26</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>experiment</td>
<td>0.78±0.06(3)</td>
<td>&lt;0$^9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rate $10^9\text{sec}^{-1}$</td>
<td>theory</td>
<td>2.6</td>
<td>8.7</td>
<td>7.4</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>experiment$^{(11)}$</td>
<td>2.62±0.06</td>
<td>6.17±0.50</td>
<td>6.43±0.50</td>
<td>6.33±0.20</td>
</tr>
</tbody>
</table>

$^{(a)}$ The value when we neglect the phase difference between $S$ and $P$ waves due to the final state interactions.

Dalitz pointed out that the best least squares fit to the experimental data on $\Sigma$ decays gives $\alpha (\Sigma^+) = -0.95$ within the validity of the $|\Delta I| = 1/2$ rule.$^{(13)}$ Taking into account the experimental value $\alpha (\Sigma^+) = 0.79 \pm 0.09$, it is necessary to postulate a small violation of the $|\Delta I| = 1/2$ rule in these decays.$^{(5)}$

For the asymmetry parameters of the decay $\Xi^- \to \Lambda\pi^-$ the experimental value is not yet settled$^{(13)}$ and the latest experimental data$^{(5)}$ is listed in Table II. In spite of considerable variation it is certainly proved by experiment that the decay $\Xi^- \to \Lambda\pi^-$ proceeds predominantly through the $S$ wave, i.e. $\gamma > 0$.$^{(10,11)}$

From Table II and the above discussions we conclude that the results are in excellent agreement with the present experimental evidence within the validity of the $|\Delta I| = 1/2$ rule. It is, finally, to be emphasized that the $P$ wave is larger than the $S$ wave for the decay $\Sigma^+ \to p\pi^0$; it would be desirable to check this by future experiments.
§3. Leptonic decays of hadrons

The ideas presented in the previous section will be extended to the semi-leptonic decay processes. For the axial vector parts, we introduce the $27$ currents $A_{\mu}^{27}$ instead of the $F$ type currents in Cabibbo’s theory. The processes with $|\Delta S|=2$ have never been found experimentally and we have the empirical iso-spin selection rule $|\Delta I|=1$ for the processes with $\Delta S=0$ and $|\Delta I|=1/2$ for $|\Delta S|=1$. Accordingly we confine ourselves to the decays allowed by this selection rule in what follows.

We describe the hadronic currents responsible for the leptonic decays in the new form of universality:

$$J_{\mu} = \cos \theta (V_{\mu}^{F}\gamma_{\mu} + A_{\mu}^{D}\gamma_{\mu} + A_{\mu}^{27}\gamma_{\mu}) + \sin \theta (V_{\mu}^{F(1)} + A_{\mu}^{D(1)} + A_{\mu}^{27(1)}),$$

where the superscripts $0$ and $1$ mean the currents with $\Delta S=0$ and $|\Delta S|=1$, respectively. The Hamiltonian responsible for leptonic processes is produced by the currents (4) coupled with the charged lepton currents.

In the leptonic decays of pseudoscalar mesons Cabibbo’s argument is completely valid in this case, since the pseudoscalar meson currents have an octet transformation property under the $SU(3)$ group. We therefore do not discuss these decays here and go on to the leptonic decay processes of baryons.

The matrix elements of the currents $J_{\mu}$ between two baryon states can be expressed in terms of three coupling parameters, namely the coupling constants of $F$ type vector, $D$ type and $27$-axial vector currents. In order to determine these parameters, we single out the $n\rightarrow p$ and $\Sigma^-\rightarrow\Lambda\beta$ decays as input. The relevant matrix elements for them are explicitly written as follows:

$$\langle p | J_{\mu} | n \rangle = \cos \theta \bar{u}_p \gamma_{\mu} \{1 + (a - \sqrt{\frac{2}{3}} b) \gamma_5 \} u_n,$$

and

$$\langle \Lambda | J_{\mu} | \Sigma^- \rangle = \cos \theta \left( \sqrt{\frac{2}{3}} a + b \right) \bar{u}_\Lambda \gamma_{\mu} \gamma_5 u_{\Sigma^-},$$

where $a$ and $b$ are the coupling constants of the $D$ type and $27$-axial vector currents, respectively.

The recent experiment on neutron $\beta$ decay$^{14}$ tells us that

$$a - \sqrt{\frac{2}{3}} b = \left( \frac{G_\beta}{G_\mu} \right) = 1.18 \pm 0.03,$$

and

$$\cos \theta = \left( \frac{G_\beta}{G_\mu} \right) = 0.977 \pm 0.001,$$

which is consistent with the Cabibbo angle derived from the leptonic decays of pseudoscalar mesons.$^{15,16}$ From the decay rate for $\Sigma^-\rightarrow\Lambda+e^-+\nu^{16}$ and Eq.
Baryon Decays and Unitary Symmetry

From Eq. (8), we get

\[ \sqrt{\frac{2}{3}} \ a + b = (0.66 \pm 0.20). \] (9)

The equations (7) and (9) give the following two solutions corresponding to the signs in Eq. (9):

Solution I: \( a = 1.03 \) and \( b = -0.18 \),

Solution II: \( a = 0.39 \) and \( b = -0.97 \). (10)

We have so far determined the three coupling parameters as shown by Eqs. (8) and (10). Making use of these values we have calculated the decay rates and the coupling types for the various baryon \( \beta \) decays. Among the above two solutions, the present experimental evidence favours the solution II, which shows a predominance of the 27-dimensional representation parts in the axial vector currents. In Table III we list a summary of our results for the \( \beta \) decays of baryons in the case of solution II.

Table III. The branching ratios and the type of interactions for the \( \beta \) decays of baryons.

In the theoretical calculation we have used the values; \( \cos \theta = 0.977 \), \( a = 0.39 \) and \( b = -0.97 \).

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Branching ratio</th>
<th>Interaction type</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>n \rightarrow p + e^+ + \bar{\nu} )</td>
<td>input ( 1 )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^- \rightarrow \Lambda + e^+ + \bar{\nu} )</td>
<td>input ( 0.75 \pm 0.28 \times 10^{-4} )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^+ \rightarrow \Lambda + e^+ + \bar{\nu} )</td>
<td>( 0.23 \times 10^{-4} )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^0 \rightarrow \Sigma^+ + e^- + \nu )</td>
<td>( &lt; 2.0 \times 10^{-16} )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^- \rightarrow \Sigma^0 + e^- + \nu )</td>
<td>( 1.3 \times 10^{-10} )</td>
</tr>
<tr>
<td>(</td>
<td>\Xi^- \rightarrow \Xi^0 + e^- + \nu )</td>
<td>( 1.1 \times 10^{-9} )</td>
</tr>
<tr>
<td>(</td>
<td>A \rightarrow p + e^- + \bar{\nu} )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>(</td>
<td>\Lambda \rightarrow p + e^- + \bar{\nu} )</td>
<td>( &lt; 2.6 \times 10^{-6} )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^- \rightarrow \Lambda + e^- + \bar{\nu} )</td>
<td>( 0.88 \times 10^{-3} )</td>
</tr>
<tr>
<td>(</td>
<td>\Sigma^- \rightarrow \Sigma^0 + e^- + \nu )</td>
<td>( 2.1 \times 10^{-4} )</td>
</tr>
<tr>
<td>(</td>
<td>\Xi^- \rightarrow \Xi^0 + e^- + \nu )</td>
<td>( 1.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>(</td>
<td>\Xi^- \rightarrow \Xi^0 + e^- + \nu )</td>
<td>( 0.46 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Table III shows that our results are in better agreement with the present experiment than Cabibbo's. It is further to be noted that in our prediction the coupling type for the \( A \_ \) decay is \( V - 1.1 \ A \) which is compatible with the larger axial vector interaction, i.e. \( A - 0.9 \pm 0.5 V^{17},^{18} \)

§ 4. Discussions

We have discussed both non-leptonic and leptonic decays of hadrons introducing the 27-axial vector currents belonging to the 27-dimensional repre-
sentation of $SU(3)$ and obtained results in good agreement with the present experiments. Interesting results are that the decay $\Sigma^+ \rightarrow n\pi^+$ proceeds through a pure $P$ wave and the decay $\Sigma^+ \rightarrow p\pi^0$ predominantly as a $P$ wave. The analysis of semi-leptonic processes predicts that the types of interactions are $V^- 1.1A$ for the $A_2$ decay and $V^+ 1.1A$ for the decay $E^- \rightarrow A + e^- + \bar{\nu}$; this will be tested accurately in the near future.

In the present paper we determined the coupling parameters independently for the non-leptonic and leptonic processes and did not discuss the relations among them. This is a question left for the future. In conclusion we emphasize that Cabibbo’s current is not the only one possible to explain the semi-leptonic decay processes.

**Acknowledgement**

The author would like to thank Dr. A. Sato for valuable discussions.

**References**

6) J. J. de Swart, Rev. Mod. Phys. 35 (1963), 916.
    P. L. Connolly et al., *Proceedings of the Sienna International Conference on Elementary Particles*, Italy (1963), p. 34.