On the Excitation of the $J^\prime=3/2^+$ Decuplet and $J^\prime=3/2^-$ Octet in the Neutrino Reaction on the Quark and the Cloud Models

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The 70-56 currents between the $J=3/2$ baryon resonance and the $J=1/2$ baryon are calculated on the $SU(12)$ model. The results of B. Sakita and K. C. Wali and ours are tabulated in Tables I (a) and (b). We have also calculated the $J=3/2$ baryon resonance-baryon currents on the cloud effect model, namely assuming the Cabibbo currents for the virtual baryon octet and the virtual meson octet in the $SU(3)$ theory. Further we take into account the Goldberger-Treiman relation, by assuming the conservation of weak vector and axial vector currents. A comparison of the $SU(12)$ model and the cloud effect model can be made from the experiments on the inelastic one pion production by neutrino-nucleon collision and the electron-neutrino angular correlation in the $\beta$ decay of $Q^-$ particles.

The aim of this paper is to investigate whether the success of $SU(6)$ as to the static properties of the baryon can be extended to our $N^*$ production by neutrino, and also to consider as an extreme opposite model, the pion cloud model which is not taken into account in the $SU(6)$ and to compare both results to clarify the validity of the $SU(6)$ theory.

§ 1. Introduction

The quark model and the $SU(6)$ theory have attained much success in the prediction of various properties of the baryon octet and the $J^\prime=3/2^+$ decuplet resonances. One of the most important consequences of this model is the ratio of the magnetic moments of proton and neutron ($\mu_p/\mu_n = -3/2$). The anomalous magnetic moments have long been believed to be explained by the meson cloud of the nucleon. It is quite embarrassing to get this correct value of the ratio in the quark model without a meson cloud effect. For the purpose of clarifying this problem, we take up a similar problem, namely the excitation of the $J^\prime=3/2^+$ decuplet and $J^\prime=3/2^-$ octet by the neutrino-nucleon collision.

It is the purpose of this note to compare the results of the ordinary cloud model with those of the relativistic quark field model in the excitation of the $J^\prime=3/2^+$ decuplet and $J^\prime=3/2^-$ octet by the neutrino-nucleon reaction. The difference of the two models may be checked by experiments on inelastic one-pion production by neutrino-nucleon collision and the electron-neutrino angular correlation in the $\beta$ decay of $Q^-$ particles.

B. Sakita and K.C. Wali derived the vector and axial vector currents between the $J^\prime=3/2^+$ decuplet and the baryon octet in the $SU(6)$ theory (that is, the current between two 56-dimensional baryons). We also evaluate the vector and axial vector currents between the $J^\prime=3/2^-$ octet and the baryon
octet on the $S\bar{U}(12)$ model\(^6\) (that is, the current between the 70-dimensional resonances and the 56-dimensional baryons). In §2 we summarize the derivation of the 70–56 current on the $S\bar{U}(12)$ model. The results of Sekita and Wali and ours are tabulated in Tables 1(a) and (b).

Our rule for the derivation of the weak current between the $J^p=3/2^+$ decuplet (or $J^p=3/2^-$ octet) and the baryon octet on the meson cloud model is as follows. We assume the conservation of weak vector and axial vector currents. In this model the $J^p=3/2^+$ decuplet and $J^p=3/2^-$ octet resonances can only interact with the lepton current through a virtual baryon and boson. For baryon- and boson- currents we take the Cabibbo current.\(^3\) To construct the boson axial vector current, we introduce the scalar octets, which have not been experimentally established, but have been conjectured for various reasons. Our current for the excitation $J^p=3/2^+$ (and $3/2^-$) can be calculated by the Feynman diagrams (Figs. 1 and 2). The divergence of the virtual integral is cut off at the baryon mass $M$. The general forms of the currents are given in Eq. (5). From the assumption of the conservation of vector and axial vector currents, we must introduce the Goldberger-Treiman terms (\(f_v\) and \(f_A\) terms in Eq. (5)) and these are evaluated by the diagram of Fig. 3. By this assumption, the \(f_v\) and \(f_A\) terms in Eq. (5) are fixed if we take for \(f_A\), \(f_T\) in Eq. (5) the values obtained by perturbation theory. We take for \(f_v\), \(f_A\), \(f_T\) and \(f_T\) the values evaluated by the perturbation method. This is consistent, because \(f_v\) (\(f_A\)) and \(f_T\) (\(f_T\)) are connected with the magnetic moment and electric charge radius in the electric current, which are fairly correctly evaluated by perturbation theory. In the conventional field theory the nucleon anomalous magnetic moment arises as the effect of the pion cloud around the nucleon, and the charge and magnetic moment distribution around the nucleon is explained as the combined effect of $\pi$, $\rho$- and $\omega$- meson clouds. Then it is expected that our \(f_v\) (\(f_A\)) and \(f_T\) (\(f_T\)) effects can be explained in a similar way.

On the other hand, in the quark model, the magnitude of the anomalous magnetic moment is considerably smaller than the experimental value and, in $SU(6)$, is assumed to be enhanced phenomenologically by the introduction of the vector meson cloud effect (by the factor $(1+2M/\mu)$, where $\mu$ is the mass of the vector meson). Therefore the \(f_T\), etc., terms calculated by the $SU(6)$ theory should be multiplied by the same factor. A check on the \(f_T\), etc., terms will give us a very important information on the structure of baryon and on the properties of the quark. At present, the quark seems to have the usual meson cloud around it.

Further a check on the Goldberger-Treiman term\(^4\) gives quite important information on the properties of the quark, which will be discussed elsewhere.

In §3, the induced terms in the $P(J^p=3/2^+)-N(J^p=1/2^+)$ currents and $D(J^p=3/2^-)-N$ currents are estimated as the contribution from the Feynman diagrams of Figs. 1 and 2. We also discuss the relation between the Goldberger-
Treiman relation (in Fig. 3) and the conservation of the vector and the axial vector currents of $P - N$ and $D - N$ currents. From this conservation of current, we fix the $f_\pi$ and $f_P$ terms uniquely.

§ 2. The evaluation of the $B(J = 3/2) - N(J^p = 3/2^-)$ current in the $SU(12)$ theory

In this section we calculate the $B(J = 3/2) - N(J^p = 1/2^+)$ currents in the $SU(12)$ theory. $N(J^p = 1/2^+)$ is assigned to the $SU(3)$ octet member of the 56-dimensional representation of $SU(6)$. There are two $J = 3/2$ unitary supermultiplets; the $N^*$ decuplet and the $N^{**}$ octet. The well-known $J^p = 3/2^- N^*$ is assigned to the $SU(3)$ decuplet member of the 56-dimensional representation of $SU(6)$. The $N^{**}$ octet may be assigned to the $SU(3)$ octet of the 70-dimensional representation of $SU(6)$. The 56–56 currents are evaluated by B. Sakita and K.C. Wali. We therefore consider the $N^{**}(J = 3/2, \tilde{J} = 1/2)$ octet $- N(J = 1/2)$ currents, and calculate the 70–56 vector and axial vector currents, for the 70 dimensional representation in the even and odd parities.

The wave functions of the $N(J^p = 1/2^+)$ octet of 56 is

$$\epsilon_{ij} \psi_k \xi_{ABD} b_0^D + \epsilon_{jk} \psi_i \xi_{BCD} b_A^D + \epsilon_{ki} \psi_j \xi_{CAB} b_B^D$$

and that of the $D(J^p = 3/2^-)$ octet of 70 is

$$\chi_{(i,j,k)} \epsilon_{ABD} b_0^D,$$

where $i, j, k = 1, 2$; $A, B, C, D = 1, 2, 3$; $\epsilon_{ij}$ and $\epsilon_{ABC}$ are Levi-Civita symbols; $\chi_i$ is a Pauli spinor; the $\chi_{(i,j,k)}$ are the spin-3/2 wave functions; the $b_A^B$ are the usual baryon octet tensors. The relativistic extensions of these wave functions are easily found to be

$$\phi_{(a,b)} = (A_{ij} C)_{ij} \psi_k \xi_{AHD} b_0^D + (A_{ij} C)_{ik} \psi_j \xi_{BCD} b_A^D + (A_{ij} C)_{kj} \psi_i \xi_{CAB} b_B^D$$
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for the $J^p=1/2^+$ octet of 56,

$$\psi_{\alpha \beta \gamma} = (A' \gamma' C)_{ij} \psi_{\alpha \beta} \epsilon_{ABD}b_d^p$$

for the $J^p=3/2^+$ octet of 70,

$$\psi_{\alpha \beta \gamma} = (A' \gamma' C)_{ij} (\gamma' \psi_{\mu})_R \epsilon_{ABD}b_d^p$$

for the $J^p=3/2^-$ octet of 70,

where $\alpha = (i, A)$, $\beta = (j, B)$, $\gamma = (k, C)$; (Suffix of Dirac spinor) $i, j, k = 1, 2, 3, 4$; $\psi$ is a Dirac spinor; $\psi_{\mu}$ is a Rarita-Schwinger field; $C$ is the charge conjugation operator; $A$ and $A'$ are the projection operators for a positive energy state and their explicit forms are

$$A = -\frac{it \gamma p + M}{2M}, \quad A' = -\frac{it \gamma p' + M'}{2M'};$$

$M$ and $M'$ are the masses of 56 and 70, respectively.

Now that we have obtained the wave functions, we can at once calculate the currents $J_{\mu}$.\[J_{\mu}A' = \overline{\psi} [\left(1 - q^2 + M^2 + M'^2\right) / 2MM'] \delta_{\mu \nu} \left(F - 3D / 2\right)_{A'} + \left(1 - q^2 + M^2 + M'^2\right) \delta_{\mu \nu} \left(F - 3D / 2\right)_{A'}

Substituting the wave functions (1) into the currents (2), and after lengthy and tedious calculations, we arrive at the following forms of the currents.

$$J_{\mu}A' = \overline{\psi} \left[\left(1 - q^2 + M^2 + M'^2\right) / 2MM'\right] \delta_{\mu \nu} \left(F - 3D / 2\right)_{A'} + \frac{1}{2MM'} q_{\nu} \left(1 - 3\gamma_{3}\right) \left(F - 3D / 2\right)_{A'} + \frac{1}{2MM'} q_{\nu} \left(1 + \gamma_{3}\right) \left(M - 3M' / 2\right) \left(F - 3M + 3M' / 2\right)_{A'} + \frac{1}{2MM'} q_{\nu} \left(1 - 3\gamma_{3}\right) \left(F - 3D / 2\right)_{A'} \psi$$

for the $J^p=3/2^-$ octet of 70 and $J^p=1/2^+$ octet of 56 currents and

$$J_{\mu}A' = \overline{\psi} \left[\left(1 + q^2 + M^2 + M'^2\right) / 2MM'\right] \delta_{\mu \nu} \left(F - 3D / 2\right)_{A'} + \frac{1}{2MM'} q_{\nu} \left(1 - 3\gamma_{3}\right) \left(F - 3D / 2\right)_{A'} - \frac{1}{2MM'} q_{\nu} \left(1 + \gamma_{3}\right) \left(F - 3D / 2\right)_{A'}$$

for the $J^p=3/2^-$ octet of 70 and $J^p=1/2^+$ octet of 56 currents.
for the \( J^p=3/2^+ \) octet of \( 70-J^p=1/2^+ \) octet of 56 currents, where \( q_\rho = p_\rho' - p_\rho ; p_\rho' (p_\rho) \) is the momentum of the Rarita-Schwinger field \( \psi_\rho \) (Dirac field \( \psi \) ; \( M' (M) \) is the mass of \( \psi_\rho \) (\( \psi \) ; \( F_A^{\alpha'} \) and \( D_A^{\alpha'} \) are the so-called \( f \) and \( d \) couplings defined by

\[
F_A^{\alpha'} = \tilde{b}_b^A b_A^b - \tilde{b}_A^b b_b^A ,
\]

\[
D_A^{\alpha'} = \tilde{b}_b^A b_A^b + \tilde{b}_A^b b_b^A - \frac{2}{3} \bar{\psi} \gamma^\alpha \psi.
\]

In the same manner the \( SU(6) \) singlet currents are given by

\[
J_\mu = \bar{\psi}_\mu \left[ \frac{-1}{2MM'} \left( q^2 + (M-M')^2 \right) \delta_{\mu\nu} + \frac{1}{2MM'} (1-\gamma_5) \right. \\
+ \frac{1}{2MM'} (M-M') (1+\gamma_5) \left. \right] \frac{1}{2MM'}
\]

for the \( J^p=3/2^- \) octet of \( 70-J^p=1/2^- \) octet of 56 currents and

\[
J_\mu = \bar{\psi}_\mu \left[ \frac{-1}{2MM'} \left( q^2 + (M+M')^2 \right) \delta_{\mu\nu} + \frac{1}{2MM'} \frac{1}{2MM'} (1-\gamma_5) \right. \\
+ \frac{1}{2MM'} (M-M') (1+\gamma_5) \left. \right] \frac{1}{2MM'}
\]

for the \( J^p=3/2^+ \) octet of \( 70-J^p=1/2^+ \) octet of 56 currents. In the derivation of Eqs. (3) and (3') we made use of the following identities:

\[
(M+M') \bar{\psi}_\mu \gamma_\mu \gamma_5 \psi + \bar{\psi}_\mu \gamma_\mu \gamma_5 \gamma_\rho q_\rho \psi + i \bar{\psi}_\rho Q_\alpha \psi = 0 ,
\]

\[
(M-M') \bar{\psi}_\mu \gamma_\mu \gamma_5 \psi + \bar{\psi}_\mu \gamma_\mu \gamma_5 \gamma_\rho q_\rho \psi + i \bar{\psi}_\rho Q_\alpha \psi = 0 ,
\]

where

\[
q_\lambda = p_\lambda' - p_\lambda , \quad Q_\lambda = p_\lambda' + p_\lambda
\]

and

\[
\epsilon_{\mu \nu \rho \lambda \phi} = - \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\lambda + \delta_{\mu \rho} \gamma_\nu \gamma_\lambda - \delta_{\mu \nu} \gamma_\rho \gamma_\lambda + \delta_{\mu \nu} \gamma_\rho \gamma_\phi .
\]
§ 3. The estimation of the B-N currents by the meson cloud model

Let us consider the matrix elements of the currents between the \( B(J = 3/2) \) baryon resonances and the baryons. The general explicit forms of the matrix elements are then,

\[
\langle B| j_\mu(0)| N \rangle = \bar{B}_i(p') \left[ f_\mu q_\nu q_s + \frac{f_\tau}{M^2} \gamma_\mu q_s + \frac{f_\mu}{M^2} \gamma_\nu q_s + \frac{f_\tau}{M^2} \gamma_\mu \gamma_\nu q_s \right] N(p),
\]

where the \( f's \) are form factors, \( q_\mu \) is the 4-momentum transfer \( (q_\mu = p_{p'} - p_p) \) and \( B_i(p') \) and \( N(p) \) represents the fields of the \( B \) and \( N \) states with 4-momenta \( p' \) and \( p \), respectively.

In the perturbation theoretical approach, the predominant parts of the matrix elements of the currents, Eq. (5), may be determined as the contribution from Figs. 1 and 2. In this subsection, we consider, as the final \( B(J = 3/2) \) state, \( \pi^0 \) \( SU(3)N^* \) decuplet and \( D(J^p = 3/2^-) - SU(3)N^{**} \) octet.*

In this section, we employ \( B(J = 3/2) \) for \( J^p = 3/2^+ \) and \( J^p = 3/2^- \) collectively. We assume \( B = P \) for the \( J^p = 3/2^+ \) state and \( B = D \) for the \( J^p = 3/2^- \) state.

To estimate the contribution to the matrix elements of Eq. (5), we adopt the following interaction Hamiltonians, where \( \pi \) is the pseudoscalar boson octet.

\[
H_{\pi N}^{(s)} = g_{\pi N} B_i^{(s)} N_i^{\pi N}
\]

for the strong \( B(J^p = 3/2^+) \pi N^- \) vertex,

\[
H_{\pi N}^{(s)} = g_{\pi N} B_i^{(s)} (a' N_i^{\pi N} + b' N_i^{\pi N})
\]

for the strong \( B(J^p = 3/2^-) \pi N^- \) vertex,

\[
H_{\pi N}^{(w)} = g_{\pi N}^{\pi N} (a N_i^{\pi N} + b N_i^{\pi N})
\]

for the strong \( N\pi N^- \) vertex,

\[
H_{\pi N}^{(w)} = \frac{G}{\sqrt{2}} (j_{\pi} \cdot j_{\pi}^{(s)} + h.c.)
\]

for the \( \pi \)-lepton vertex of Fig. 2 and

\[
H_{\pi N}^{(w)} = \frac{G}{\sqrt{2}} (J_{\pi} \cdot J_{\pi}^{(s)} + h.c.)
\]

for the weak baryon lepton vertex of Fig. 1, where \( J_{\pi} \) and \( j_{\pi} \) are the currents for the baryon and the pseudoscalar mesons, respectively. Roman letters, \( p, q, r, \ldots \), denote the \( SU(3) \) indices and Greek letters, \( \mu, \nu, \ldots \), denote the Lorentz space indices. Equations (6d) and (6e) are fixed from the usual Cabibbo cur-

* Hereafter, the parity of \( N^{**} \) octet will be specified as odd.
rents$^3$ of $SU(3)$. The explicit current form for $J_\mu$ is then written as,

$$J_{\mu r}^p = (a\tilde{N}_q^{\nu r} \gamma_\mu N_r^p + b\tilde{N}_r^{\nu r} \gamma_\mu N_q^p) + c(\tilde{N}_q^{\nu r} N_r^p - \tilde{N}_r^{\nu r} N_q^p),$$

(7)

where $a=2b=\sqrt{2}(G_A/G_V)$ and $c=1/2$ in the case of $F/D=1/3$ for the axial vector currents. We take the pure $F$ type in the vector currents between the baryons. For boson currents

$$j_{\mu r}^p = (a\sigma_q^{\nu r} \gamma_\mu \pi_r^q + b\sigma_{-r}^{\nu r} \gamma_\mu \pi_q^p) + c(\pi_q^{\nu r} \gamma_\mu \pi_r^q - \pi_r^{\nu r} \gamma_\mu \pi_q^p),$$

where the scalar octet $\sigma_q^p$ is introduced for axial boson currents. We leave the $F$ and $D$ mixture for the $D(J^P=3/2^-)-N$ strong currents by the following parametrization:

$$J_{\mu i} = \bar{D}_{i\mu} \gamma_\mu [\alpha D_i + (1-\alpha) F_i] N,$$

(7')

where $i$ is the $SU(3)$ index of Goldberger-Treiman and $D$'s and $F$'s are defined in Eq. (4). The parameters in Eq. (6b) are expressed in terms of $\alpha$ as follows:

$$a' = \sqrt{2}, \quad b' = \sqrt{2}(2\alpha-1).$$

For the tentative derivation of the relative coupling constant in Eqs. (6a) and (6b), we take Miyamoto's model$^{3,4}$ of $SU(3) \times SU(3)$ and assume (6, 3) and (3, 6) for the $B(J=3/2)$ state. Then the ratio of the two coupling constant are determined as follows:

$$g_p g_D = \sqrt{3}.$$

In this case the $F/D$ ratio of the $D-N$ currents is also fixed to be $1/3$.

Now let us obtain the effective interaction Hamiltonian for the $\nu+N\rightarrow l+B$ scattering in Fig. 1 by the conventional method. The result is

$$H_I^{\text{eff}} = T(H_I^{(3)} H_I^{(\nu')} H_I^{(\nu')})$$

(8a)

$$= g g_{\nu'} G_{\nu} T[\tilde{N}_q^{\nu r} \gamma_5 \{aN_r^p \gamma_\mu \pi_r^q + bN_r^p \gamma_\mu \pi_q^p\} \{a\tilde{N}_q^{\nu r} \gamma_\mu \pi_r^q + b\tilde{N}_r^{\nu r} \gamma_\mu \pi_q^p\} + c(\tilde{N}_q^{\nu r} N_r^p - \tilde{N}_r^{\nu r} N_q^p)]$$

(8b)

for $B=P$. In the case of $B=D$, we must replace the last factor by the following one:

$$\tilde{B}_{\mu r}^p \gamma_5 \{a'N_r^p \gamma_\mu \pi_r^q + b'N_r^p \gamma_\mu \pi_q^p\}.$$

In the reduction of the hadron currents of Eq. (8) to Eq. (3), we use the following propagator for the pion octet, etc.,

$$\langle T[\pi_q^{\nu r} \gamma_\mu \pi_r^p]\rangle = \frac{1}{k^2 + \mu^2 - i\epsilon} \left[ \delta_q^{\nu r} \delta_q^{\nu r} - \frac{1}{3} \delta_q^{\nu r} \delta_q^{\nu r} \right],$$

(9)
On the Excitation of the $J^p=3/2^+$ Decuplet and $J^p=3/2^-$ Octet

where $k$ is the momentum of the pion and $\mu$ is its mass. Using the propagators of Eq. (9), one can straightforwardly reduce Eq. (8) to the following:

$$
H_{\Gamma_{tt}} = -\frac{2}{3} \frac{g_{\nu\gamma} G_{\nu}}{\sqrt{2}} \int d^4r \int d^4s \int d^4t <B| \frac{i}{i\gamma \cdot r - m} \frac{1}{i\gamma \cdot s - m} (a - 2b) \gamma_5 + c (a + b) \\
\times \left( \frac{1}{i\gamma \cdot r - m} \frac{1}{i\gamma \cdot s - m} \right) B^\mu_\nu N_\mu^\prime \gamma_\nu N > \delta^{\alpha\beta}(t + r - p)\delta^{\gamma\delta}(r + q - s)\delta^{\eta\zeta}(t + s - p') \\
\times T\langle l | j_{\mu\nu}^{\mu\nu}(q) | \nu \rangle,
$$

(10)

where $q = p' - p$. It should be noticed that the axial-vector part of the $P-N$ current cannot contribute to Eq. (10) in the case $F/D = 1/3$, i.e. $a = 2b = \sqrt{2} (G_A/G_V)$, since the factor $(a - 2b)$ in Eq. (10) vanishes. Equations (10) and (5) determine the form factors $f'$ as the contribution from Fig. 1 for the $B = P$ state. The contribution from Fig. 2 is estimated similarly. In the calculation of the Feynman diagrams of Figs. 1 and 2, we use a sharp cutoff at $q^2 = -M^2$ in the case of divergence of the integrals. The results are tabulated in Table I. In Table I, we do not include the Goldberger-Treiman contribution.

Table I. The coupling constant in Eq. (3) on the $SU(6)$ model and the cloud model (Cloud) without including the G-T contribution.

(a) $P(J^p=3/2^+)-N(J^p=1/2^+)$ currents

<table>
<thead>
<tr>
<th></th>
<th>$f_\alpha$</th>
<th>$f_\gamma$</th>
<th>$f_\nu$</th>
<th>$f_\tau$</th>
<th>$f_\rho$</th>
<th>$f_\sigma'$</th>
<th>$f_\sigma$</th>
<th>$f_\tau'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(6)$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.685</td>
<td>+1.0</td>
<td>+0.305</td>
</tr>
<tr>
<td>Cloud</td>
<td>-0.071</td>
<td>-1.49</td>
<td>-2.65</td>
<td>+1.38</td>
<td>+0.170</td>
<td>+1.29</td>
<td>-1.29</td>
<td>-1.01</td>
</tr>
</tbody>
</table>

(b) $D(J^p=3/2^+)-N(J^p=1/2^+)$ currents

<table>
<thead>
<tr>
<th></th>
<th>$f_\alpha$</th>
<th>$f_\gamma$</th>
<th>$f_\nu$</th>
<th>$f_\tau$</th>
<th>$f_\rho$</th>
<th>$f_\sigma'$</th>
<th>$f_\sigma$</th>
<th>$f_\tau'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(6)$</td>
<td>-1</td>
<td>+0.732</td>
<td>0</td>
<td>0.055</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cloud</td>
<td>+0.100</td>
<td>+1.58</td>
<td>-2.94</td>
<td>-1.10</td>
<td>+0.58</td>
<td>-2.10</td>
<td>+8.66</td>
<td>+1.03</td>
</tr>
</tbody>
</table>

We take $g_{\mu\nu} = g_{\nu\mu}$ in the Eqs. (6a) and (6b).

The Goldberger-Treiman term comes from Fig. 3 by the partial conservation of the vector and axial vector currents, Eq. (5); there appear the following relations between $f_\alpha, f_\sigma, f_\gamma$ and between $f_\rho, f_\sigma', f_\tau$.

$$
\langle B | \partial_\rho j^{(\nu)}_\rho \gamma_\nu N > = \vec{B}_\nu(p') q_\nu \left[ f_\rho + f_\sigma' + \frac{M + M'}{M} f_\tau \right] \gamma_\nu N(p) \\
= \vec{B}_\nu(p') q_\nu \left[ f_\rho + f_\sigma' + \frac{M + M'}{M} f_\tau \right] \gamma_\nu N(p) = 0
$$

for the $B = P$ state,

$$
\langle B | \partial_\rho j^{(\nu)}_\rho \gamma_\nu N > = \vec{B}_\nu(p') q_\nu \left[ f_\rho + f_\sigma + \frac{4M}{M} f_\tau \right] N(p) = 0
$$
for the $B=D$ state,
\[
\langle B | \partial_\mu f^{(\mu)}_\rho | N \rangle = \bar{B}_\rho(p') q_r \left( f_\tau + f_\sigma + \frac{\Delta M}{M} f_\nu \right) N(p) = 0
\] (11)
for the $B=P$ state and
\[
\langle B | \partial_\mu f^{(\mu)}_\rho | N \rangle = \bar{B}_\rho(p') q_r \left( f_\tau + f_\sigma + \frac{M+M'}{M} f_\nu \right) N(p) = 0
\] for the $B=D$ state, where $\Delta M = M' - M$.

The contribution from Fig. 3 can be determined as follows:

\[
fa = \sqrt{2} \ g_{\ast NN} f_\pi,
\] (12a)
for the $B=P$ state and
\[
f\nu = \sqrt{2} \ g_{\ast NN} f_\pi,
\] (12b)
for the $B=D$ state.

This is the so-called Goldberger-Treiman relation. By the use of the Goldberger Treiman relation for the $N-N$ current, we have the result

\[
f_\pi = \sqrt{2} \ \frac{M}{g_{\ast NN}} G_A.
\]

Then Eqs. (12a) and (12b) are obtained as

\[
f_\sigma = \frac{g_\nu'}{g_{\ast NN}} G_A,
\] (13a)
for the $B=P$ state and
\[
f_\nu = \frac{g_\nu'}{g_{\ast NN}} G_A.
\] (13b)

Then, if we use the perturbation theoretical value for $f_\pi$ and $f_\nu$, and also use the value of Eq. (13) for $f_\sigma$ and $f_\nu$, we can uniquely determine $f_\nu$ and $f_\pi$. These values for $f_\nu$ and $f_\pi$ are tabulated in Table II.

Table II. Goldberger-Treiman contribution.

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<thead>
<tr>
<th></th>
<th>$f_\pi$</th>
<th>$f_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P-N$ current</td>
<td>$+2.68$</td>
<td>$+0.200$</td>
</tr>
<tr>
<td>$D-N$ current</td>
<td>$-18.0$</td>
<td>$+2.2$</td>
</tr>
</tbody>
</table>

§ 4. Discussion

In the preceding sections, we made a comparison between the quark model of $SU(6)$ and the perturbation theoretical cloud model in Tables I and
II. At present there is no experimental evidence for the existence of the quarks, but the internal $SU(6)$ symmetry of the strong interaction seems to play an important role in the interpretation of strong interaction phenomena, though at the phenomenological stage. It is an important analysis, to compare the result of the $SU(6)$ theory and the meson cloud model in the one pion production in neutrino-baryon collisions and the $\beta$ decay of $\Omega^-$. This will be discussed elsewhere. It seems that a quark has a meson cloud around it, and therefore, it is anticipated that the electric form factor, our $f_\nu$ ($f_A$, $f_T$ and $f_{T'}$), derived from $SU(6)$ should be multiplied by the meson cloud factor $(1+2M/\mu)$.

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References

5) M. Gell-Mann, Physics 1 (1964), 63.