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Estimating the Variance of Airborne Snow Water Equivalent Estimates Using Computer Simulation Techniques

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Estimates of snow water equivalent derived from measurements of the attenuation of terrestrial gamma radiation vary as a function of several known sources of error. In this study, a computer simulation of the physically-based model used to obtain estimates of snow water equivalent is developed and the variance in estimates of snow water equivalent is investigated using simulation techniques. Several of the principle sources of error in the estimating model are simulated and studied for various conditions under which snow water equivalent estimates are obtained. The simulation results indicate that the error in airborne snow water equivalent estimates made in both forested and open environments with up to 60 cm of snow water equivalent is less than 12 per cent of the estimate for typical flight lines. The results of this study can aid National Weather Service hydrologists in improving river and flood forecasts, water supply forecasts, and spring flood outlooks for large areas of the United States and Canada.

Introduction

The Office of Hydrology of the National Weather Service in the National Oceanic and Atmospheric Administration has developed and currently maintains an operational Airborne Gamma Radiation Snow Survey Program based in Minneapolis, Minnesota. The primary purpose of the program is to make airborne estimates of snow water equivalent and soil moisture over a network of 1,400 flight lines covering portions of 25 states and 7 Canadian provinces. These estimates are used by hydrologists in Weather Service Forecast Offices, River Forecast Centers, and other U.S. and Canadian Federal, state, and provincial agencies for river and flood forecasts, water supply forecasts, and spring flood outlooks.

The technique used to obtain airborne snow water equivalent estimates is designed to control as many sources of error in the estimate as possible; however, many sources of error remain. In this research, we investigate the magnitude of the errors in the estimates of snow water equivalent by developing a computer simulation of the physically-based model used to obtain the snow water equivalent estimates. We simulate many conditions under which estimates of snow water equivalent are obtained and estimate the variance in the estimate of the snow water equivalent for each condition from replications of the simulation. The major sources of errors investigated are: 1) the uncertainties associated with radiation count rates, 2) the decreased signal caused by increased snow water equivalent, 3) the uncertainty associated with the estimates of ground-based soil moisture measurements, 4) the variation due to differences in the lengths of flight lines, and 5) biases caused by variance in the snow water equivalent and by forest biomass along the flight line.

Airborne Measurement Technique

The airborne measurement technique uses the attenuation of natural terrestrial gamma radiation by the mass of the snow cover to make airborne estimates of snow water equivalent over a flight line which is typically 16 km long and 300 m wide covering an area of approximately 5 km^2. Consequently, each estimate is a mean areal measure integrated over the 5 km^2 area of the flight line. The gamma radiation flux near the ground originates primarily from the natural ^{40}K, ^{238}U, and ^{208}Tl radioisotopes in the soil. In a typical soil 96 per cent of the gamma radiation is emitted from the upper 20 cm of soil (Zotimov 1968). After a measure of the background (no snow cover) radiation and soil moisture is made over a specific flight line, a second measurement of these parameters is made over the flight line when snow is present. The attenuation of the radiation signal due to the snowpack is used to estimate the mean areal amount of water in the snow cover over the flight line (Fritzsche 1982).

The measurement technique is designed to account for as many sources of error as possible. Any variation of flight altitude between the times that the background and over-snow data are collected is accounted for by continuously recording temperature, pressure, and radar altitude. The air mass over the flight line is calculated from these three measures and used to normalize the background and over-snow gamma counts to an air mass of 17 g cm^{-2}. Water vapor in the column of air is also accounted for by monitoring the partial vapor pressure sensed by the pressure transducer. Additionally, soil moisture is estimated for each flight line when the background and the over-snow gamma data are collected. These estimates are used to adjust the snow water equivalent estimate for differences in the soil moisture between the two times when the data are collected.

Various sources of error in airborne snow water equivalent estimates remain, however, and many have been identified. Extraneous radiation, which may cause inaccuracies, is contributed to the spectra by the Compton tails associated with the photopeaks of higher energy, the cosmic radiation component, the aircraft and fuel, the pilots, and the detection system itself (Fritzsche 1982). Additionally, errors in recording air temperature, air pressure, and radar altitude and in estimating soil moisture (four factors used to correct for the intervening air mass and soil moisture) may contribute to errors in the snow water equivalent estimate. Finally, the natural variation in count rates of the potassium, thorium, and uranium in the soil add to the uncertainty in the snow water equivalent estimate.

Other sources of error in the estimate of snow water equivalent have recently been identified and investigated. Carroll and Carroll (1989A) studied the effect of variance in the snow water equivalent on the estimate. Estimates of snow water equivalent derived from measurements of the attenuation of terrestrial gamma radiation are systematically biased downward if variability in the snow water equivalent exists in the snowpack along the flight line. The degree of underestimation is a function of both the shape and variance of the distribution of the snow water equivalent. Additionally, estimates of snow water equivalent are systematically biased downward if substantial amounts of forest biomass exist in the region where the radiation measurements are obtained. Carroll and Carroll (1989B) have examined the effects of forest biomass on the estimates and found that the degree of underestimation is a function of the amount of biomass and the amount of potassium in the biomass.

Vogel *et al.* (1985) simulated some of the principle sources of error for airborne measurements made over forested watersheds with as much as 600 mm of snow water equivalent and showed that the error is less than 12 per cent. The simulated results agree closely with the empirical errors derived from ground snow survey data collected in a forest environment with 480 mm of snow water equivalent (Carroll and Vose 1984). In addition, the simulation technique was used to assess the effect of some of the principle sources of error on airborne measurements made over agricultural environments with 20 mm to 150 mm of snow water equivalent. The results indicate that the error is, in part, a function of snow water equivalent and ranges from 4 to 10 per cent. Again, the errors derived from the simulation agree closely with the errors derived using airborne and ground-based snow survey data collected over an agricultural environment (Carroll *et al.* 1983).

Past studies of the errors in the estimates of snow water equivalent have focused primarily on only two sources of error: those associated with the radiation count rates and those associated with the measurement of soil moisture content. Each of these sources was investigated in isolation without regard to how it might interact with other sources of error. In this study we investigate the effects of several sources of error and consider the possible interaction of one source of error with another.

Table 1 – Values of Random Variables Used in Simulation

Background Count Rates (counts per minute)				
Potassium	Thorium	Uranium	Gross Counts (Upper Detector)	Gross Counts (Lower Detector)
4259	1246	1286	4681	39674

Background and Over-snow Values	Altitude (m)	Pressure (Bars)	Temperature (C)
Background	156.04	0.956	13.20
Over-snow	155.00	0.965	–4.10

The Simulation Procedure

The values of the random variables involved in the simulation study are given in Table 1. These values were selected to be representative of those found in practice. The average values of the background count rates of potassium, thorium, uranium, and gross counts and both background and over snow average values of temperature, air pressure, radar altitude, soil moisture, and cosmic count rates are mean values computed from selected flight lines typical of those where estimates of snow water equivalent are obtained. The average values of the over-snow count rates of potassium, thorium, uranium, and gross counts were estimated from a recursive set of simultaneous equations for different amounts of snow water equivalent. The equations were estimated from over-snow data obtained from flight lines which have similar and typical background count rates.

In each iteration of each simulation, a randomly generated error component was added to each of the random variables. The distributions from which the errors were generated were selected to be representative of the true distributions as guided by the theory and empirical evidence of the process. For each condition simulated, 1,000 replications were generated. If any of the replications produced a negative value for any of the stripped, normalized count rates, the results from that simulation were not reported. The equations of the physically-based model simulated in this study are given by Fritzsche (1982).

The simulation study was performed for many different conditions under which estimates of snow water equivalent are obtained. In the initial simulations, the conditions which were represented were all combinations of: 1) flight times (5 seconds, 30 seconds, 1 minute, 2 minutes, and 5 minutes which represent flight line lengths of 300 m, 1.8 km, 3.6 km, 7.2 km, and 18 km respectively), 2) two values of the coefficient of variation of soil moisture (0, and 0.25) which is varied to account for the uncertainty associated with the measurement of soil moisture, and 3) various amounts of snow water equivalent. Estimates of the mean snow water equivalent and the associated variance for each estimate were computed from a weighted average of the stripped, normalized, gross counts, thorium counts, and potassium counts. This entire set of simulations was repeated two more times with the exception that the estimates of the snow water equivalent and the associated variances in the estimates were computed first from the stripped, normalized, gross counts only and second from the stripped, normalized, potassium counts only.

In the next set of simulations, the effects of forest biomass over the flight line were modeled. The estimating model was modified to account for the amount of biomass (in these simulations, we assumed an average of 4 grams of biomass per square centimeter of the flight line (Emerick 1982)) and the ratio of the potassium content in one gram of biomass to the potassium content in 20 cm^3 of surface soil (we assumed the average of this ratio to be 0.002 (Carroll and Carroll 1989B)). The distributions of these variables is assumed to be approximately normal with a coefficient of variation of 0.125. The larger the magnitude of these variables, the greater the bias in the estimate of the snow water equivalent. The larger the coefficient of variation, the larger the variance in the estimate. This set of simulations was completed for all combinations of three flight times (1, 2, and 5 minutes) and for various amounts of snow water equivalent being simulated. The coefficient of variation of soil moisture was set to zero. The estimates of snow water equivalent were obtained from the stripped, normalized, potassium counts because biomass over the flight line affects only the potassium count rate (Carroll and Carroll 1989B). The effects of biomass on estimates of snow water equivalent obtained from a weighted average of the gross counts, thorium counts, and potassium counts are not as great as those observed in these simulations.

The final set of simulations was designed to model the effect of variance in the snow water equivalent over the flight line. The distribution of the snow water equivalent along the flight line was assumed to be normal with a standard deviation of 20 cm. It was also assumed that the estimate of the variance of the snow water equivalent was obtained from 50 ground samples. If the standard deviation of the distribution of the snow water equivalent increases, the downward bias in the estimate of the snow water equivalent is increased and the variance in the corrected estimate is increased. This set of simulations was completed for all combinations of three flight times (1, 2, and 5 minutes) and for various simulated levels of snow water equivalent. The coefficient of variation of soil moisture was set to zero. The estimates of the snow water equivalent were obtained from a weighed average of the stripped, normalized, gross counts, thorium counts, and potassium counts.

All of the simulations were run using the Statistical Analysis System (SAS) software on an IBM 4381 computer.

Results and Discussion

The salient results of the computer simulation procedure are depicted in Figs. 1 through 7. Fig. 1 provides the results of the simulations designed to estimate the means and standard deviations of airborne snow water equivalent estimates using: 1) a weighted average of the estimates obtained from stripped, normalized, count rates in the gross count, potassium, and thorium photopeak windows, 2) data collected for 5 seconds, 30 seconds, 1 minute, 2 minutes, and 5 minutes intervals (*i.e.,* flight line lengths from 300 m to 18 km), and 3) an assumed calibration soil moisture coefficient of variation of zero. In Fig. 1, the standard deviation of the estimate of the snow water equivalent is plotted against the mean estimate of the snow water equivalent for the five flight times. The simulated standard deviations given in Fig. 1 vary as a function of snow water equivalent and the primary sources of error which include: 1) flight line length, 2) air mass measurement error, and 3) radiation counting statistics errors. The standard deviations given in Fig. 1 represent an error (*i.e.,* coefficient of variation) of airborne snow water equivalent of less than 10 per cent for all snow depths for all simulations where the flight line length is longer than 3.6 km (*i.e.,* 1 minute). In an effort to isolate the major airborne errors in the measurement technique, errors in the ground-based calibration soil moisture measurements were excluded in Fig. 1.

A flight time of 5 seconds is included in Figs. 1, 2, and 3 to indicate the errors associated with the practical limits of airborne radiation data collection. The detection system collects radiation and air mass data in 5-second intervals (which represents a flight line length of 300 m). Consequently, it is possible to calculate an airborne soil moisture or snow water equivalent value for a flight line section 300 m long by 300 m wide. The 5-second data have substantial radiation counting statistics error because of the extremely short data collection interval. In an effort to reduce the 5-second counting statistic errors represented in Figs. 1, 2, and 3, it is possible to fly a flight line repeatedly and accumlate the radiation data collected over multiple flights for the same 5-second data collection interval. In this way, it is possible to reduce the 5-second counting error and still retain the 300 m by 300 m resolution. For example, if 5-second data were accumulated for 12 data collection passes over the same flight line on the same day (to minimize radon gas contamination), each 5-second data interval (composed of data from all 12 passes) would have an error equivalent to one pass over a flight line 3.6 km long which requires one minute to fly.

Fig. 2 provides the standard deviation of airborne snow water equivalent measurements made under the same conditions as those simulated in Fig. 1 with the exception that the coefficient of variation of the ground-based soil moisture data is assumed to be 25 per cent. This represents the natural variability of soil moisture along the length of a typical flight line.

When obtaining estimates of snow water equivalent, various advantages and disadvantages are associated with the estimates obtained from each of the three of

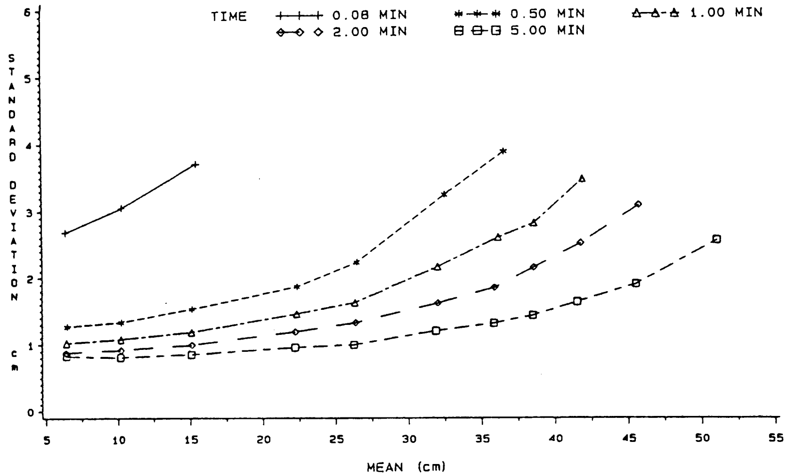

Fig. 1. Plot of standard deviation versus mean of weighted snow water equivalent estimates, soil moisture coefficient of variation equal 0.

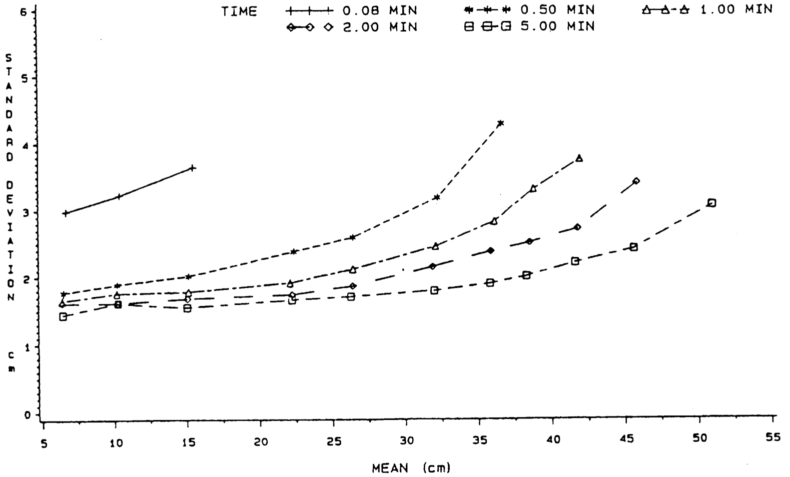

Fig. 2. Plot of standard deviation versus mean of weighted snow water equivalent estimates, soil moisture coefficient of variation equal 0.25.

the photopeak windows used in the simulations given in Figs. 1 and 2. Estimates made from the gross count window have the advantage associated with high count rates but the disadvantage of being confounded due to extraneous radon that is difficult to isolate and remove from the raw radiation data. Estimates derived from the potassium window have the advantage of a comparatively high count rates (higher than thorium but less than the gross count window) but the disadvantage of confounding due to Compton scattered counts generated from higher energy ra-

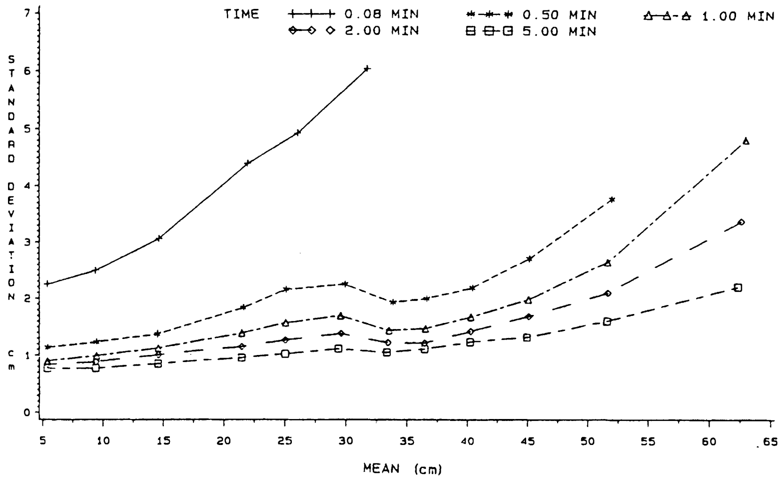

Fig. 3. Plot of standard deviation versus mean of snow water equivalent estimates calculated from gross count window, soil moisture coefficient of variation equal 0.

don. Estimates generated from the thorium window have the advantage of being unaffected by radon but the disadvantage of low count rates. In snow packs of less than 20 cm of water, a statistical weighting of the three photopeaks is used to provide a single-best, weighted average airborne snow water equivalent estimate.

Fig. 3 gives data that were simulated under the same conditions as the data in Fig. 1 except that radiation data from only the gross count window were used to estimate both the snow water equivalent and the variance of the estimate of snow water equivalent. The gross count window covers a larger portion of the radiation spectrum and consequently provides an order of magnitude more data than the potassium window – which typically provides twice as much data as the thorium window. Consequently, the gross count window can be used in deep snow packs to estimate snow water equivalent where the potassium and thorium signals are substantially attenuated and consequently introduce large errors in the estimate. The reduced errors of airborne snow water equivalent estimates derived from only the gross count window are given in Fig. 3.

Forest biomass simultaneously acts as both a source of radiation (from the uptake of potassium) and an attenuator of the terrestrial gamma radiation signal. The effects of forest biomass tend to cause a systematic underestimate in the airborne snow water equivalent estimate if not accounted for. Fig. 4 gives the results of the simulations made under the assumptions regarding the nature of the forest biomass (*i.e.*, the density and potassium content of the biomass) described above. In Fig. 4, the mean snow water equivalent estimates, both corrected and uncorrected for biomass, are plotted against the raw potassium counts for an 18 km flight line. The raw potassium counts indicate the amount of snow water equivalent being simulated; an inverse relationship exists between the number of potassium counts and

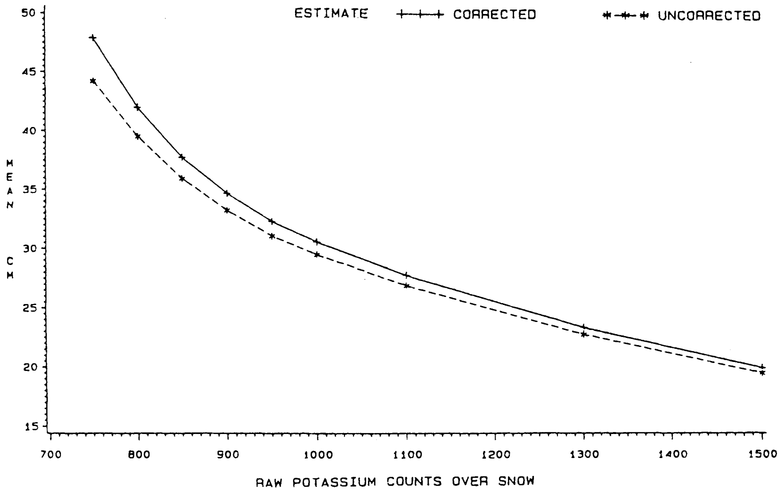

Fig. 4. Plot of mean snow water equivalent estimates, both uncorrected and corrected for biomass, versus raw potassium counts over snow for flight time of 5 minutes.

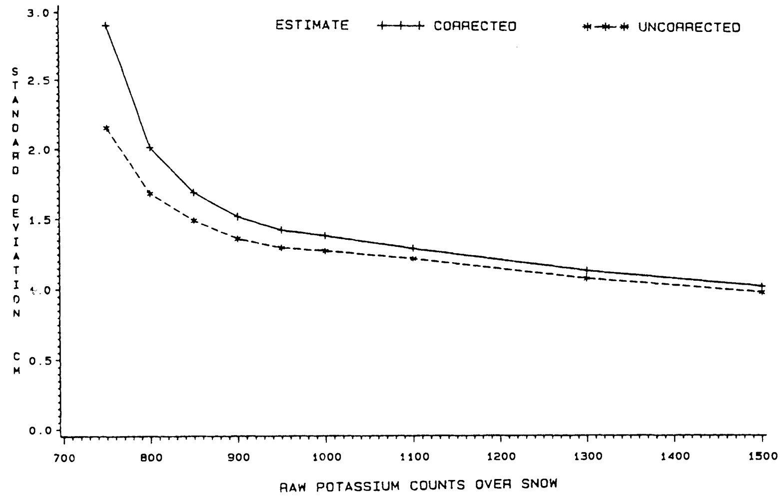

Fig. 5. Plot of standard deviation of snow water equivalent estimates, both uncorrected and corrected for biomass, versus raw potassium counts over snow for flight time of 5 minutes.

the amount of snow water equivalent. Under the assumed conditions, the mean snow water equivalent is not greatly biased by forest biomass except in deep snow-cover conditions (*i.e.*, 50 to 60 cm of water).

The data in Fig. 5 were generated under the same conditions as those in Fig. 4. In Fig. 5, the standard deviations for estimates of snow water equivalent that are both corrected and uncorrected for forest biomass are plotted against the raw potassium

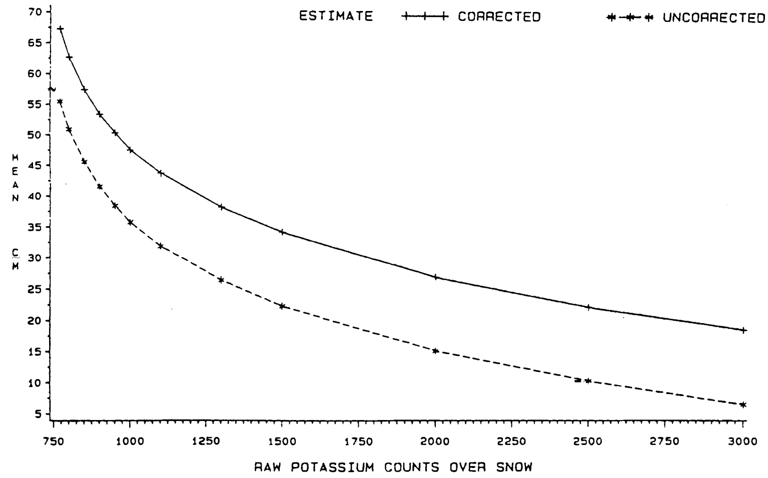

Fig. 6. Plot of mean snow water equivalent estimates, both uncorrected and corrected for variance in snow pack versus raw potassium counts over snow for flight time of 5 minutes.

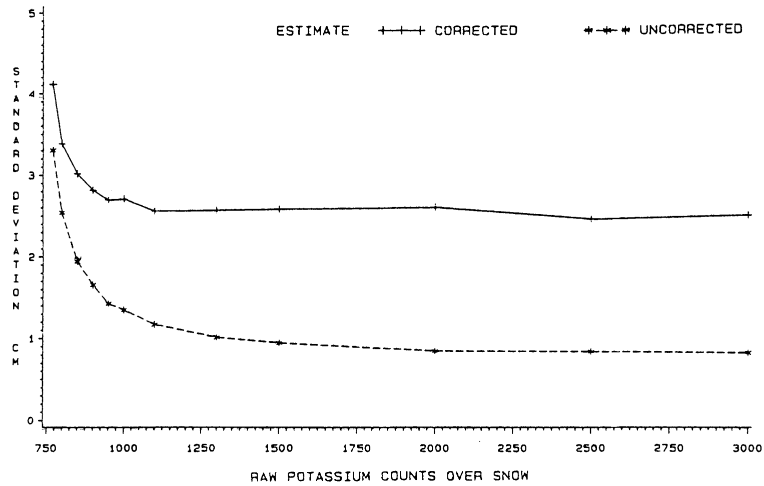

Fig. 7. Plot of standard deviation of snow water equivalent estimates, both uncorrected and corrected for variance in snow pack, versus raw potassium counts over snow for flight time of 5 minutes.

counts. The standard deviation in the corrected estimate is larger because of the uncertainty associated with the estimates of the amount of forest biomass and the amount of potassium in the biomass. As with the mean estimate of snow water equivalent, the standard deviation of the estimate is not dramatically affected by forest biomass except in deep snow conditions.

Variability in the snow water equivalent along the length of a flight line introduces a systematic downward bias in the estimate of snow water equivalent. The downward bias is a function of the shape and variance of the distribution of the snow water equivalent and can be corrected if these factors are known. Fig. 6 shows plots of the mean snow water equivalent estimate both corrected and uncorrected for the variance in the snow water equivalent plotted against the raw potassium counts for an 18 km flight line. In the simulations, the standard deviation of the snow water equivalent was assumed to be 20 cm. Because the correction for the bias is not a function of the amount of snow water equivalent, the difference between the corrected and uncorrected estimate is nearly constant for all count rates. In actual snow cover conditions, the standard deviation of the snow water equivalent is likely to be lower than 20 cm (the assumed value in the simulations) for shallow snow packs, and hence, the corrected and uncorrected estimate will tend to be closer under these conditions.

The data in Fig. 7 were simulated under the same conditions as those in Fig. 6. In Fig. 7, the standard deviations for estimates of snow water equivalent that are both corrected and uncorrected for variance in the snow pack are plotted against the raw potassium counts. The corrected estimates have a higher standard deviation because they include the uncertainty associated with the estimate of the variance in the snow water equivalent.

Summary

The technique used to make airborne snow water equivalent estimates using natural, terrestrial gamma radiation can be used for snow packs with as much as 60 cm of water equivalent. The error increases with water equivalent as a result of the reduced radiation signal due to attenuation of the terrestrial signal by the water equivalent. Nonetheless, the coefficient of variation of the airborne snow water equivalent estimate remains under 10 per cent for flight lines 3.6 km or longer.

A systematic bias, resulting in an underestimate of the true water equivalent, is introduced by forest biomass covering the flight line and by the standard deviation of the water equivalent along the flight line. It is possible to correct for the systematic underestimate given knowledge, or assumptions, about: 1) the quantity of forest biomass, 2) the ratio of the amount of potassium in the biomass to that in the soil, and 3) the standard deviation of the water equivalent along the flight line. Typically, the systematic biases generate a substantial underestimate of the airborne snow water equivalent estimate only in deep snow packs. The airborne technique can make reliable, real-time snow water equivalent estimates in both open and forested regions with water equivalents up to 60 cm with an error of less than 12 per cent. To make reliable measurements in regions with more than 60 cm of water equivalent would require: 1) more detection crystals, 2) a lower, slower airborne platform, and 3) the derivation of new system calibration for attenuations in areas with an excess of 60 cm of water.

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