On the properties of stochastic intermittency in rainfall processes

A. Molini, P. La Barbera and L.G. Lanza

Department of Environmental Engineering, University of Genova, Via Montallegro 1, 16145, Genova, Italy (E-mail: *molini@diam.unige.it*)

Abstract In this work we propose a mixed approach to deal with the modelling of rainfall events, based on the analysis of geometrical and statistical properties of rain intermittency in time, combined with the predictability power derived from the analysis of no-rain periods distribution and from the binary decomposition of the rain signal. Some recent hypotheses on the nature of rain intermittency are reviewed too. In particular, the internal intermittent structure of a high resolution pluviometric time series covering one decade and recorded at the tipping bucket station of the University of Genova is analysed, by separating the internal intermittency of rainfall events from the inter-arrival process through a simple geometrical filtering procedure. In this way it is possible to associate no-rain intervals with a probability distribution both in virtue of their position within the event and their percentage. From this analysis, an invariant probability distribution for the no-rain periods within the events is obtained at different aggregation levels and its satisfactory agreement with a typical extreme value distribution is shown.

Keywords Binary signal; intermittency; laminar phases; rainfall; time series; Weibull distribution

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Introduction

The rainfall process presents a deeply irregular evolution in time and space. When observing pluviometric time series, such behaviour emerges in the form of long no-rain (or low-rain) periods alternated with very intense and unstable intervals, resembling intermittency. The term intermittency derives from the theory of non-linear dynamical systems where it precisely indicates a particular group of non-linear phenomena, whose signal alternates periodic (regular, laminar) and chaotic (irregular, turbulent) behaviour in an erratic fashion (Kantz and Shreiber, 1997). An outline definition of intermittency can also be given in terms of probability of extreme events: for an intermittent signal the occurrence of high amplitude events is more probable than for a Gaussian process and the spectral density function moves from the typical structure of a red noise (where the low frequencies are most represented) to that of a blue-noise (characterised by an increasing power law for the largest frequencies) while the influence of laminar periods decreases.

This type of "fat-tail" distributions and red-noise spectra also characterise a variety of natural phenomena presenting irregular and discontinuous patterns in space and time. Moreover, signals with alternating persistent laminar periods (the inter-arrival process in the case of rainfall) and highly bursting phases (namely the rainfall events) can be generated by stochastic processes with infinite degrees of freedom or even by chaotic systems governed by few independent modes. It is not always easy to distinguish between these two dynamical behaviours. From a theoretical point of view the hypothesis that space-time rainfall belongs to the class of chaotic processes, or that it simply displays a few characteristic features within its apparently irregular structure, is fascinating and it has fed a wide literature in the last two decades ranging from the first works on the fractal nature of rain fields (Lovejoy and Mandelbrot, 1985; Lovejoy and Schertzer, 1985) to the more recent papers on the multifractal structure of rainfall (Deidda, 2000) and the scaling properties of space-time rainfall (Kumar and Foufoula-Georgiou, 1993a; Kumar and Foufoula-Georgiou,1993b; Kumar and

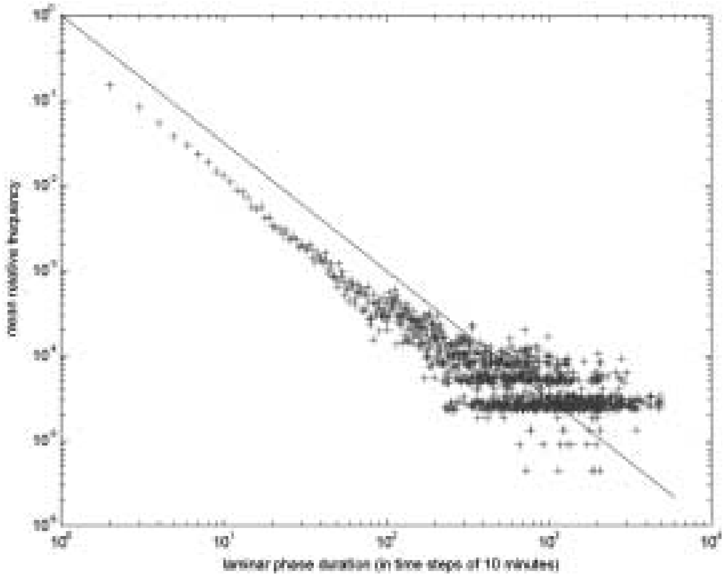

Figure 1 Average frequency distribution of laminar phases for a 10 years long time series, recorded at the Department of Environmental Engineering of the University of Genova (aggregation=10 minutes), compared with the power law distribution with exponent −1.5, characteristic of on-off intermittent systems

Foufoula-Georgiou, 1994). At the same time much effort has been devoted in the literature towards developing reliable procedures for testing the hypothesis of multifractality or scaling behaviour in rainfall data and understanding the limits of application of methods borrowed from the typical approach of the analysis of non-linear time series (see e.g. Kedem *et al.*,1987). Recently, the observed variability of rainfall in time was tested against on-off intermittency, a particular form of intermittence well known in the study of non-linear dynamical systems (Heagy *et al.*, 1994; Platt *et al.*,1994; Kantz and Shreiber, 1997; Platt *et al.*, 1992), by Toniolo *et al.* (2000). On-off intermittency, that occurs in some coupled systems where a forcing component acts on a forced one by means of a given operator called "skew product", presents a power law distribution of the laminar phases and a characteristic red-noise power spectrum that is typical of the rainfall process too (Figure 1).

However, the analysis performed by Toniolo *et al.*(2000) with structure functions and statistical tools like "shuffling" and the amplitude adjusted Fourier phase randomisation (AAFFT) seems to support the conclusion that no definitive evidence of multifractality in rainfall time series can be drawn from the observed data sets. In the present work, intermittency is regarded as a random variable and defined as the percentage of no rain periods within a rainfall event, looking at the process in a purely stochastic way.

This is called in hydrology "internal intermittency" to distinguish it from the inter-arrival process ("external intermittency"). In this way we focus on the stochastic properties of no-rain periods within the phases of most intense activity of the system, by neglecting the inter-arrival process. For this purpose a simple geometrical filter for the extraction of rainfall events from the observed time series is preliminarily applied and the stochastic properties of the events observed at various aggregation scales are analysed.

A filter for the extraction of rain events

A first step in identifying the internal intermittent structure of rainfall time series consists in separating the high activity phases of the phenomenon (the events) from the long no-rain periods (inter-arrivals) by means of some filtering procedure, robust enough to capture all events even though based on general criteria. The selected method is a double threshold

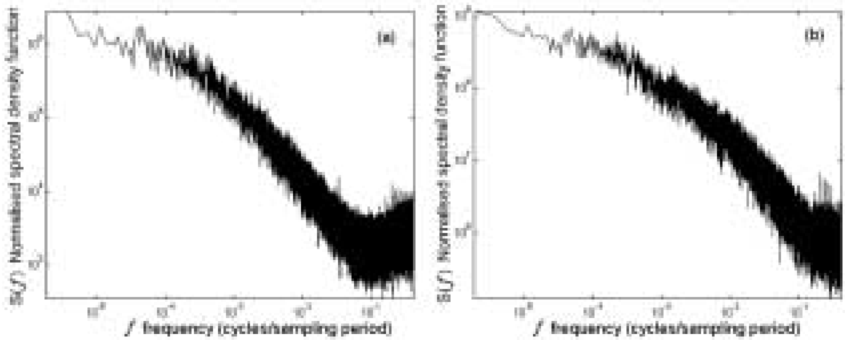

Figure 2 Mean normalised spectral density functions obtained from the ten years time series recorded at the University of Genova for the binary (a) and the original (b) signal

geometric filter that, in its simplicity, permits a fast implementation of event extraction. Calibration of the filtering procedure was undertaken by preliminary estimation of two parameters that are directly linked to the climatology of the site. The climatologic thresholds adopted are the maximum duration of the event (one day) and the minimum inter-arrival period (6 hours), and are assumed to characterize the climatology of the sub-Appennine region where the rain gauge station is located and the typical time scales of meso-scale meteorological events that are responsible for such a climate regime. The numerical figures for the filtering parameters have been selected on the basis of a number of tests, resulting in numerical values that ensure a yearly average number of events and a rainfall depth of the same amplitude as the real ones. The filter extracts rainfall events from the time series by means of an iterative comparison between the average duration of two consecutive rain pulses and the distance (in terms of no-rain steps) that separates the pulses, constrained to the two reference values of the minimum and maximum event duration. Extraction of the events has been performed starting from the binary (rain/no-rain) signal derived from a ten years pluviometric time series, recorded at high resolution (1 minute) in Genova (Italy). Some results of the spectral analysis performed both on the binary and on the original rain signal are reproduced in Figure 2. Obviously, the binary signal, whose realisations are constrained only to two possible results (rain-no rain) presents more pronounced characteristics of periodicity with respect to the amplitude signal, and this fact is highlighted by the compared spectral analysis of the two signals, where we can observe a stronger increase of the power spectrum at high frequencies for the binary component than for amplitudes (Figure 2).

Although the filter adopted certainly influences the statistics related to the intermittency process, we have assumed that when the filtering procedure is able to reproduce to coherent values for the typical event duration and interarrival periods of the examined region, it probably provides coherent values for related variables too. The assumption still needs an accurate validation based on sensitivity analysis, which is however beyond the objectives of the present paper.

Internal intermittency in time

Rain events have been extracted from a ten years long time series recorded at high resolution (1 minute) at the rain gauge station of the University of Genova (Department of Environmental Engineering) and an average number of 60 events/year has been obtained, with durations ranging between half an hour and 1 day. The observed frequency distribution of no-rain periods within events at different aggregation levels (from 5 minutes to

6 hours) has been compared with several theoretical distributions. In particular, the observed frequency distribution of intermittency seems to fit quite closely the Weibull theoretical distribution:

$$f(\quad) = \frac{}{}\;\frac{}{}^{\,-1} \exp - \frac{}{} \qquad (1)$$

whose cumulative density function can be written as:

$$F(\quad) = 1 - \exp - \frac{}{} \qquad \text{if} \quad > 0 \qquad (2)$$

so that the expected value and the variance of intermittency within rainfall events can be expressed as:

$$E[\quad] = \quad 1 + \frac{1}{} \qquad (3)$$

$$Var[\quad] = \;^{2} \quad 1 + \frac{2}{} - \quad 1 + \frac{1}{} \;^{2} \qquad (4)$$

where   is the percentage of no-rain periods,   is the Euler function,   is the shape parameter and   the scale factor. For   close to 1 the Weibull distribution tends to become exponential while for   close to 2 it approximates the Rayleigh distribution, so that the deviation of   from unity, at different aggregation levels, is a measure of the tendency of stochastic intermittency to become exponential rather than Rayleigh distributed. In Figures 3 and 4 the observed cumulative distribution of intermittency (at aggregation level of 5 and 10 minutes respectively) is compared with the corresponding Weibull theoretical cumulative distribution functions. Here the dotted bands represent the limits of acceptance of the discretization error due to the sampling of the rainfall signal, which increases towards the lowest intermittency values and the bands therefore diverge. Moreover at both 5 and 10 minutes aggregation scales, parameters   and   closely fluctuate around 2.1 and 0.3 and such figures are quite persistent at all the investigated aggregation scales in time (ranging from 5 minutes to 6 hours) so that the probability distribution parameters seem to be invariants of the rainfall process. This fact is highlighted in Figure 5 where the Weibull parameters   and   are represented as a function of different aggregation levels; however,

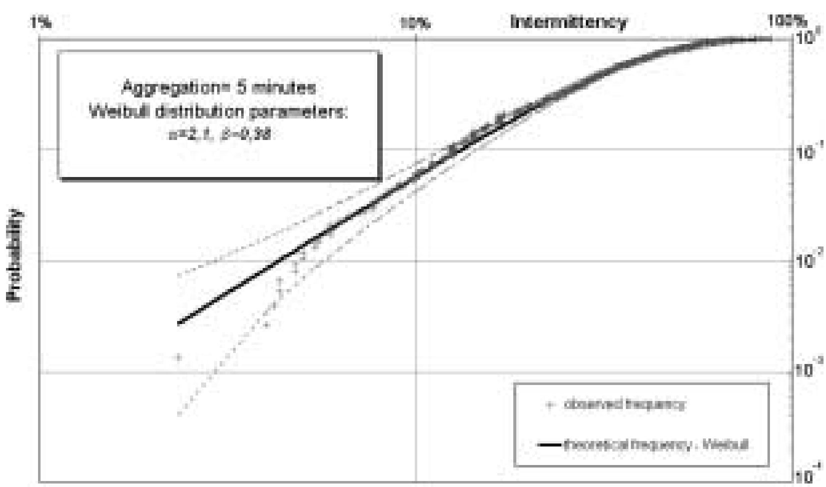

Figure 3 Comparison between the observed cumulated distribution of intermittency (aggregation level 5 minutes) and the corresponding Weibull theoretical distribution (after Molini, 2000a,b)

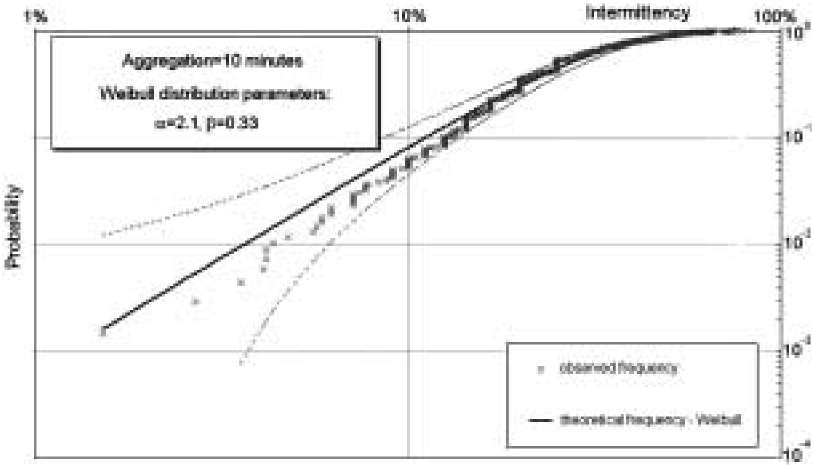

Figure 4 Comparison between the observed cumulated distribution of intermittency (aggregation level 10 minutes) and the corresponding Weibull theoretical distribution (after Molini, 2000a,b)

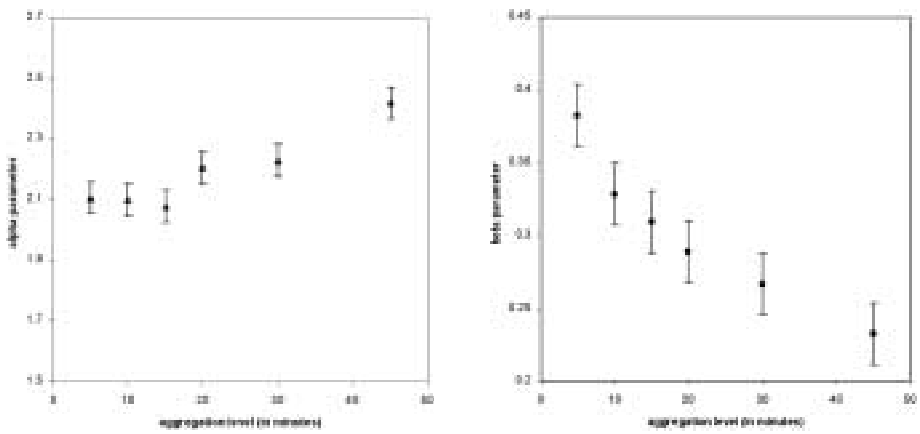

Figure 5 Variation of the Weibull parameters alpha (left) and beta (right) at different aggregation levels

while the parameter seems to obey a linear trend within its range of variation, the evolution of seems to follow a logarithmic law. The steadiness of Weibull parameters, far from being a signature of any scaling property of the intermittency process within rain events, can be very useful for the reproduction of the structure of rainfall signals and for hydrological applications, such as time disaggregation techniques in time. At the same time, since fluctuates around 2 we can conclude that, at all the investigated aggregation scales intermittency tends to follow a Rayleigh distribution. The Weibull model is a classic extreme value distribution, and its agreement with the observed frequencies of the intermittency problem is probably a consequence of the filtering procedure adopted when selecting no rain periods within the rainfall events by neglecting the long quiescent periods. As mentioned in the introduction, in fact, the events represent the very bursting periods of activity within the rainfall process, and therefore a sort of extreme process with respect to the laminar phases. The results obtained seem to be in accordance with the physical properties of the process, though some limitations of the adopted procedure must be kept in mind.

First of all, the filter adopted is strongly dependent on the climatology of the site, so that the same analysis in different sites would require a preliminary definition of the filter's two parameters, based on a suitable climatological analysis.

Second, the geometrical filter, due to its simple interactive structure, can be insufficiently efficient in the extraction of short duration events that are the most difficult to distinguish from each other.

Conclusions

The rainfall signal extracted from a ten years long time series sampled at high resolution (1 minute) has been analysed in order to obtain invariant characteristics linked to the binary structure of the rain signal (alternating rainy and no-rainy periods) and to the amplitude signal during the activity periods for the system. Following a preliminary spectral analysis a specific study of the no-rain periods within rainfall events extracted from the pluviometric time series of Genova was undertaken in order to define a probabilistic distribution for internal intermittency. Comparing the observed frequency of intermittency with some common probability distributions used for the description of hydrological processes, a good agreement of experimental data with the Weibull theoretical distribution has been obtained.

This is a classic extreme value distribution, and its agreement with the observed frequencies of intermittency is probably a consequence of the filtering procedure adopted when selecting no rain periods within the rainfall events and neglecting the long quiescent periods.

At the same time the steadiness of parameters   and   for the Weibull distribution has been observed at all the aggregation scales investigated (ranging from 5 minutes to 6 hours).

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