Pulsar motions in our Galaxy

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ABSTRACT

Pulsar motions in our Galaxy from their birth until age 2 Gyr are studied statistically via Monte Carlo simulation of $2 \times 10^5$ pulsars with the best available representation of the Galactic potential. We find that the distribution of heights above the Galactic plane for pulsars with characteristic ages less than about 8 Myr could be well fitted by a Gaussian function. For older pulsars, an extra exponential function is necessary to fit the distribution. The scaleheight of the Gaussian component increases linearly with time until about 40 Myr. The height distribution becomes stabilized after about 200 Myr. These results are not sensitive to initial height or radial distributions. Taking the relationship between the initial velocity and height distribution, we found from the latest pulsar catalogue that the height distribution of pulsars younger than 1 Myr directly implies a mean initial velocity of $280 \pm 96 \text{ km s}^{-1}$. Comparison of a simulated sample of pulsars with the currently available millisecond pulsars shows that their 1D initial velocity dispersion should be most probably $60 \pm 10 \text{ km s}^{-1}$.

Key words: pulsars: general.

1 INTRODUCTION

Pulsars are high-velocity objects in our Galaxy (Lyne & Lorimer 1994). They are born in supernova explosions near the Galactic plane where their progenitors live, but move away very fast from the plane. The evolution of pulsar heights was considered by Bhattacharya et al. (1992), Hartman et al. (1997) and Mukherjee & Kembhavi (1997), but has not been explicitly described in most pulsar population syntheses; therefore, a picture of pulsar height evolution has not been very clear. We want to demonstrate in this paper by 3D simulations how pulsars move in our Galaxy.

Birth velocities of both normal and millisecond pulsars (MSPs) have been investigated by many authors. For normal pulsars, Lyne & Lorimer (1994) studied transverse speeds of 29 pulsars younger than 3 Myr and obtained a mean birth velocity of $V_B = 450 \pm 90 \text{ km s}^{-1}$. Taking into account selection effects and using all available pulsar proper motion data in population synthesis, Lorimer, Bailes & Harrison (1997) found a mean birth velocity of $V_B \sim 500 \text{ km s}^{-1}$. Hansen & Phinney (1997) considered the distribution of proper motions, including all available upper limits, and concluded that the mean birth velocity $V_B$ should be smaller, only 250–300 km s$^{-1}$. Cordes & Chernoff (1998) performed a detailed analysis for measured velocities of 49 young pulsars. They favoured a two-component Gaussian model in three dimensions with characteristic velocities$^1$ $\sigma_z$ of 175 and $700 \text{ km s}^{-1}$. Arzoumanian, Chernoff & Cordes (2002) infer the velocity distribution of radio pulsars detected in 400-MHz surveys, taking into account beaming, selection effects and luminosity evolution. They also favour a two-component Gaussian model with $\sigma_z \sim 90$ and $500 \text{ km s}^{-1}$.

For MSPs, Lorimer (1995) accounted for known survey selection effects and obtained the local surface density of MSPs, birth rate and a lower limit of the mean birth velocity of $V_B \geq 80 \text{ km s}^{-1}$. The result was confirmed by Cordes & Chernoff (1997), who obtained a velocity perpendicular to the Galactic plane of $v_z = 52^{+17}_{-11} \text{ km s}^{-1}$, i.e. a 3D velocity dispersion of $\sigma_z = 84 \text{ km s}^{-1}$, using likelihood analysis on previous survey data for MSPs plus selection effects. Later, Lyne et al. (1998) found that population syntheses of MSPs with Maxwellian initial 1D velocity dispersion of $80 \pm 20 \text{ km s}^{-1}$ agree best with the data of newly discovered MSPs in the Parkes Southern Sky Survey together with previous MSPs. The 3D mean birth velocity is correspondingly $130 \pm 30 \text{ km s}^{-1}$. These results were later confirmed by Toscano et al. (1999) using more MSP proper motions obtained from timing measurements.

Previously, Narayan & Ostriker (1990, hereinafter NO90) tried to model the observed pulsar populations. They deduced an analytic formula for the height distribution by assuming a Gaussian distribution at all times and solving the energy conservation equation in the $z$-direction (perpendicular to the Galactic plane). However, the height distribution changes from time to time when pulsars run away from the Galactic plane. The integrated $z$-distribution of pulsars with different ages may not be Gaussian. We will show by our Monte Carlo simulation that the scaleheight evolution is much more complicated than Gaussian. Analytic solutions obtained from an over-simplified Galactic acceleration model can hardly represent a realistic situation, especially over long times with non-linear...
evolution of pulsar heights. By simulations, we are able to track the height distribution of a pulsar sample for any form of the acceleration.

In our work, the initial velocity dispersion for MSPs is obtained by comparison of the height distributions between simulated and observed pulsar samples. We also tried to derive the initial velocity dispersion of normal pulsars by comparing their scaleheights and characteristic ages. In Section 2 we will describe the simulation procedures together with input parameters and governing equations. Several possibilities for parameter choice are also discussed. In Section 3, we show the simulation results, mostly regarding how the pulsar scaleheight evolves. We demonstrate in Section 4 two applications of our simulations – the determination of birth velocity dispersions of both normal pulsars and MSPs. The influence of selection effects is also discussed. Conclusions are given in Section 5.

2 SIMULATION DETAILS

In our simulations, we take a Galactocentric rectangular coordinate system, where the x- and y-axes are orthogonal in the Galactic plane and the z-axis is perpendicular to the plane. The dynamic status of a pulsar at any time can be described by \((x, y, z; v_x, v_y, v_z)\), where \(x, y\) and \(z\) are position coordinates, and \(v_x, v_y\), and \(v_z\) are the velocities along each axis. Obviously, the Galactocentric radius \(R = \sqrt{x^2 + y^2}\). The distance between the Sun and the Galactic centre is taken as \(R_0 = 8.0\) kpc. Simulation results should not depend on the coordinate system taken. Before calculating pulsar positions at any time \(t\), the initial positions and velocities of pulsars and the Galactic acceleration should be discussed.

2.1 Initial height distribution

Initial positions of pulsars should follow the positions of their progenitors, e.g. OB stars, or the distribution of supernova remnants. Maíz-Apellániz (2001) has listed all previous determinations of scaleheights of the OB star disc in either an exponential or a Gaussian height distribution. He obtained a solid measurement of the vertical distribution of B-B5 stars in the solar neighbourhood as

\[
P_z(z_{\text{ini}}) = \frac{1}{\sqrt{2\pi h_{\text{ini}}}} \exp\left(-\frac{z_{\text{ini}}^2}{2h_{\text{ini}}^2}\right)
\]

(1)

with a scaleheight of \(h_{\text{ini}} = 63\) pc from Hipparcos data, though a single-component, self-gravitating and isothermal disc model with a sech function is as good as the Gaussian. Here we ignore the halo component, which is about 4 per cent of stars and it is not clear whether this component is related to MSPs. In our simulations, we assume this height distribution to be adequate for any places in the disc.

We also tried an exponential distribution \(P_z(z_{\text{ini}}) = (1/2h_{\text{ini}})\exp(-|z_{\text{ini}}|/h_{\text{ini}})\) with a scaleheight of \(h_{\text{ini}} = 60\) pc and a flat distribution in an infinitely thin disc \((z_{\text{ini}} = 0)\). We found that our results are not sensitive to different initial z-distributions, as all of them lead to the same or very similar dynamic behaviour after one million years. In the following we will use a Gaussian distribution in equation (1), and assume it is valid at all Galactocentric radii.

2.2 Initial radial distribution

The radial density distribution of newly born neutron stars is not at all clear. Narayan (1987) made a Gaussian fit to the observed radial density profile obtained by Lyne, Manchester & Taylor (1985) using a normalized function

\[
\rho_R(R) = \frac{1}{64\pi^2} \exp\left[-\left(\frac{R}{8}\right)^2\right] \text{(kpc)}^{-2},
\]

(2)

where \(\rho_R\) is the radial density. Bailes & Kniffen (1992) pointed out that another function with \(\rho_R \rightarrow 0\) as \(R \rightarrow 0\) is also consistent with the observed data. However, most of the subsequent work took Narayan’s fit as a starting distribution in \(R\) (e.g. Lorimer et al. 1993; Johnston 1994; Mukherjee & Kembhavi 1997). This distribution reflects the observed distribution in \(R\) rather than the initial distribution, which is most probably related to the exponential distribution of OB stars in the Galactic plane expected in our Galaxy and other spiral galaxies (Bahcall 1986; Paczynski 1990, hereinafter P90). Note here that the radial probability distribution \((P_R)\), which is used for discussions below and plots in Fig. 1, should have the form

\[
P_R dR = \frac{2\pi R Pr dR}{\int_0^\infty 2\pi R Pr dR}.
\]

(3)

For Narayan’s function\(^2\), \(P_R = (R/32)\exp[-(R/8)^2]\). In fact, this function has a natural deficit for \(R \rightarrow 0\), which is a salient feature of the observed data that Johnston (1994) tried to explain.

In our simulations we have tested a few probability functions, as shown in Fig. 1.

\[(1)\text{ The exponential distribution:}\]

\[
P_R(R_{\text{ini}}) = \frac{1}{R_{\text{exp}}} \exp\left(-\frac{R_{\text{ini}}}{R_{\text{exp}}}\right).
\]

(4)

hereafter \(R_{\text{ini}} = \sqrt{x^2 + y^2}\) is the initial Galactocentric radius. The re-scaled characteristic radius \(R_{\text{exp}} = 4.7\) kpc\(^3\), as given in Hartman et al. (1997).

\[(2)\text{ The Gamma distribution:}\]

\[
P_R(R_{\text{ini}}) = a_R \frac{R_{\text{ini}}}{R_{\text{exp}}} \exp\left(-\frac{R_{\text{ini}}}{R_{\text{exp}}}\right).
\]

(5)

\(^2\text{We note that in the literature some authors claim to have used the Gaussian radial distribution of Narayan (1987), but by mistake they have used }\rho_R(R)\text{ to represent }P_R.\text{ The prefactor of }P_R, R/32,\text{ is }R\text{-dependent, while that of }\rho_R(R)\text{ is a constant, }\left(64\pi^2\right)^{-1}.\)

\(^3\text{The authors implicitly used }R_0 = 8.5\text{ kpc in context. Here we scaled it to }R_0 = 8.0\text{ kpc. Likewise in the following cases 3 and 4.}\)
with \( R_{\text{exp}} = 4.5 \) kpc and 0.4 kpc \( \lesssim R_{\text{ini}} \lesssim 25 \) kpc, following \( \Phi \). This radial distribution corresponds to an exponential radial density. Here \( \sigma = 1.0683 \). See also Gonthier et al. (2002).

(3) The Gaussian distribution:

\[
P(R_{\text{ini}}) = \frac{1}{\sqrt{2\pi R_{\Phi}}} \exp \left( -\frac{R_{\text{ini}}^2}{2R_{\Phi}^2} \right),
\]

with a rescaled \( R_{\Phi} = 4.5 \) kpc following Lorimer et al. (1993). Many authors (e.g. Hartman et al. 1997) used this distribution later.

(4) The offset Gaussian distribution:

\[
P(R_{\text{ini}}) = \frac{R_{\text{ini}}}{R_{\Phi}^2} \exp \left( -\frac{R_{\text{ini}}^2}{2R_{\Phi}^2} \right),
\]

with rescaled \( R_{\Phi} = 1.7 \) kpc and \( R_{\text{of}} = 3.3 \) kpc given by Hartman et al. (1997).

(5) Narayan’s distribution:

\[
P(R_{\text{ini}}) = \frac{R_{\text{ini}}}{R_{\Phi}^2} \exp \left( -\frac{R_{\text{ini}}^2}{2R_{\Phi}^2} \right),
\]

here \( R_{\Phi} = 4.5 \) kpc is taken from the normalized factor of the Gaussian distribution.

(6) The linear distribution:

\[
P(R_{\text{ini}}) = \frac{2R_{\text{ini}}}{R_{\text{ini}}^2 - R_{\Phi}^2},
\]

so that in a given range of Galactocentric radii \( R_1 < R_{\text{ini}} < R_2 \) (we took \( R_1 = 0.4 \) kpc and \( R_2 = 25 \) kpc) the pulsar surface density is taken to be a constant.

We have simulated pulsar motions for all the above cases and will make comparisons with the evolved radial distribution, while for detailed studies we will concentrate on the Gamma distribution (equation 5).

2.3 Initial velocities

The initial velocities can be written as \( \mathbf{v}_{\text{ini}} = \mathbf{v}_{\text{birth}} + \mathbf{v}_{\text{rot}} \), where \( \mathbf{v}_{\text{birth}} \) is the birth velocity and \( \mathbf{v}_{\text{rot}} \) the rotation velocity of the Galaxy. The physical origin of the birth velocity is not clear hitherto, which might be generated from the disruption of a binary (Gunn & Ostriker 1970), rocket effects (Helfand & Tademaru 1977) or asymmetric supernova explosion (Dewey & Cordes 1987). We will not go into further detail, but assume instead that the birth velocity is isotropic and follows a Maxwellian distribution

\[
P_r(v_{\text{birth}}) = \sqrt{\frac{2}{\pi \sigma_{\text{birth}}^2}} \exp \left( -\frac{v_{\text{birth}}^2}{2\sigma_{\text{birth}}^2} \right),
\]

where \( \sigma_{\text{birth}} \) is the 1D velocity dispersion, corresponding to a mean velocity of \((8/\pi^{1/2})\sigma_{\text{birth}}\) or a 3D velocity dispersion of \((3^{1/2})\sigma_{\text{birth}}\). We will show the dynamical results for \( \sigma_{\text{birth}} = 100, 200, 300 \) and 400 km s\(^{-1}\) in our simulations. \(^4\)

2.4 Galactic potential and acceleration

Pulsars accelerate in the Galactic gravitational potential, the acceleration being \( \mathbf{g} = -\nabla \phi \). The potential \( \phi \) is determined by the mass distribution in our Galaxy, \( \nabla^2 \phi = 4\pi G \rho \), where \( G \) is the gravitational constant and \( \rho \) is the mass density as a function of position. One can find that

\[
\nabla \cdot \mathbf{g} = -4\pi G \rho.
\]

\(^4\)Note here \( 1 \) km s\(^{-1}\) \( \approx 1 \) pc Myr\(^{-1}\).

In fact, equation (11) can be rewritten as

\[
\rho = \frac{4}{4\pi G} \frac{\partial g_z}{\partial z} + \frac{1}{R} \frac{\partial (R g_R)}{\partial R}
\]

in a cylindrical coordinate system. Here \( g_z \) is the acceleration in the \( z \)-direction and \( g_R \) in the \( R \)-direction. In order to simulate pulsar dynamics, it is very important to find the best representation of the Galactic potential.

2.4.1 Adopted Galactic potential model

We used the potential model of disc/bulge originally proposed by Miyamoto & Nagai (1975):

\[
\phi_i(R, z) = \frac{G M_i}{\left( R^2 + [a_i + (z^2 + b_i^2)^{1/2}]^{1/2} \right)^{1/2}}.
\]

Here, \( a_i, b_i \) and \( M_i \) are model parameters, and \( i = 1 \) stands for the bulge and \( i = 2 \) for the disc. Using the density distribution of the halo given by (Kuijken & Gilmore 1989, hereinafter KG89):

\[
\rho = \frac{\rho_h}{1 + (r/r_c)^2},
\]

where \( r^2 = x^2 + y^2 + z^2 \), \( P90 \) derived the potential for the halo component as

\[
\phi_h = -\frac{G M_h}{r_c} \left[ \frac{1}{2} \ln \left( \frac{1 + r^2}{r_c^2} \right) + \frac{r}{r_c} \tan^{-1} \frac{r}{r_c} \right],
\]

where \( M_h = 4\pi \rho_h r_c^2 \) and \( \rho_h \) and \( r_c \) are model parameters. \( P90 \) determined all the parameters (see Table 1) for the disc, bulge and halo potentials to make a good agreement with the rotation curve, local volume density and the column density in the range \( z = \pm 700 \) pc. He took \( \rho_0 = 8.0 \) kpc. This potential expression has been used for simulations of pulsar motions by Hansen & Phinney (1997), Cordes & Chernoff (1998) and Gonthier et al. (2002). Since the rotation curve for \( R < 0.4 \) kpc cannot be well described by the formula for the potentials, as Sofue & Rubin (2001) show, all our simulations have been limited to \( R > 0.4 \) kpc.

2.4.2 Galactic potential previously used

Previously, several models for Galactic mass distribution and 1D acceleration have been used to simulate pulsar motions in the \( z \)-direction.

(1) NO90 modelled the mass density in the Galaxy as a thin layer at \( z = 0 \) with surface density \( \Sigma \) and a uniform halo with density \( \rho_h \), and obtained the acceleration in the \( z \)-direction as

\[
g_z = 4\pi G \rho \left( z + \frac{\Sigma}{2 \rho_h} \right).
\]

Table 1. Potential parameters given by \( \Phi \), with subscripts ‘1’ and ‘2’ corresponding to bulge and disc, respectively, and subscript ‘c’ corresponding to halo.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 ) (kpc)</td>
<td>0</td>
</tr>
<tr>
<td>( b_1 ) (kpc)</td>
<td>0.277</td>
</tr>
<tr>
<td>( M_1 ) (M(_\odot))</td>
<td>( 1.12 \times 10^{10} )</td>
</tr>
<tr>
<td>( a_2 ) (kpc)</td>
<td>3.7</td>
</tr>
<tr>
<td>( b_2 ) (kpc)</td>
<td>0.2</td>
</tr>
<tr>
<td>( M_2 ) (M(_\odot))</td>
<td>( 8.07 \times 10^{10} )</td>
</tr>
<tr>
<td>( r_c ) (kpc)</td>
<td>6.0</td>
</tr>
<tr>
<td>( M_c ) (M(_\odot))</td>
<td>( 5.0 \times 10^{10} )</td>
</tr>
</tbody>
</table>
which can be further used to determine the disc surface density. Gilmore (1989), has the form

either directly from CI87 or using parameters given by Kuijken & Gilmore (1999, hereinafter F98) for ISM distribution studies. The model of KG89 seems to have been much better constrained by high z and obtained the acceleration in the z-direction as

\[ g_z = 1.04 \times 10^{-3} \left( \frac{1.26z}{\sqrt{z^2 + 0.18^2}} + 0.58z \right). \]

which can be further used to determine the disc surface density.

Their formula for \( g_z \) has been used by Bhattacharya et al. (1992) and Hartman & Verbunt (1995) to simulate pulsar motions in the z-directions for the investigation of pulsar magnetic field decay, and extended by Ferrière (1998, hereinafter F98) for ISM distribution studies. The model of KG89 seems to have been much better constrained at high z than that of NO90. See Fig. 2 and discussions by Hartman & Verbunt (1995).

(3) The potential model for the disc/spheroid and nucleus/bulge, either directly from CI87 or using parameters given by Kuijken & Gilmore (1989), has the form

\[ \phi_{db} = \frac{GM}{\left( a + \sum_{i=1}^{3} b_i \left( z^2 + h_i^2 \right)^{1/2} + b^2 + R^2 \right)^{1/2}}, \]

\[ \phi_{bd} = \frac{GM_{bh}}{(b^2 + R^2)^{1/2}}, \]

which has been used by Lorimer et al. (1993, 1997), Hartman et al. (1997) and Mukherjee & Kembhavi (1997) for pulsar population synthesis. The model parameters \( a, b, h_i, \beta \), and mass \( M \) for different components were constrained by the rotation curve data.

(4) DB98 have constructed a set of mass models for our Galaxy using all available observational constraints. All of their models consist of the spheroidal bulge, halo and three disc components, namely the thin and thick stellar discs and the interstellar medium disc. The density for bulge and halo in the model is described by the spherical density distribution

\[ \rho_\phi = \rho_0 \left( \frac{m}{r_0} \right)^{-\gamma} \left( 1 + \frac{m}{r_0} \right)^{-\beta} \exp \left( -m^2/r_i^2 \right). \]


---

**Figure 2.** Acceleration \( g_z \) calculated for \( R = 4 \) and 8 kpc for the models of Carlberg & Innanen (1987, hereinafter CI87), Dehnen & Binney (1998, hereinafter DB98), F98, KG89, NO90, P90 and P90.

This model was also used by Itoh & Hiraki (1994) and tried by Hartman & Verbunt (1995).

(2) KG89 rewrote equation (12) as

\[ \rho = -\frac{1}{4\pi G} \left[ \frac{\partial g_z}{\partial z} + 2(A^2 - B^2) \right] \]

for disc galaxies following Mihalas & Binney (1981). Here, \( A \) and \( B \) are the Oort constants. After considering the double-exponential discs and the spherical components (halo, bulge and corona), they modelled the distribution of matter in the disc via tracer stars at high \( z \) and obtained the acceleration in the z-direction as

\[ g_z = 1.04 \times 10^{-3} \left( \frac{1.26z}{\sqrt{z^2 + 0.18^2}} + 0.58z \right). \]

which can be further used to determine the disc surface density.

Their formula for \( g_z \) has been used by Bhattacharya et al. (1992) and Hartman & Verbunt (1995) to simulate pulsar motions in the z-directions for the investigation of pulsar magnetic field decay, and extended by Ferrière (1998, hereinafter F98) for ISM distribution studies. The model of KG89 seems to have been much better constrained at high z than that of NO90. See Fig. 2 and discussions by Hartman & Verbunt (1995).

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\[ \phi_{db} = \frac{GM}{\left( a + \sum_{i=1}^{3} b_i \left( z^2 + h_i^2 \right)^{1/2} + b^2 + R^2 \right)^{1/2}}, \]

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which has been used by Lorimer et al. (1993, 1997), Hartman et al. (1997) and Mukherjee & Kembhavi (1997) for pulsar population synthesis. The model parameters \( a, b, h_i, \beta \), and mass \( M \) for different components were constrained by the rotation curve data.

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\[ \rho_\phi = \rho_0 \left( \frac{m}{r_0} \right)^{-\gamma} \left( 1 + \frac{m}{r_0} \right)^{-\beta} \exp \left( -m^2/r_i^2 \right). \]


---

**Figure 3.** The contours of \( g_z \) (upper panel) and \( g_x \) (lower panel) calculated from potentials of DB98, P90 and CI87 with modified parameters by CI87 and KG89. The contour levels are at \( 2^\circ \) pc Myr\(^{-2} \), with \( n = 0, \pm 1, \pm 2, \pm 3 \), and with notes near \( n = 0 \).

Here \( m = (R^2 + q^{-2}z^2)^{1/2} \), and the density of discs is given by

\[ \rho_\phi(R, z) = \frac{\Sigma_d}{2\pi z_d} \exp \left( \frac{-R_d}{R_d - \frac{R}{R_d - \frac{z}{z_d}}} \right). \]

where \( \rho_\phi, R_d, q, \gamma, \beta, r_i, \Sigma_d, z_d, R_d \) and \( R_m \) are model parameters. One set of the best fitted parameters that we tried in this paper was derived by satisfying a total local surface density of 52.1 M\( \odot \) pc\(^{-2} \) in the solar neighbourhood and a circular velocity of \( v_c(R_0) = 217 \) km s\(^{-1} \) with \( R_0 = 8 \) kpc (Model 2 in their table 3).

2.4.3 **Discussion of Galactic potential models**

Obviously most models of \( g_z \) (see Fig. 2) are acceptable in the regions only around the sun and near the mid-plane, which can not be used to simulate pulsar motions farther than a few kpc from the Sun in our Galaxy. The change of acceleration (both \( g_z \) and \( g_x \)) with \( R \) and \( z \) (see Fig. 3) must be considered. Previous results based on simulations with only \( g_z \) acceleration (e.g. Bhattacharya et al. 1992) should therefore be treated with caution.

An accurate model for the mass distribution and acceleration is needed for better understanding of pulsar motions in our Galaxy. The rotation curves for the models of DB98, P90 and CI87 with parameters modified by Kuijken & Gilmore (1989) are shown in Fig. 4. We noticed that the potential given by DB98 is almost the same as that of P90, though newer constraints were used. Our simulations for both potentials produced very similar results. The main difference between these two models is the rotation curve peak at \( R \approx 300 \) pc, which was not considered by DB98 but P90 did consider the rotation
We also noticed that the mass model of CI87, even with parameters modified by Kuijken & Gilmore (1989), cannot produce the proper rotation curve at high $z$ and large $R$. The very slow change of the potential near the Galactic centre (Fig. 3) is not reasonable. The accelerations given by DB98 and P90 at small $R$ and at high $z$ (see Fig. 3) seems to be better constrained by observational data (see Fig. 4). Therefore the models of DB98 and P90 are recommended here. Paczyński's potential is preferable owing to its simple analytic form.

2.5 Governing equations and calculation method

The initial velocities and positions of the pulsars were generated randomly according to the discussions above. The rotation velocity was calculated from the Galactic potential. The motion of a pulsar is therefore only governed by the Newtonian kinetic equation as,

\[
\begin{align*}
\frac{dr(t)}{dt} &= v(r) \\
\frac{dv(r)}{dt} &= g(r),
\end{align*}
\]

where $r(t)$, $v(r)$ and $g(r)$ are 3D vectors. The equations above were solved numerically in three directions using the 5th-order Runge–Kutta method with adaptive control stepsizes (using the subroutine RKQS) (Press et al. 1992, p. 712). The initial stepsize is $dt = 10^{-4}$ Myr, and calculations of position and velocity in 3D then
3 RESULTS AND DISCUSSION

Limited by the computation resources available, we simulated $2 \times 10^8$ pulsars at age $t = 0$, and set their initial positions and velocities according to the details discussed in Section 2. We then traced their motions up to an age of 2 Gyr, and derived statistics on their locations every 0.1 Myr. We first present results from Gaussian distributions of the initial height $z$, and Gamma-distributions for initial $R$, which will be called the standard initial conditions hereafter. We then compared those with the results from other initial conditions.

3.1 Results from standard initial conditions

3.1.1 The scaleheight evolution

The heights of simulated pulsars are binned with equal height interval every 0.1 Myr. We used two data sets: the average height of the $i$th bin, $z(i)$, and the number of pulsars in the $i$th bin, $N(i)$. The Levenburg–Marquardt method was then employed to fit these two data sets to given functions (Press et al. 1992, p. 678) to obtain the height distribution.

For pulsars with an age less than 8 Myr, the height distribution can be well fitted by a single Gaussian function (Fig. 6: $t = 2$ Myr and 7 Myr), i.e.

$$N(i) = A \exp \left[ -\frac{z(i)^2}{2 h_z^2} \right].$$

Here $h_z$ is the scaleheight and $A$ is amplitude. We found that $h_z$ increases linearly with $t$ (Fig. 7), which can be represented as

$$h_z = h_0 + \sigma t,$$

where $h_0$ and $\sigma$ are fitting coefficients, listed in Table 2.

The relation between $\sigma$ and $\sigma_{\text{birth}}$ should be discussed. At small $t$, one could take

$$z(t) = z_{\text{ini}} + v_{\text{ini}} t$$

as a simplification, where $z_{\text{ini}}$ is height at time $t$. To justify this approximation, we calculated the turnover time of pulsars, which was defined as the time when pulsars change the sign of $v_z$, i.e. start to move towards the Galactic plane for the first time. It is approximately a quarter of the oscillation period for a pulsar moving up and down. As can be seen from Fig. 8, most pulsars change their motion directions (i.e. the sign of $v_z$) at $\sim 15$ Myr, a time not sensitive to initial velocities. Therefore, for a Maxwellian initial 3D velocity distribution, it is natural that the distribution of height at small age $t$ can have a Gaussian form, and that the scaleheight increases with time as $h_z \sim \sigma_{\text{birth}} t$. We noticed that roughly $\sigma_{\text{birth}} \sim \sigma$, a relation that can be used to determine 1D initial velocity dispersions. We emphasize that formula (26) is only a first-order approximation. The effect of the deceleration of the Galactic potential did work to a certain extent, which made $\sigma$ slightly smaller, i.e. $\sigma < \sim \sigma_{\text{birth}}$ (Table 2).

As time increases, the central peak in the height distribution gets more and more prominent (Fig. 6: $t = 10$ Myr). A single Gaussian function cannot fit the distribution satisfactorily. We found that a Gaussian plus an exponential function provides a much better fit,
\( N(i) = A \exp \left( \frac{z^2(i)}{2h_g^2} \right) + B \exp \left( -|z(i)|/h_e \right). \) (27)

where \( h_g \) and \( h_e \) are scaleheights, and \( A \) and \( B \) are amplitudes. The exponential component mainly models the smaller heights and the Gaussian component accounts for the larger ones. The scaleheight of the Gaussian component, \( h_g \), increases linearly with time until \( t \sim 40 \) Myr (Fig. 9).

When \( t > \sim 40 \) Myr, the height distribution gets more concentrated towards lower heights and \( h_g \) decreases gradually (Fig. 9). When \( t > \sim 200 \) Myr, the height distribution tends to stabilize, and both \( h_g \) and \( h_e \) show no large variations. We noticed that the larger the initial velocity dispersion, the longer the stabilization time and the larger the resulting scaleheight.

### 3.1.2 Scaleheights at different radii

Earlier, we have shown the scaleheight of the \( z \)-distribution of all pulsars in the range \( 0.4 \) kpc \( \leq R \leq 25 \) kpc. It is not clear whether the \( z \)-distribution changes with \( R \).

The height distributions in four ranges of \( R \) does not show significant discrepancies (Fig. 10).

### 3.1.3 Evolution of the radial distribution

We assume that all simulated pulsars are initially located in the range \( 0.4 \) kpc \( < R < 25 \) kpc with a maximum at \( R = 4.5 \) kpc. During the simulation, we trace only those pulsars in the region \( 0.4 \) kpc \( < R < 25 \) kpc. As mentioned above, the Galactic potential near the Galactic centre may not be realistic. This might cause exotic pulsar motion trajectories (Carlberg & Innanen 1987) so that we ignore all pulsars moving close to the centre (\( R < 0.4 \) kpc). Pulsars moving out to \( R > 25 \) kpc are classed as ‘escaped’ pulsars, since the Galactic gravitation is too weak to bound them. The number of escaped pulsars increases until 200 Myr (Fig. 11). The larger the initial velocities, the more likely the pulsars are to escape. For \( \sigma_{\text{birth}} = 400 \) km s\(^{-1}\), more than 60 per cent of pulsars escape after 100 Myr. Obviously, a huge number of pulsars, even after their radio emission turns off, have gone into the Galactic halo or into intergalactic space, which could be a very important ingredient of the dark-matter halo.

We found that there is almost no evolution of the radial density and radial probability distribution (Fig. 12) if the initial radial distribution is a Gamma-distribution. The different velocities do not change the distribution although numbers of remained pulsars change a lot. Concerning the form of the radial distribution, there is a deficit towards the Galactic centre, as Johnston (1994) discussed. A peak appears at \( \sim 4.5 \) kpc, not too different from the initial distribution. However, the surface density does not have such a deficit, in contrast to the observed pulsar density distribution newly determined by Lorimer (2004).

### 3.2 Results for other initial conditions

#### 3.2.1 Alternative initial height distributions

Two kinds of initial height distribution have been tried for simulations of pulsar motions: an exponential height distribution and a flat distribution of all \( z_{\text{ini}} = 0 \).
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Figure 12. The $R$-distribution (lower panel) and the normalized radial density (upper panel) at $t = 0$ (initial) and $t = 2$ Gyr.

Figure 13. The change of the index $\alpha$ in a generalized fitting function $N(i) = A \exp\left\{ -\left( \frac{|z(i)|}{h_a} \right)^\alpha \right\}$.

For those non-Gaussian initial height distributions, a Gaussian function did not always give a good fit for $t < 1$ Myr. So we tried another function for the height distribution:

$$N(i) = A \exp\left\{ -\left( \frac{|z(i)|}{h_a} \right)^\alpha \right\},$$

where $\alpha = 1$ corresponds to an exponential function and $\alpha = 2$ a Gaussian function. We examined a few initial height distribution cases. As shown in Fig. 13, for the initial Gaussian distribution of heights or the flat distribution with $z_{ini} = 0$, $\alpha$ is always nearly 2, so that we can always fix $\alpha = 2$ in the fit. For the exponential distribution of initial heights, the index $\alpha$ increases from 1 to 2 gradually, which can be approximately described by the function

$$\alpha = 2 - \exp\left( -\frac{t}{0.35\text{Myr}} \right).$$

When $t$ is greater than 1 Myr, the height distributions can be described by a Gaussian or a Gaussian plus an exponential function, as shown in Section 3.1.1. We therefore conclude that the pulsar height distribution is insensitive to the initial distribution after $t > 1$ Myr.

3.2.2 Alternative initial radial distributions

As we introduced in Section 2.1, alternative radial distributions can be used for simulations: (1) Gamma, (2) Gaussian, (3) offset Gaussian, (4) exponential, (5) Narayan and (6) uniform density distribution.

We first checked the evolution of height distribution, and found that all these radial distributions produce similar results, as Fig. 14 shows. That is to say, the height distribution is insensitive to the initial radial distribution.

The evolution of radial distributions has been checked using both the density and radial distributions (see Fig. 15). Obviously pulsars at large $R$ feel a smaller Galactic potential and hence are easy to escape after some years. The final distribution seems to be more prominent at small $R$ where the Galactic potential is much larger due to the presence of the Galactic bulge.

We indeed see a fall-off at small $R$ in both the density distribution and radial distribution from our simulations with all kinds of initial radial distributions. However, compared with the observed deficit at small $R$ recently determined by Lorimer (2004), the simulated deficit is much smaller in all cases of radial distributions. Note that we simulate simply moving neutron stars and take no account of their radio emission. It is likely that the beaming effect and the evolution of pulsar radio emission probably have to be considered carefully for a fair comparison. We can conclude from our simulation, however, that there should be a large number of evolved neutron stars near the Galactic centre, which may no longer be observable as radio pulsars.

4 APPLICATIONS

All the above simulations are made in ideal conditions for pulsar motions in the Galactic potential. Previous authors have simulated currently observed pulsar populations, so our results give a complementary image of pulsars moving in our Galaxy. As just mentioned, for a detailed comparison between the simulation results and the observed sample, one has to consider beaming and the evolution of the radio emission of pulsars. We understand that the selection effects on pulsar surveys are quite severe for a few reasons. For example, in any flux-limited survey, more luminous pulsars are easy to detect, even if they are far away. Faint pulsars can be detected only if they are very nearby. It is not clear how the luminosity of pulsars evolves with age, though evidence available shows that young pulsars tend to be brighter. Due to dispersion smearing, only nearby millisecond pulsars are easy to be discovered, mostly at high latitudes.
4.1 Young pulsars: initial velocity dispersion derived from the $z$-distribution

Most pulsar surveys have been conducted near the Galactic plane where young pulsars are predominantly found. As our simulations show, the scaleheight is linearly increasing for young pulsars in the form of $h_z = h_0 + a \tau$ for $\tau < 8$ Myr, which may be used to determine the initial velocity of normal pulsars.

Nevertheless, at these two age extremes, i.e. young pulsars and millisecond pulsars, we do not have to synthesize populations for comparison of the velocity distributions or height distributions. As we do not expect the pulsar luminosity to evolve much in 1 Myr, and also previous surveys have been mostly concentrated on the Galactic plane where young pulsars live, the sample of very young pulsars suffer much less from selection effects in the surveys. For millisecond pulsars, the height distribution is very stable after 200 Myr (see Figs 9 and 14), so that millisecond pulsars of any age should follow the same height distribution.

For young pulsars ($\tau < 8$ Myr), $z = z_{init} + v_{init} \tau$. If $z_{init} < -0.06$ kpc is ignored, a pulsar with a $z$-velocity of 400 km s$^{-1}$ will reach 0.4 kpc at 1 Myr, but 4 kpc at 10 Myr. Previous pulsar surveys, like the Parkes Multibeam Survey which discovered about half of all known pulsars, mainly covered $|b| \leq 5^\circ$. If the average pulsar distance is about 6 kpc, all pulsars of $|b| \leq 5^\circ$ would have a $|z| < 0.5$ kpc at or nearer than this average distance. Only pulsars with $\tau < 8$ Myr and a $z$-velocity less than $v_{z} \approx \frac{z}{\tau} = 0.5$ kpc/8 Myr = 63 km s$^{-1}$ can be picked up in such a survey. The detected older pulsars must have much smaller $z$-velocities, otherwise they would have to be much farther away than 6 kpc.

Pulsars younger than 1 Myr cannot move too far away even with a large initial $z$-velocity because of their small ages. As a result, the combined sample would not suffer much selection effect due to survey regions. This is why Lyne & Lorimer (1994) chose 1 Myr as the cut-off age of the sample. Now we try to use much more pulsars of $\tau < 1$ Myr to estimate $z$-velocity. About 244 pulsars have been divided into three bins, with roughly the same number of pulsars in each. The 1D velocity dispersion in the $z$-direction derived from the scaleheights of $z$-distributions of three sub-samples is $175 \pm 56$ km s$^{-1}$, as shown in Fig. 16. This corresponds to a 3D velocity dispersion of $303 \pm 97$ km s$^{-1}$ or a mean velocity of $280 \pm 96$ km s$^{-1}$.

To compare with previous velocity estimates, we should use the model for pulsar distance due to Taylor & Cordes (1993), and should

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**Figure 15.** The density distribution (normalized $\rho_R$) and the radial distribution ($P_R$) at $t = 0$ yr (upper panels) and $t = 2$ Gyr (lower panels) for various initial distributions.

**Figure 16.** The scaleheights versus characteristic ages of currently known pulsars. Solid lines represent linear fits. The numbers of pulsars in the corresponding bins are indicated in the plot.
5 CONCLUSIONS

We have presented a generalized statistical picture of how pulsars move in our Galaxy. The potential given by P90 can be a good representation of the mass distribution within our Galaxy, and has a simple analytic formula. We found that the final height distributions are not sensitive to the forms of the initial z- and R-distributions. The height distribution can be well fitted by a Gaussian function within ~8 Myr, and the scaleheight increases linearly with time. After that, an extra exponential function is required to fit the height distribution. The height distribution stabilizes after about 200 Myr.

The height distribution of pulsars younger than 1 Myr directly implies a mean initial velocity of 280 ± 96 km s\(^{-1}\). Comparison of the simulated sample of millisecond pulsars with the observed sample suggests the 1D initial velocity dispersion of MSPs to be most probably 60 ± 10 km s\(^{-1}\), consistent with estimates given by previous authors.

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Figure 17. K–S test: cumulative probability distributions of simulated and observed samples (upper panel), and the significance level for different initial velocities (lower panel).

Not include the pre-scaling factor from \( R_0 \). We then find a 1D velocity dispersion of 187 ± 64 km s\(^{-1}\), or a 3D velocity dispersion of 324 ± 110 km s\(^{-1}\) and a mean velocity of 299 ± 102 km s\(^{-1}\). These values are consistent with estimates given by Hansen & Phinney (1997) – a mean pulsar velocity of 250 km s\(^{-1}\) – or by Lyne & Lorimer (1994) – a mean of 450 ± 90 km s\(^{-1}\).

4.2 Initial velocity dispersion of MSPs

MSPs have a more complex evolutionary history than normal pulsars. Generally, MSPs are old neutron stars spun up by mass and momentum transferring from the companions (e.g. Alpar et al. 1982). Their ‘birth’ velocity is either the velocity of the binary system or its sole velocity after the disruption of the binary system. Another consecutive problem is their ages. Characteristic ages from period and period derivative probably do not reflect their true ages. Are they chronologically old? Hansen & Phinney (1997) demonstrated that pulsars older than 10\(^8\) yr show asymmetric drifts. Toscano et al. (1999) confirmed the asymmetric drifts of MSPs, which justify that MSPs are really dynamically old enough to be virialized. There is no doubt that MSPs are really old objects in the Milky Way.

From our simulation, the height distribution of old pulsars (e.g. \( t > 200 \) Myr) is stabilized. MSPs of all ages follow the same height distribution. For the comparison of the z-distributions of the simulated and observed samples, we did not take into account selection effects for the MSP discovery. The observed sample consists of 48 MSPs from the latest pulsar catalogue after discarding the MSPs in globular clusters. The simulated sample are old pulsars within 3 kpc from the Sun in a number of sets of simulations with initial 1D z-velocity dispersion from 30 to 180 km s\(^{-1}\) (with steps of 5 km s\(^{-1}\)). The K–S test is employed to check whether these two distributions are from the same parent distribution. We find that \( \sigma_{\text{birth}} = 60 \pm 10 \) km s\(^{-1}\) gives the largest probability (Fig. 17). If the velocities of MSPs follow a Maxwellian distribution, the 1D velocity dispersion is most probably 60 ± 10 km s\(^{-1}\) or the mean velocity dispersion most probably 96 ± 16 km s\(^{-1}\), consistent with previous results (e.g. Lyne et al. 1998).
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