A plausible Galactic spiral pattern and its rotation speed

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ABSTRACT
We report calculations of the stellar and gaseous response to a Milky Way mass distribution model including a spiral pattern with a locus as traced by K-band observations, superimposed on the axisymmetric components in the plane of the disc. The stellar study extends calculations from previous work concerning the self-consistency of the pattern. The stellar response to the imposed spiral mass is studied via computations of the central family of periodic and nearby orbits as a function of the pattern rotation speed, Ω_p, among other parameters. A fine grid of values of Ω_p was explored, ranging from 12 to 25 km s⁻¹ kpc⁻¹. Dynamical self-consistency is highly sensitive to Ω_p, with the best fit appearing at 20 km s⁻¹ kpc⁻¹. We give an account of recent independent pieces of theoretical and observational work that are dependent on the value of Ω_p, all of which are consistent with the value found here: the recent star formation history of the Milky Way, local inferences of cosmic ray flux variations and Galactic abundance patterns. The gaseous response, which is also a function of Ω_p, was calculated via 2D hydrodynamic simulations with the ZEUS code. For Ω_p = 20 km s⁻¹ kpc⁻¹, the response to a two-armed pattern is a structured pattern of four arms, with bifurcations along the arms and interarm features. The pattern qualitatively resembles the optical arms observed in our Galaxy and other galaxies. The complex gaseous pattern appears to be linked to resonances in stellar orbits. Among these, the 4:1 resonance plays an important role, as it determines the extent of the stellar spiral pattern in the self-consistency study presented here. Our findings seemingly confirm predictions by Drimmel & Spergel (2001), based on K-band data.

Key words: ISM: structure – Galaxy: fundamental parameters – Galaxy: kinematics and dynamics – Galaxy: structure – galaxies: spiral.

1 INTRODUCTION
The comparison of near-infrared and optical images of external galaxies reveals interesting differences. Striking examples are M81 and NGC 2997 [see pictures in Elmegreen (1981) and Block et al. (1994), respectively]. It is common to observe a smooth, simple two-armed K-band pattern but a more complex pattern in the optical blue band, often suggesting more arms and bifurcations (segments of arms that appear to be connected to a K-band arm but are not detectable in the infrared). A two-armed smooth structure underlying a more complex morphology also appeared in the work of Grosbøl, Pompei & Patsis (2002); in a K-band study of 53 nearby spirals, most galaxies displayed a grand-design, two-armed, symmetric pattern in their inner regions which often breaks up into tighter-wound, multiple arms further out. Non-linear effects were invoked to explain such morphology.

In recent work, data from COBE-DIRBE have shed light into the Milky Way spiral pattern. Drimmel (2000) and Drimmel & Spergel (2001) have presented a comprehensive picture of what this pattern is like, presenting emission profiles of the Galactic plane in the K band and at 240 µm. The former data set, which suffers little absorption and traces density variation in the old stellar population, is dominated by a two-armed structure with a minimum pitch angle of 15.5°. At 240 µm, the pattern is consistent with the standard four-armed model, that corresponding to the distribution of the youngest stellar populations delineated by Hα regions.

The conventional picture of the spiral pattern of our Galaxy maps at least four arms, named Norma, Crux–Scutum, Carina–Sagittarius and Perseus (for a recent review, see Vallée 2002, who also reports a likely pitch angle of 12° for this pattern). Additionally, features such as the Orion spur at the solar neighbourhood have been revealed (Georgelin & Georgelin 1976). Drimmel (2000) laid down, from the comparison, the hypothesis that the four-armed structure is the gas response to the two-armed ‘stellar’ pattern.

Assuming that indeed the K-band data is by far a better tracer of mass than the optical data of spiral structure, in this work we...
model the spiral pattern from the locus and pitch angle of Drimmel (2000) and study its self-consistency, a requirement we consider it must satisfy. Because the answer strongly depends on the pattern speed, this study should yield a value for this fundamental Galactic parameter.

The value of the pattern speed of the Galaxy has been a matter of controversy for a long time. From the values proposed by Lin, Yuan & Shu (1969) of $\Omega_0 = 11-13$ km s$^{-1}$ kpc$^{-1}$, numbers in the range of 10–60 km s$^{-1}$ kpc$^{-1}$ have been used in the literature. In a previous paper (Pichardo et al. 2003, hereafter P1), we explored the stellar dynamics in a full axisymmetric model for our Galaxy, superimposing our modelling of mass distribution for the spiral pattern: the locus of Drimmel (2000), the optical locus of Vallée (2002) and a superposition of both. P1 did not assume the usual simple perturbing term (a cosine term for the potential) that had been used in the literature in the modelling of spiral arms; it is precisely the very prominent spiral structure in red light that suggests to us that such structure should be considered an important galactic component worthy of a modelling effort beyond the simple perturbing term. P1 modelling consists of a superposition of oblate spheroids for the spiral. Two different values of $\Omega_0$ were tried, $\Omega_0 = 15$ and 20 km s$^{-1}$ kpc$^{-1}$. The best self-consistency was achieved with $\Omega_0 = 20$ km s$^{-1}$ kpc$^{-1}$ for the locus and pitch angle of Drimmel (2000). Fig. 7 of P1, a mosaic of our self-consistency test (explained below) for parameters including the global spiral mass and loci, exhibits a remarkably good response for this case which rules out the other three cases in the panel. In fact, the behaviour at $\Omega_0 = 20$ km s$^{-1}$ kpc$^{-1}$ is so good that one can hardly envision an improvement on general theoretical grounds. The question that comes to mind is, given the spiral locus and pitch angle of Drimmel (2000), our adopted mass distribution and the spiral mass implied by observations of external galaxies applied to those parameters, now fixed, are there other values of $\Omega_0$ which also satisfy our self-consistency criterion to that accuracy? In this Letter, we extend the self-consistency calculations to a finer range of values of $\Omega_0$ in order to answer that question. Once the best value is found, we put it to the test in two ways, calculating the gaseous response to find out whether it is consistent with the observed optical Galactic spiral pattern, which amounts to a test of Drimmel’s hypothesis. Finally, we review recent work using independent data sets which are sensitive to the value of $\Omega_0$. It is another way of testing our prediction against ‘nature’, and not only versus different modelling approaches, as the subjects of those independent studies are quite different from our Galactic modelling effort.

A continued line of work by Contopoulos and collaborators (see, for example, Patsis, Grosbøl & Hiotelis 1997 and references therein) has provided the framework to study the response of gaseous discs to spiral perturbations. In that paper, a comparison between smoothed particle hydrodynamics (SPH) models with Population I features observed on B images of normal, grand design galaxies, showed that the 4:1 resonance generates a bifurcation of the arms and interarm features. Furthermore, Contopoulos & Grosbøl (1986, 1988) had shown that the central family of periodic orbits do not support a spiral pattern beyond the position of the 4:1 resonance, which thus determines the extent of the pattern. From linear theory, weak spirals can extend their pattern up to corotation. A phenomenological link between resonances, the angular speed, and the stellar and gas patterns in spirals is complemented by the study of Grosbøl & Patsis (2001) using deep $K$-band surface photometry to analyse spiral structure in 12 galaxies. They find that the radial extent of the two-armed pattern is consistent with the location of the major resonances: the inner Lindblad resonance (ILR), the 4:1 resonance, corotation and the outer Lindblad resonance (OLR). For galaxies with a bar perturbation, the extent of the main spiral was better fitted assuming it is limited by corotation and the OLR. Using $H_o = 75$ km s$^{-1}$ Mpc$^{-1}$, the pattern speed was found to be for the entire sample of the order of 20 km s$^{-1}$ kpc$^{-1}$ and, remarkably, was not a sensitive function of morphological type or total mass.

In the following section, we describe our results for the stellar orbital response to the imposed spiral pattern, through which $\Omega_0$ is determined.

2 ORBITAL SELF-CONSISTENCY MODELLING, INFERRING $\Omega_0$

As in P1, our axisymmetric Galactic model is that of Allen & Santillán (1991), which includes a bulge and a flattened disc proposed by Miyamoto & Nagai (1975), together with a massive spherical dark halo. We coupled to this mass distribution a spiral pattern modelled as a superposition of inhomogeneous oblate spheroids along a locus that fits the $K$-band data of Drimmel (2000), with a pitch angle of 15.5°. P1 describes in detail the parameters of the spheroids, which briefly are that the minor axis of the spheroids is perpendicular to the Galactic plane and its length is 0.5 kpc, and that the major semi-axes have a length of 1 kpc. Each spheroid has a similar mass distribution. Different density laws, linear and exponential, were analysed, finding no important differences.

The total mass in the spiral is fixed such that the local ratio of spiral to background (disc) force have a prescribed value. Seeking sensible values for this ratio, we used the empirical result of Patsis, Contopoulos & Grosbøl (1991), where self-consistent models for 12 normal spiral galaxies are presented, a sample including Sa, Sb and Sc galaxies. Their Fig. 15 shows a correlation between the pitch angle of the spiral arms and the relative radial force perturbation. The forcing, proportional to the pitch angle, is increasing from Sa to Sc types in a linear fashion. For our pitch angle of 15.5°, the required ratio for self-consistency is between 5 and 10 per cent. As shown in P1, the ratio is a function of galactocentric distance $R$. The authors consider strong spirals to be those in which the ratio is 6 per cent or more.

We found that, in order to obtain relative force perturbations in the 5 to 10 per cent range, our model requires a mass in the spiral pattern of 0.0175 $M_D$, where $M_D$ is the mass of the disc. With that choice, our model predicts a peak relative force of 6 per cent, and an average value, over $R$, of 3 per cent. Other spiral masses were explored, but the analysis favours this case, borderline but on the weak side of the limit separating linear (weak) and non-linear (strong) regimes considered by Contopoulos & Grosbøl (1986, 1988). It is worth noticing that previous results were obtained using rather simplified galactic models, in which the relative amplitude of the spiral perturbation was taken as a fixed few per cent of the axisymmetric force at all radii. To illustrate a reference value, Yuan (1969) proposed 5 per cent.

We follow the technique of Contopoulos & Grosbøl (1986) to compute the ratio of the average density response and the imposed density, $\rho_i/\rho(R)$, calculating a series of central periodic orbits and using flux conservation between every two successive orbits. A dynamically self-consistent model will be one in which $\rho_i/\rho(R)$ does not deviate significantly from unity at any $R$; it will be a potential in which the orbits of stars will produce density enhancements in phase with the imposed pattern. Hence, $\rho_i/\rho(R)$ is a merit function for the dynamical self-consistency of the proposed spiral pattern. We then sample the unknown parameter $\Omega_0$ throughout a fine grid of values, and determine its optimal value as the one for which $\rho_i/\rho(R)$ is as
proposed spiral locus is shown with open squares. A set of periodic orbits are traced with continuous lines, and the maxima in the response density are the filled (black) squares. The frame of reference is the rotating one where the spiral pattern is at rest.

flat and as close to unity as possible over the range [3.3, 12] kpc for $R$. This yields $\Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1}$ as a prediction of our analysis. The response is quite sensitive to $\Omega_p$: nearby values such as 19 or 21 km s$^{-1}$ kpc$^{-1}$ showed noticeable differences. The range explored spanned values from 12 to 25 km s$^{-1}$ kpc$^{-1}$.

In Fig. 1, we show the results for the case $\Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1}$. On the Galactic plane, the assumed spiral pattern and a set of stellar periodic orbits are drawn, with response density maxima shown by black squares. Notice the close coincidence of these with the locus of the imposed pattern within the boxy orbit which marks the 4 : 1 resonance. In the old kinematic-wave interpretation of orbital support for the spiral, one can see support inside the resonance, 4 : 1 resonance. In the old kinematic-wave interpretation of orbital support for the spiral, one can see support inside the resonance, 4 : 1 resonance.

Having determined shape, number of arms and total mass content of the spiral pattern from observations, and having inferred the optimal $\Omega_p$ from dynamical self-consistency, we now study the response of the gaseous disc to such a spiral potential through hydrodynamical simulations.

Fig. 2 shows the gas response to the imposed pattern with $\Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1}$. The locus is indicated with open squares. We performed 2D numerical simulations in polar coordinates using the ZEUS code (Stone & Norman 1992a,b), without including gas self-gravity. The numerical grid covers 2π radians and a radial range from 1 to 15 kpc; however, we disregard results internal to $R = 3.3$ kpc, due to the influence of boundary conditions (BC). The calculation is made in the rotating spiral pattern frame of reference. Resolution is 500 × 500 zones and the BC are inflow–outflow (inner to outer) in $R$ and periodic in the azimuth $\phi$. The temperature was fixed at 8000 K, and the simulation is isothermal, given the short cooling time-scales compared with the dynamical time-scales. The disc reaches a nearly steady state rapidly, which was followed in this case for 3 Gyr. The system is initialized with velocities from the Galactic model rotation curve, adding the spiral source terms through an input table for ZEUS, and an exponential gas density law with a radial scale length of 15 kpc and a local value of about 1.1 cm$^{-3}$ (see Martos & Cox 1998).

In between the imposed two-armed pattern, another two-armed pattern emerges, which displays a sharp shock with a maximum strength along the spiral between, roughly, 5 and 7 kpc. This position is quite close to the 4 : 1 resonance. On each side, this new ‘optical’ two-armed pattern ends up in a corotation island. In the figure, the solar position is along a radial line from the Galactic centre (the origin of both the inertial and the rotating frames) at 20′ from the primed (rotating) $x'$ axis (Freudenreich 1998). Following the ‘K-band’ pattern, there is a slightly offset gaseous response to the imposed potential, making the gas response a four-armed pattern.

A number of caveats apply to our gas simulation: one is that the strength of the shocks will considerably diminish in a full 3D, MHD simulation (the inclusion of the vertical direction and magnetic field was considered in Martos & Cox 1998). Recent simulations in full, realistic Galactic models are scarce: Gómez & Cox (2002) employed the Galactic model of Dehnen & Binney (1998). However, their value of $\Omega_p$ (12 km s$^{-1}$ kpc$^{-1}$) places the 4 : 1 resonance beyond 22 kpc, far out from the observed pattern extent. For that $\Omega_p$, bifurcations and much of the rich structure is removed and the

3 CALCULATING THE GASEOUS RESPONSE TO THE BEST-FITTING POTENTIAL

Figure 1. Self-consistency analysis for $\Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1}$. The proposed spiral locus is shown with open squares. A set of periodic orbits are traced with continuous lines, and the maxima in the response density are the filled (black) squares. The frame of reference is the rotating one where the spiral pattern is at rest.

Figure 2. Simulation with the ZEUS code of the gas response to a spiral pattern with $\Omega_p = 20 \text{ km s}^{-1} \text{ kpc}^{-1}$ (open squares), shown in the rotating frame of the spiral pattern after 2.55 Gyr of evolution. The arrows give the velocity field, their size being proportional to the speed, with the maximum speed shown being 212 km s$^{-1}$. Dense zones correspond to dark regions.
pattern becomes ring-like (Yáñez & Martos, in preparation). Another important component not included yet in these simulations is the Galactic bar, which we recently modelled (Pichardo, Martos & Moreno 2004, hereafter P2). Englmaier & Gerhard (1999) studied the gas dynamics in the bar potential determined by data. Their value of \( \Omega_p = 60 \text{ km s}^{-1} \text{kpc}^{-1} \). This work reproduces many features of the inner Galaxy. There is then a large difference in the angular speeds of the bar and the spiral. The bar–spiral coupling was considered by Bissantz, Englmaier & Gerhard (2003) in a comprehensive study that assembles the available data sets and SPH simulations in gravitational potentials determined from the near-infrared luminosity of the bulge and disc, and in some cases, an outer halo and a spiral model for the disc. This work contains models allowing for different pattern speeds for the bar and the spiral. Their optimal values, at the same position of the bar corotation and bar orientation adopted here, are found to be 60 and 20 km s\(^{-1}\) kpc\(^{-1}\) for the bar and the spiral, respectively.

For the spiral pattern, Drimmel & Spergel (2001) find that the arm strength begins to fall off at about 0.85 \( R_\odot \). This is just too close to the \( R = 7 \text{ kpc} \) position of the 4:1 resonance (for our determined \( \Omega_p \)) to dismiss it as an accident. It seems to favour a scenario in which the details of the coupling bar–spiral are not crucial to the dynamics at radii greater than 7 kpc or so. P2 proposes three different models for the Galactic bar mass distribution: an inhomogeneous ellipsoid, a prolate spheroid and a superposition of ellipsoids fitting the observed boxy isophotes. There are strong model-dependent kinematics near the bar. Follow-up work indicates to us that, while there is a large dispersive effect on orbits inside or near the bar region, at the solar position the effect is minor. For instance, bar-induced vertical dispersive motion occurs only inside \( R = 4 \text{ kpc} \). However, see Mülbauer & Dehnen (2003), who conclude that at least the lowest-order deviations from axisymmetric equilibrium in the local kinematics can be attributed to the bar. The comparison with the effects of the spiral structure is deferred to a future paper. We argue that the result obtained by Bissantz et al. (2003), in a study incorporating dynamics of the bar and the spiral, is a strong support for our claim that, even without the presence of the bar in our model, our predicted value should provide a valid comparison to first order with independent studies relying on \( \Omega_p \) at the solar position and beyond.

4 RECENT INDEPENDENT DETERMINATIONS OF \( \Omega_p \)

From the data in Fig. 2, one can directly obtain the gas surface density along a circle with the radius of the Sun’s orbit. In the most simplistic circular approximation (the orbit has radial excursions of the order of 2 kpc), there are two main peaks of similar densities, and several local maxima of lower density. The mass contrast is consistent with \( K \)-band observations (Kranz, Slyz & Rix 2001), which give an arm/interarm density contrast for the old stellar population of 1.8 to 3 for a sample of spiral galaxies. Interestingly, these two peaks have surface densities in reasonable agreement with the expected threshold density for star formation, a value of approximately \( 10 \text{ M}_\odot \text{ pc}^{-2} \) (Kennicutt 1989), in which we are considering the reduction in the shock compression due to the magnetic field and to 3D dynamical effects. Other local maxima are factors of 3 or more below that value, making any associated burst of star formation a less likely event.

With \( \Omega_p = 20 \text{ km s}^{-1} \text{kpc}^{-1} \), the time baseline in the rotating frame for a circular orbit of that radius is very approximately 1 Gyr at our assumed solar radius. We found that the two density peaks mentioned above are separated in time by \( \approx 0.5 \text{ Gyr} \). We expect then such a periodicity for enhanced star formation. We note that if self-gravity were included, the density contrast would increase, making the predicted periodicity for star formation more evident.

It is interesting to compare this prediction with recent results of direct inferences of the star formation history of the solar vicinity. Hernandez, Valls-Gabaud & Gilmore (2000) analyse the colour–magnitude diagram of the solar neighbourhood as seen by the Hipparcos satellite, and using Bayesian analysis techniques derive the star formation history over the last 3 Gyr. These authors find an oscillatory component with a period of 0.5 ± 0.1 Gyr. By studying the age distribution of young globular clusters, de la Fuente Marcos & de la Fuente Marcos (2004) obtain a periodicity in the recent star formation history at the solar circle of 0.4 ± 0.1 Gyr. It is remarkable that two distinct and independent assessments of the recent star formation history at the solar circle yield a periodicity which is perfectly consistent with the density arm crossing period we derive in this study.

Shaviv (2002) finds that the cosmic ray flux reaching our Solar system should periodically increase with each crossing of a Galactic spiral arm. Along the same lines of last section, over a time baseline of the past 1 Gyr, we added our estimated magnetic field compression from Martos & Cox (1998) to plot the expected synchrotron flux variations moving along the solar circle and assuming the mass distribution fixed in time. We find six local maxima with fluxes that are higher than the flux of today. This is the same number of peaks satisfying that condition in Shaviv’s work, who plots the ratio cosmic ray flux/the cosmic ray flux of today obtained from a sample of 42 meteorites, which they relate to climate changes in Earth. Shaviv (2002) reports a period of 143 Myr for the episodes (crossings) from meteorites data, which leads to a value of \( \Omega_p \) = \( \Omega_\odot \) ± 9.1 ± 2.4 km s\(^{-1}\) kpc\(^{-1}\), which is marginally consistent with our preferred value.

Andrievsky et al. (2004) report on the spectroscopic investigation of 12 Cepheids situated at Galactic radii of 9 to 12 kpc, where they find an abrupt change in metallicity. The region 10 to 11 kpc appears to be the most important, and the change in metallicity is explained in terms of the assumed position of corotation. That position is precisely the location of corotation in our modelling, if the value \( \Omega_p = 20 \text{ km s}^{-1} \text{kpc}^{-1} \) is adopted. However, it is worth noticing that the assumed \( R_\odot \) is 7.9 kpc, 0.6 kpc less than the value for this fundamental parameter in our Galactic model. This difference might not alter their results for the change in metallicity level in the vicinity of corotation, given the large width of the density enhancement caused by the corotation islands at about \( R = 11 \text{ kpc} \) in two extended regions of our Fig. 2.

5 DISCUSSION

As found by Contopoulos & Grosbøl (1988), self-consistency is improved by introducing velocity dispersion; this is a realistic effect that can only be neglected by arguing that non-linearity dominates in strong spirals. For our Galaxy, the observations of Drimmel & Spergel (2001) suggesting the termination of the spiral at the position of the 4 : 1 resonance indicate a marginally strong spiral in the framework of Contopoulos & Grosbøl (1988). On the opposite side, our best self-consistent model is found at the lowest spiral mass considered in P1, 0.0175 \( M_\odot \), for which no stochastic motion was found. From this fact, a weak spiral and a linear regime come to mind. A possible solution to this issue could be that, while the strength of the spiral begins to fall at the 4 : 1 resonance, termination occurs at corotation, as predicted for the weak case in that...
framework. However, the quite different modelling of the galactic – particularly, spiral – mass distribution used in their studies and ours could make an interpretation of our results in their framework an unfair one. At any rate, the response at our best $\Omega_p$ is so flat that there appears to be no need to invoke velocity dispersions. In an analysis based on periodic orbits, such as ours, such dispersion will be small. Other close values of $\Omega_p$ could be improved by this effect in a study out of the scope of this work, involving many orbits departing from the periodic ones and hence subjected to larger velocity dispersions.

We conclude that a two-armed spiral pattern satisfying observational restrictions from K-dwarf distributions yields an optimal dynamically self-consistent model, for values close to $\Omega_p = 20 \, \text{km s}^{-1} \, \text{kpc}^{-1}$. The fact that various independent estimates of this quantity, sensitive to highly distinct physics, yield values for $\Omega_p$ in agreement with our estimation gives us confidence in the result. In regard to the gaseous response, we notice that the independent studies we quote for comparison are not only consistent with the value found here for $\Omega_p$, but also with a density distribution corresponding to a four-armed gaseous pattern with structural features reminiscent of optical observations of our Galaxy and other spirals.

ACKNOWLEDGMENTS

We thank an anonymous referee for helpful comments which significantly improved the clarity of the final presentation. E. Moreno, M. Martos, B. Pichardo, M. Yañez acknowledge financial support from CoNaCyT grant 36566-E, and UNAM-DGAPA grant IN114001.

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This paper has been typeset from a TEX/LATEX file prepared by the author.