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A Model for Water Exchange Between the Baltic Sea and the Gulf of Riga

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A model for water exchange of a basin connected to the open sea by two channels has been worked out. The model is applied on the Gulf of Riga, where the water exchange processes *versus* strong landbased inflow of nutrients are the most important factors in formation of the trophic status of the Gulf.

The model's response both to stationary external conditions and nonstationary impulses (changes in the sea level, wind and riverine inflow) is analysed and possibilities for matter exchange calculations are discussed. It appeared that the water exchange depends strongly on the configuration and position of the straits. The water exchange in such a two-channel system is considerably different (and stronger) than in the case of a one-channel version. The main external force for the Gulf appeared to be the wind.

Introduction

The Gulf of Riga is a (relatively small) semi-enclosed water-basin in the eastern part of the (relatively large) Baltic Sea bordering on Estonia and Latvia (Fig. 1). The Gulf (area 16,395 km^2, volume 424 km^3) being under the influence of heavily loaded river inflow (averaging 32 km^3 a year) is characterized by considerably higher nutrient concentrations than the open sea.

It is evident, that the trophic status of the Gulf depends on the efficiency of nutrient sedimentation within the Gulf and on the intensity of the water exchange via the straits. However, the connection goes through two straits and therefore the water exchange processes are rather complicated to explore. The bigger strait, the Irben

Strait has the width of 27 km, the sill depth is 21 m and the area of the minimal cross-section – 0.37 km^2. Another outlet is actually a system of straits, called Väinameri ("the sea of straits", in Estonian). The Suur Strait is the southernmost, the narrowest (5 km) and the deepest (21 m) part in this system. The flow in the Suur Strait is nearly unidirectional over the cross-section and our direct current measurements cover about 45% of the period 1993-95. The measurements in the Irben Strait are not equally representative since the strait is wider, the velocities smaller and the two-layer or two-directional water-exchange scheme is frequent. The existence of two channels in the Gulf does not enable the application of the simple models known for water-exchange description. In some papers the Suur Strait has been declared to be of negligible importance and simply "closed" up (*e.g.*, Võsumaa *et al.* 1995). Though the area of the cross section of the Suur Strait is 9 times smaller than that of the Irben Strait, we are going to prove that the role of the Suur Strait is far more important than the often expected 10%.

The major task of the study is to present a relatively simple water-exchange model and investigate the water-exchange processes in a system where a large and a small water body are connected by two channels. Such a model could also be used in other similar water bodies, where the water-exchange goes through two separately standing channels (*e.g.* Marmara Sea, Gulf of Mexico). The further aim of modelling is to cover the gaps in our field data sets for the Suur Strait and to estimate the resultant flows in the Irben Strait and to present budget calculations for the whole Gulf for whatever period of interest.

Model Description

The reaction of the basin (connected with to open sea by one channel) to the changes of the sea levels outside the basin has been discussed *e.g.* by LeBlond and Mysak (1978). Astok and Otsmann (1981) applied the one-channel model for the Baltic Sea. For the Gulf of Riga at least the two-channel version should be used.

The new two-channel model has firstly been outlined by Otsmann *et al.* (1996). Later we have analysed also the model version composed from two sub-basins and four straits: The northern outlet, previously treated as one channel, was divided into one additional basin (Väinameri) connected to the other basins by three straits (see also Fig. 1). However, speaking merely about the Gulf, the behaviour of such a system did not differ substantially from the two-channel version and in the current paper we discuss only the two-channel version.

The configuration of the Gulf, the positions of the straits and model parameters are presented in Fig. 1. The model input (forcing functions) are the sea level outside the Gulf, H, wind stress, τ, in the channels and river inflow into the Gulf, Q_J.

We use a barotropic model. The baroclinic model should be used in case the difference between the barotopic and the baroclinic pressure gradients were remark-

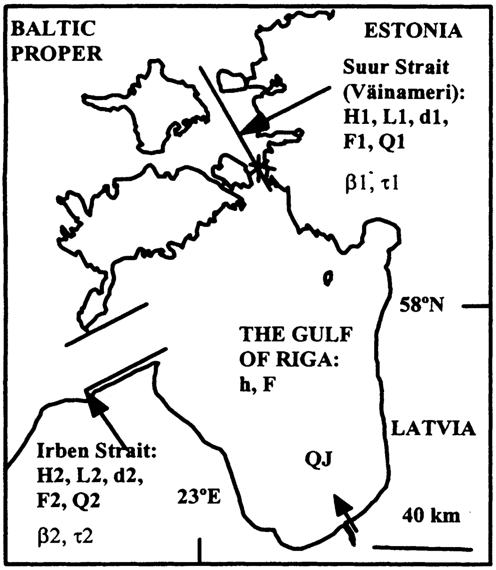

Fig.1. The scheme of the Gulf of Riga and the model parameters.
* – position of the most frequent field measurements.

able. The difference in salinity between the Gulf of Riga and the Baltic Proper is 1.5 PSU and the sea-level differences of about 0.012 m would suffice to balance the baroclinic gradient. According to Kõuts and Håkansson (1995) the measured sea-level differences exceed this value at least ten times. It should be possible to introduce the baroclinic pressure term if reliable input data were available, but so far they were not. – Our idea was to develop a highly applicable model. The baroclinic circulation which certainly exists within the Irben Strait does not affect the volumes Q_1 and Q_2 found by the model, since these volumes represent average volumes over the cross-section on the lengths of the straits L_1 and L_2.

The influence of air pressure differences, Δp_a, is negligible due to the small area of the Gulf. For that reason Δp_a have not been taken into account in this model either.

The model outputs include the flow volumes, Q_1, Q_2, in the straits and the average water level in the Gulf, h.

The model is based on the motion equations in straits integrated along the coordinates x, y, z and the water balance equation for the Gulf

$$\frac{dQ_1(t)}{dt} + RQ_1(t) + \frac{gF_1}{L_1}h(t) = f_1(t) \tag{1}$$

$$\frac{dQ_2(t)}{dt} + RQ_2(t) + \frac{gF_2}{L_2}h(t) = f_2(t) \tag{2}$$

$$F\frac{dh(t)}{dt} = Q_1(t) + Q_2(t) + Q_J(t) \tag{3}$$

where

$$f_i(t) = F_i\left(\frac{g}{L_i}H_i(t) + \frac{\tau_i(t)}{d_i}\right), \quad i = 1,2 \tag{4}$$

i = 1 – Suur Strait; $i = 2$ – Irben Strait, t – time,
F – area of the Gulf of Riga (the value used: 1.63×10^{10} [m^2]),
F_i – cross-section of the channel ($F_1 = 4.5\times10^4$; $F_2 = 4.0\times10^5$ [m^2]),
L_i – length of the channel ($L_1 = 7.5\times10^4$; $L_2 = 4.8\times10^4$ [m]),
d_i – channel depth ($d_1 = 10.5$; $d_2 = 19$ [m]),
R – friction coefficient (3.5×10^{-5} [s^{-1}]), g – gravity acceleration,
Q_i – volume flow in the channel; $u_i = Q_i/F_i$,
Q_j – river inflow (1×10^3 [m^{3}s^{-1}]),
H_i – water level in the Baltic Proper,
h – horisontally average water level in the Gulf,
τ_i – wind stress projection on direction of channels: $\beta_1 = 155°$; $\beta_2 = 65°$,

$$\tau_i = |\vec{\tau}|(-1)\cos(\beta_i - \varphi), \qquad |\vec{\tau}| = c\left(\frac{\rho_a}{\rho_w}\right)|\vec{u}_a|^2 \tag{5}$$

ρ_a – air density, ρ_w – water density, u_a – wind velocity, φ – wind direction,
c – nondimensional coefficient ($c = 1.2\times10^{-3}$).

$RQ_i(t)$ in the left side of the Eq. (1) describes the influence of the bottom friction on slow motions (velocities less than 10 cm/s). This linear approximation could be used in small motion velocities, since the description of the motion with nonlinear friction, $r|u_i|u_i$, in the analytical case is more complicated but essentially of the same nature. In the numerical solution the nonlinear friction was used in the realistic forcings. Combining Eqs. (1) and (2) with Eq. (3) we shall obtain the equations for $Q_i(t)$, $h(t)$ and $Q(t) = Q_1(t)+Q_2(t)$

$$\left(\frac{d^2}{dt^2} + R\frac{d}{dt} + \omega_0^2\right)h(t) = \frac{f_1(t)+f_2(t)}{F} + \frac{1}{F}\left(\frac{d}{dt} + R\right)Q_j(t) \tag{6}$$

$$\left(\frac{d^2}{dt^2} + R\frac{d}{dt} + \omega_0^2\right)Q(t) = \frac{d}{dt}\left(f_1(t)+f_2(t)\right) - \omega_0^2 Q_j(t) \tag{7}$$

$$\left(\frac{d^2}{dt^2} + R\frac{d}{dt} + \omega_{01}^2\right)Q_1(t) = \frac{d}{dt}f_1(t) - \omega_{01}^2\left(Q_2(t)+Q_j(t)\right) \tag{8}$$

$$\left(\frac{d^2}{dt^2} + R\frac{d}{dt} + \omega_{02}^2\right)Q_2(t) = \frac{d}{dt}f_2(t) - \omega_{02}^2\left(Q_1(t)+Q_j(t)\right) \tag{9}$$

where

$$\omega_0^2 = \omega_{01}^2 + \omega_{02}^2, \qquad \omega_{0i}^2 = \frac{gF_i}{L_iF}, \qquad i = 1,2 \tag{10}$$

The form of Eqs. (6)-(9) are known in mechanics as the forced oscillation equations (*e.g.* Landau and Lifschitz 1973). The right sides of the equations represent the sum of external forces evoking the oscillations. Parameters ω_0, ω_{0i} are the eigen-oscillation frequencies of the system in the absence of frictional forces. The given model parameter values (excluding friction) yield the periods of the eigen-oscillations $T_0 = 2\pi/\omega_0 \approx 23.6$ hours, $T_{01} = 2\pi/\omega_{01} \approx 92$ h, $T_{02} = 2\pi/\omega_{02} \approx 24.4$ h (Fig. 2C). The oscillations with the period of 24 h are common in the current velocity records obtained for the Irben Strait (Petrov 1979). The oscillations with period of 3-4 days could also be found in the spectra calculated from the velocities of the Suur Strait (Suursaar *et al.* 1996), but the influence of atmospheric synoptical activities should also be considered in that case.

The model displays that the reaction of the flows in the straits to the change of the external forcings is very different. Note also that the eigen-oscillation frequency of the system of the Gulf with two channels is higher than the frequencies of individual channels:

$\omega_0 > \max(\omega_{01}, \omega_{02})$ and $T_0 < \min(T_{01}, T_{02})$. If $Q_1(t) \equiv 0$ or $Q_2(t) \equiv 0$ (in Eqs. (8) and (9)), we get the equation describing the one-channel basin flows.

Model Behaviour: Response to Quasistationary Conditions and Impulses

Although stationary conditions could be rarely observed in the Gulf, the analysis of the stationary case is justified, since it takes more than one day for the flow to pass through the strait and the role of oscillations with periods of less than one day is considerably smaller than the motions induced by atmospheric synoptical activities (~3-4 days). For the model the water exchange could be considered quasistationary in the frequencies $\omega^2 << \omega^2_{01}$. For that frequency band the derivative (by time) could be omitted in the Eqs. (1)-(2).

The agreement with the quasistationary case could be seen in the curves of phase-shift and amplification (Fig. 2). The phase-shift vanishes and the amplification function stays almost constant in frequencies $\omega<\omega_{01}$ when $\omega \rightarrow 0$. Consequently, the flows and the sea level follow the temporal changes of the external forces. Thus, the solutions of Eqs. (1)-(3) are similar to the stationary case where the constant external forcings are replaced with the forcings slowly changing in time (the external forcings should be averaged over the scale $T \geq 3$ days).

The maximum flows (and the difference in the sea levels $|h(t)-H(t)|=0$) appear in the wind directions of $\varphi=164°$ and $164°+180°$ (Fig. 3 and Fig. 4A): the forces of the wind stress are balanced with the frictional forces. In case of one channel such an intensive wash-through does not exist, since the difference in the sea levels is in balance with the wind stress and therefore $Q=0$. For wind directions $\varphi=74°$ and $254°$ the flow is missing in the straits, but the sea level deviations inside the Gulf are max-

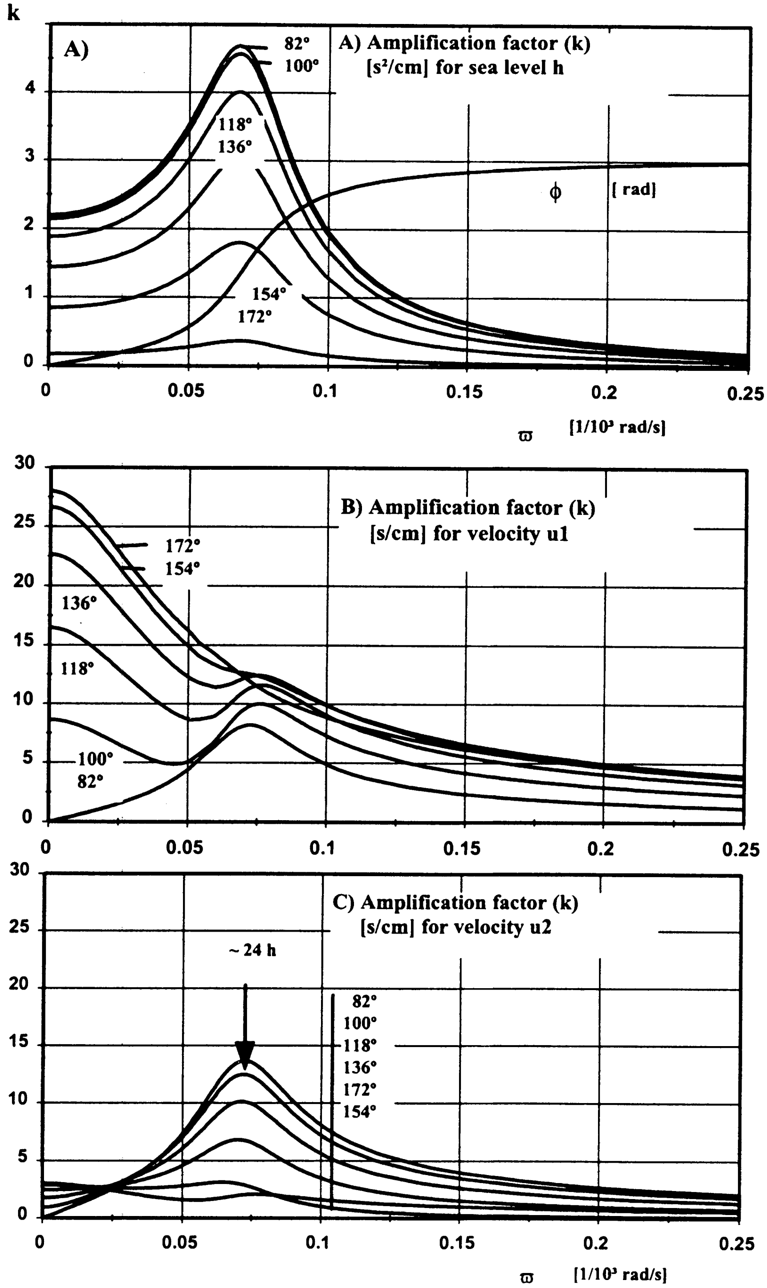

Fig. 2. Dependence of the amplification (resonance) factors from the frequency (ω) in different wind directions for sea level (A) and velocities in the Suur (B) and Irben Strait (C). Φ – the phase-shift function (for A). Wind stress: $|\tau| = 1$ cm^2/s^2.

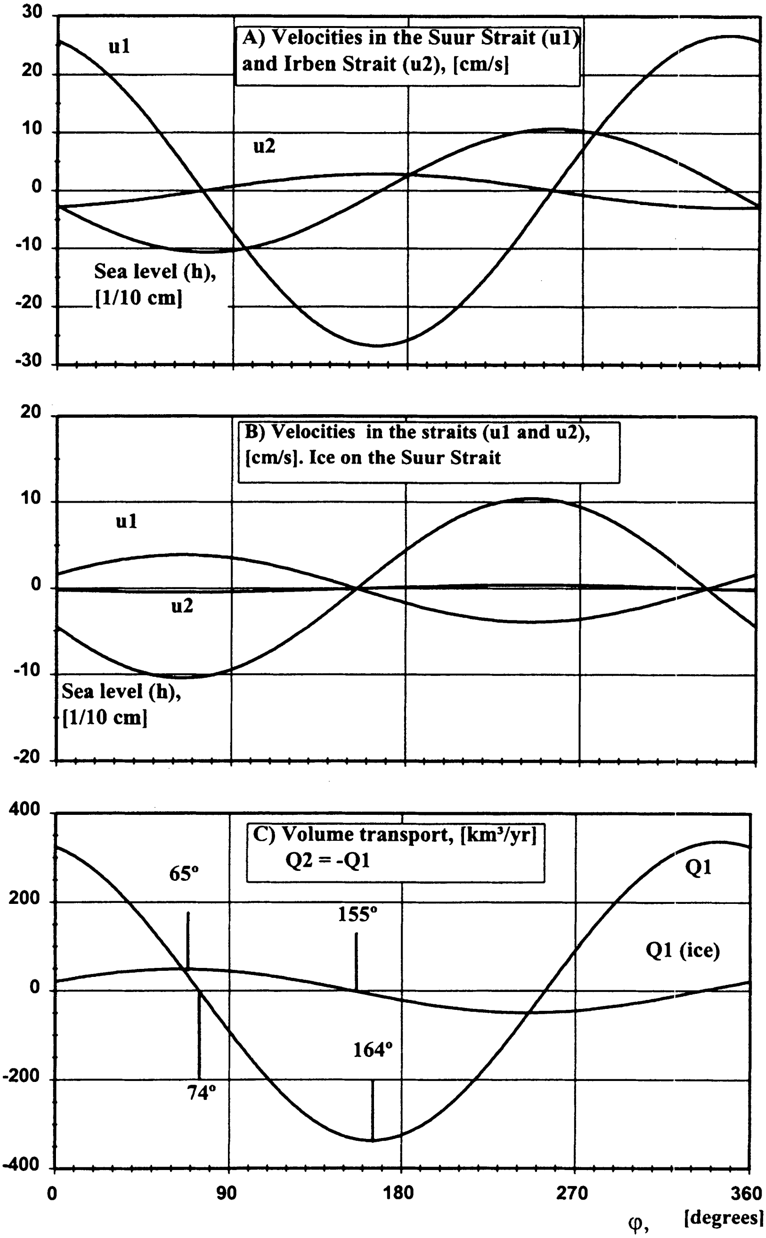

Fig. 3. Dependence of velocities and volume transports in the Suur Strait and the Irben Strait on the wind direction (φ, module = 7 m/s) and on existence of ice in the Suur Strait area. $Q_j = H_1 = H_2 = 0$.

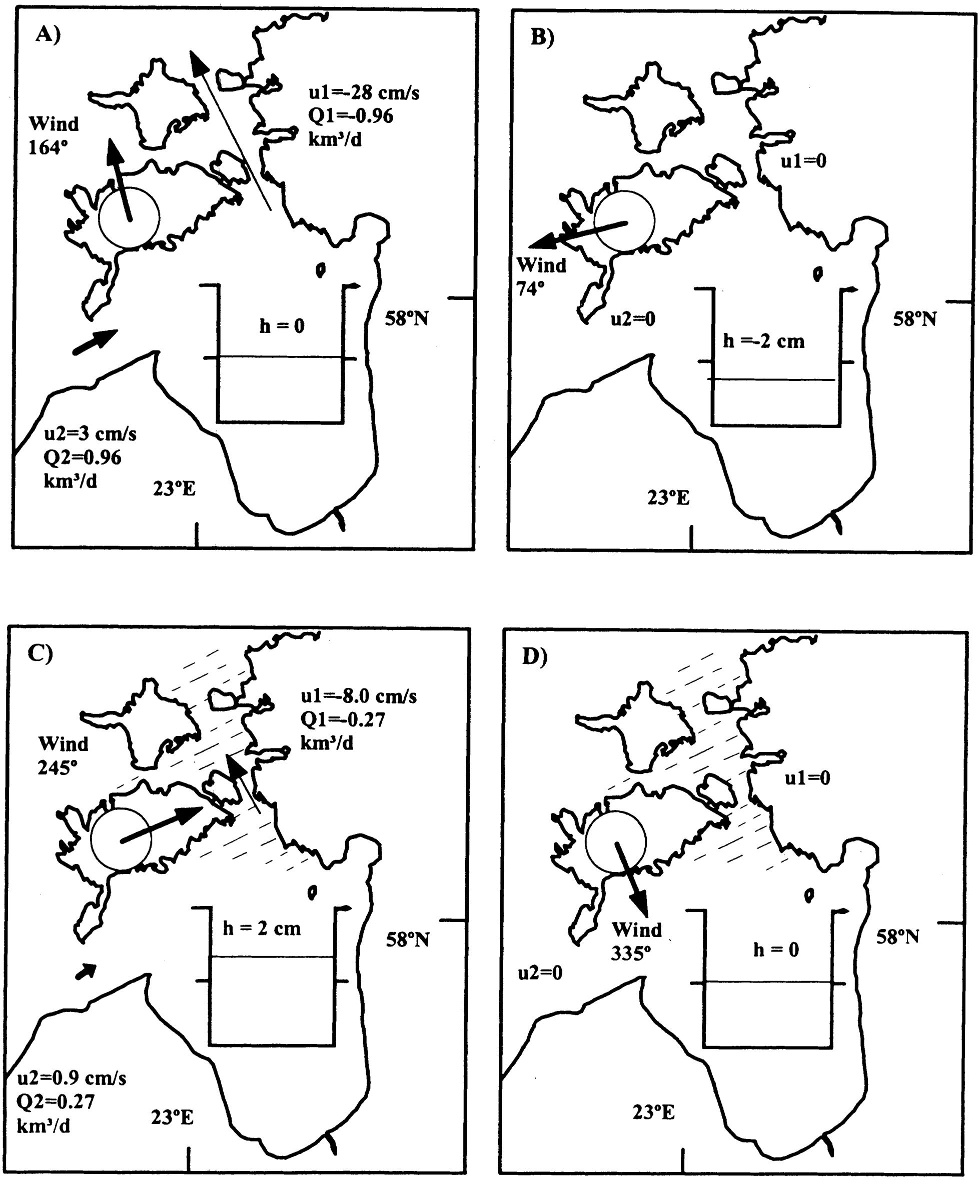

Fig. 4. Water exchange and mean sea level in the Gulf of Riga for some special wind directions (τ = 1 cm^2/s^2, Q_2=-Q_1, see also Fig. 3). A, B – situation without ice; C, D – ice on the Väinameri area (typical winter situation). The situation with the wind from 344° is similar to (A), but with opposite flows in the straits; 254° is similar to (B), but with the sea level h=2 cm; 65° is similar to (C), but with the sea level *h* =-2 cm and 155° is identical to (D).

imal then (Fig. 4B). When one of the straits (Suur Strait) is covered by ice, the volume flows are smaller and the "optimal" wind directions for the exchange are 65° and 245° (Fig. 3B,C and Fig. 4C), but compared to the previous case the flows appear in different straits in reversed order. It is interesting to note that the flow does not exist both in 155° and 335°, and the sea level, h, is then 0. On the contrary, the level h=0 is coupled with the maximal velocities in the absence of ice (Fig. 4A *vs.* 4D).

The response of the Gulf to the alternating external conditions and impulses is reflected in the model outputs. But the form of the frequency-relationship $k(\omega)$ (Fig. 2), and hence the displacement and the form of the resonance peak depend only on the ω_0 and R, *i.e.*, on the measures of the channels and the Gulf and frictional forces.

The response of the flows to the rapid increase in the wind velocity and reaction to the rise of the sea level outside the Gulf are quite different (Fig. 5A, B). Should the open sea level increase close to one channel, the response on that channel would be maximal (Fig. 5B). Should both the H_1 and H_2 have an equal increase, the flows Q_1 and Q_2 would be very weak, and the level h becomes equal to the level $H_1 = H_2$. The directions 74° (Fig. 5A), 164°, 65° and 155° are the same exceptional directions which also appear prominently in Figs. 2 and 3.

The increase in inflows through the rivers (similar to the spring flood) affects mainly the flows in the Irben Strait (Fig. 5C), because the Suur Strait has smaller cross-section and higher resistance. (Note that the magnitude of Q_1 is at least 10 times smaller than in Q_2 in that case). However, it is not correct to draw the simple conclusion that also in the realistic conditions "the Daugava River flows out through the Irben Strait".

Verification and Application

The extensive field work carried out in the Väinameri region in 1993-96 has been described by Suursaar and Astok (1996). The model was calibrated and verified against the field data gained in Oct-Nov. 1993 (Fig. 6) and in Jan-March 1995 (Fig. 7). Generally, the model inputs are the measured wind stresses above the straits averaged over the area of these straits, and the sea levels measured at the entrance of the strait (closest to the open sea). In the first calibration and verification case the wind measured at one point (by Aanderaa authomatic weather station located at the height of about 10 m in Viirelaid Isle, Suur Strait) was used. The wind stresses for both straits (τ_1(t), τ_2(t), Fig. 6A) were calculated from the same wind data, but representing the different projections. The wind data measured with 10 minutes time interval were averaged on 1-hour interval. The water level data were not used, since their quality was low (daily averages interpolated into 1h interval).

The model parameters ω_{01} and ω_{02} were found by the average measures of the straits. The term $r|u_i|u_i$ in the motion equations describes frictional forces. It includes

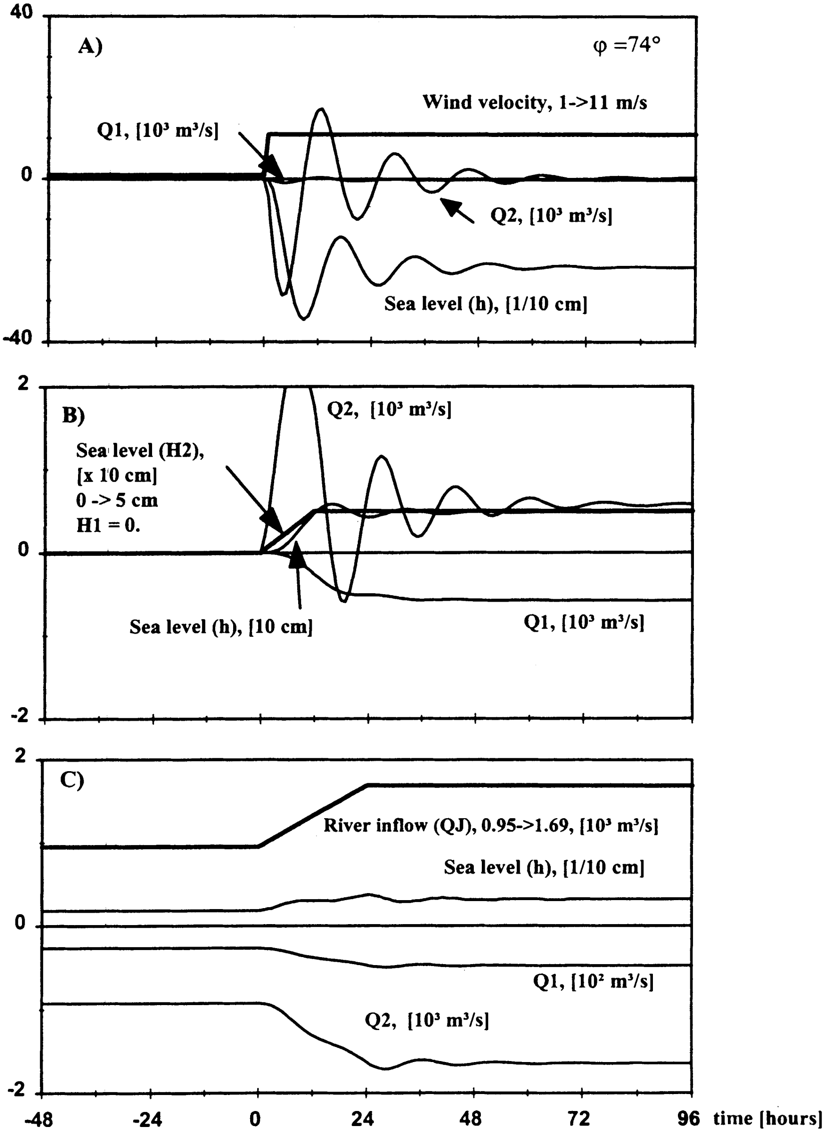

Fig. 5. Some examples of model behaviour. Response of volume flows (Q_i) and sea level (h) to the increase in the wind velocity (A), increase in open sea level H_2 (B) (5 cm for 12 h) and in river flood (C).

the information on the bottom friction, nonlinearity of motions inside the straits (due to changes in cross-sections) and other factors reacting on the increase or decrease of the average velocity. Quadratic friction has been used, where the parameter r is taken as constant and found by calibration. The coefficient r was found from the cut of measurements (3-4 days) where both the wind and the flow velocities in the Suur

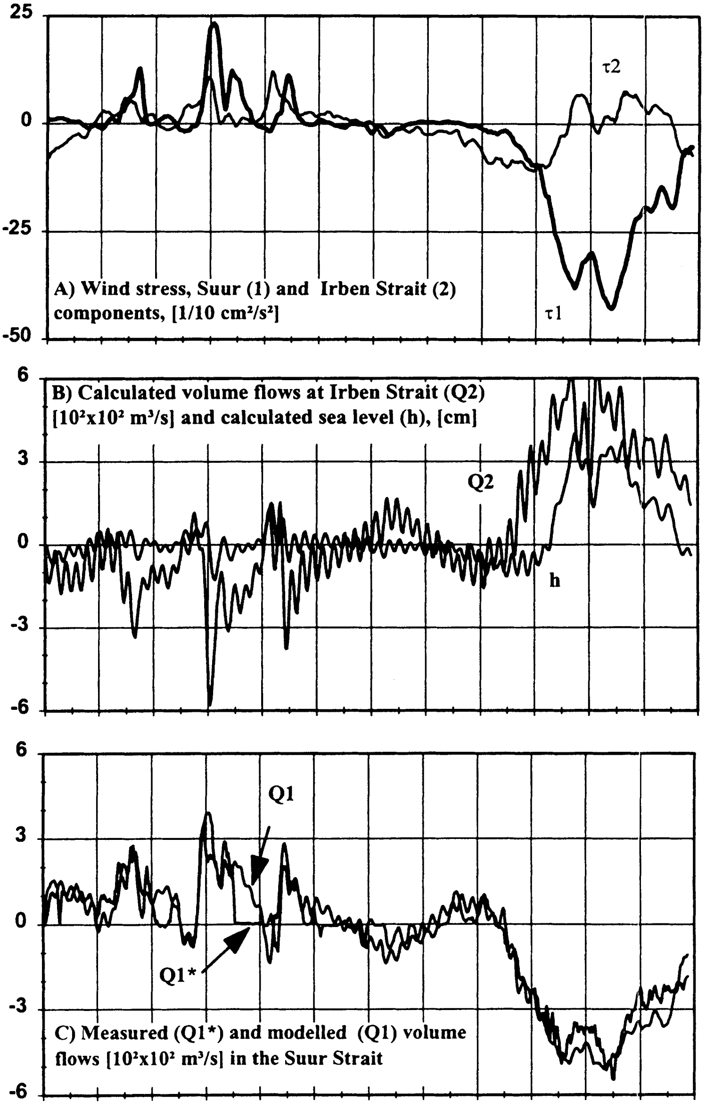

Fig. 6. Comparison of the modelled and measured velocities in the Suur Strait (C) and some other modelled values (B) in the realistic external forcings (A). Note the malfunction (entangling) of measuring equipment on 8th day (Q_1* in C).

Strait were quasistationary. For verification the currents measured in the Suur Strait ($u_1(t)$) by Aanderaa current (RCM-7) meter were used. The mooring station was deployed at 12 m (the depth in that part of the strait is 19 m) and the data measured every 10 minutes were averaged over one-hour period similarily to the wind data.

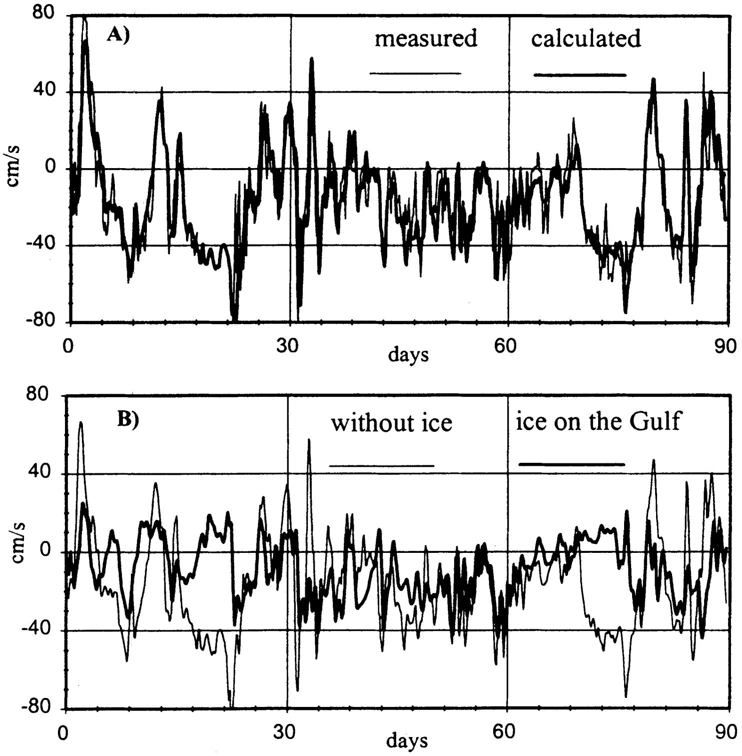

Fig. 7. Comparison of measured and modelled velocities (A) in the Suur Strait, January-March 1995. The comparison of modelled velocities in the case of ice-cover and ice-free situation is shown in B for the same period.

According to estimations based on velocity profilings (Suursaar *et al.* 1996) the values obtained at the depth of 10-12 m could be taken as average velocities over the whole cross-section of the Suur Strait.

It should be noted, that quite a good correlation coefficient (0.95) between the u_1 and u_1* (Fig. 6) was obtained without using the sea-level data and the actual data of the wind stress above the other strait. Better accordance between the measured and modelled velocities could be testified in the Fig. 7A. Here, both the sea-level data and HIRLAM model winds above all the channels were used for input.

Fig. 7B shows the comparison of the same record with the situation when there was ice on the Gulf. For modelling it means that the wind stress above the Suur Strait is absent – And the differences are drastic. Both the direct measurements and modelling have shown how different in magnitudes the water exchange in the Gulf of Riga could be. Especially in winter the water exchange depends strongly on the existence of ice-cover in the northern part of the Gulf of Riga. It shows, once again, the leading role of the wind stress in the water-exchange processes of the Gulf of Riga.

We would like to stress, that verification against the field data in the Irben Strait is not necessary. It is also very labour-consuming to get water-exchange estimates for that strait based on direct measurements, since different from the Suur Strait the structure of the velocity fields is very heterogeneous in the Irben Strait. At least 3 current-meters and a long measuring period is needed. Moreover the dominant signal with 24-h period actually does not contribute to the real water exchange a lot. This "real water exchange", specified as cumulative or integral flows for a certain longer period, comprises only a small proportion of the total sum of in- and outflows. Consequently, the reliability of that estimate is low. On the other hand, the estimates for the Suur Strait seem highly reliable, since the flow is spatially quite homogeneous and the temporal variability could be successfully described by the model.

According to our direct measurements (gaps preliminary filled using wind coefficients) the total sum of inflows in the Suur Strait is about 130 km^3/yr and 160 km^3/yr for outflows (Suursaar *et al.* 1996). The integral flows (*i.e.*, the "real exchange") were found to be only 50-80 km^3/yr in both directions. Consequently, the roughly equal resulted flow in the Irben Strait appears in opposite directions simultaneously. Using salt conservation law (known also as the Knudsen's method) it is possible to find all the flow components needed for budget calculations in the Gulf. Not to speak of the qualitative role (the straits "work" together!), the share of the Suur Strait was quantified as about 32% of the water exchange (integral flows), the share of the Irben Strait as about 57% leaving about 11% for the river inflows.

Conclusions

1) A two-channel model for water exchange has been developed. The model shows, that water exchange depends considerably on the direction and configuration of the channels. Due to the existence of two channels the water exchange in the case of Gulf of Riga is more intensive than if there were only one channel. The straits "work" together.
2) The system of the Gulf of Riga amplifies oscillations with a period of about 24 hours, which can be noted significantly in the velocities in the Irben Strait.
3) The main factor for the water exchange in the Gulf of Riga is wind. The sea-level difference is secondary factor. The water exchange is the most intensive with SSE and NNW winds. In winters with ice-cover the water-exchange scheme is different: the velocities are 3-4 times smaller and the water exchange is the most intensive with WSW and ENE winds, instead.
4) The described model can be used for water and nutrient exchange calculations and for providing boundary conditions for the ecological models of the Gulf of Riga. The model gives integral (resulted) flows for both straits. The flows in the Irben Strait must be only splitted into simultaneous in- and outflowing components (using *e.g* Knudsen's method).

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